Computer Algebra
Independent Integration Tests

Summer 2023 edition

6-Hyperbolic-functions/6.5-Hyperbolic-secant/178-6.5.2-e-x-^m-a+b-sech-c+d-x^n-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [84]. This is test number [178].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (84)	0.00 (0)
Mathematica	95.24 (80)	4.76 (4)
Fricas	73.81 (62)	26.19 (22)
Maple	59.52 (50)	40.48 (34)
Mupad	55.95 (47)	44.05 (37)
Giac	52.38 (44)	47.62 (40)
Maxima	46.43 (39)	53.57 (45)
Sympy	42.86 (36)	57.14 (48)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

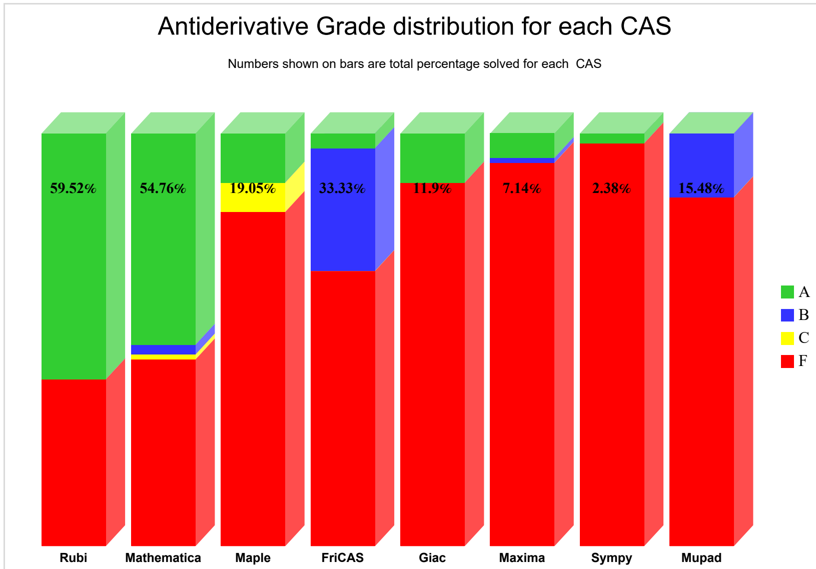
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

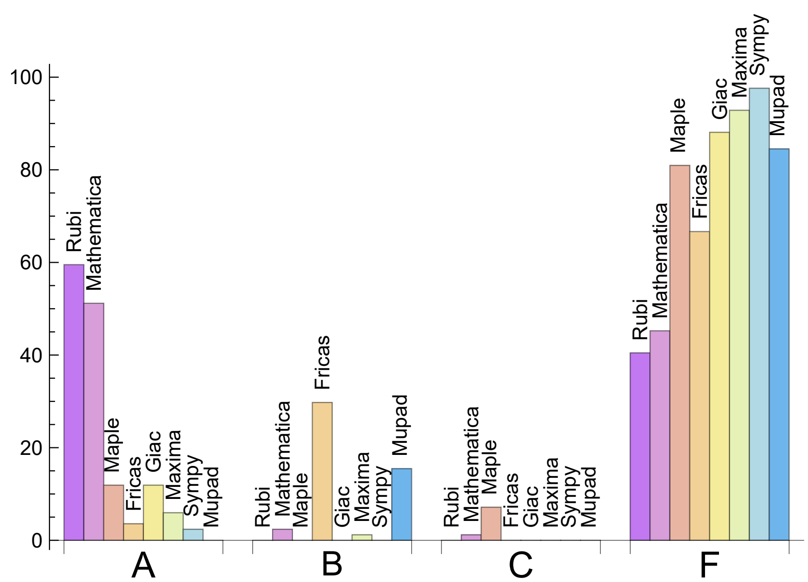
System	% A grade	% B grade	% C grade	% F grade
Rubi	59.524	0.000	0.000	40.476
Mathematica	51.190	2.381	1.190	45.238
Maple	11.905	0.000	7.143	80.952
Giac	11.905	0.000	0.000	88.095
Maxima	5.952	1.190	0.000	92.857
Fricas	3.571	29.762	0.000	66.667
Sympy	2.381	0.000	0.000	97.619
Mupad	0.000	15.476	0.000	84.524

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Fricas	22	90.91	9.09	0.00
Maple	34	100.00	0.00	0.00
Mupad	37	0.00	100.00	0.00
Giac	40	100.00	0.00	0.00
Maxima	45	57.78	2.22	40.00
Sympy	48	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.27
Rubi	0.31
Maxima	0.45
Giac	0.61
Maple	0.74
Mupad	1.96
Sympy	5.19
Mathematica	13.70

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	18.67	0.96	17.00	0.94
Giac	30.77	1.03	20.00	1.11
Mupad	64.66	1.52	22.00	1.22
Maple	70.26	1.22	18.00	1.00
Maxima	114.21	5.74	59.00	3.00
Rubi	282.39	1.00	72.50	1.00
Mathematica	292.99	1.16	52.00	1.10
Fricas	623.32	3.81	44.00	2.15

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

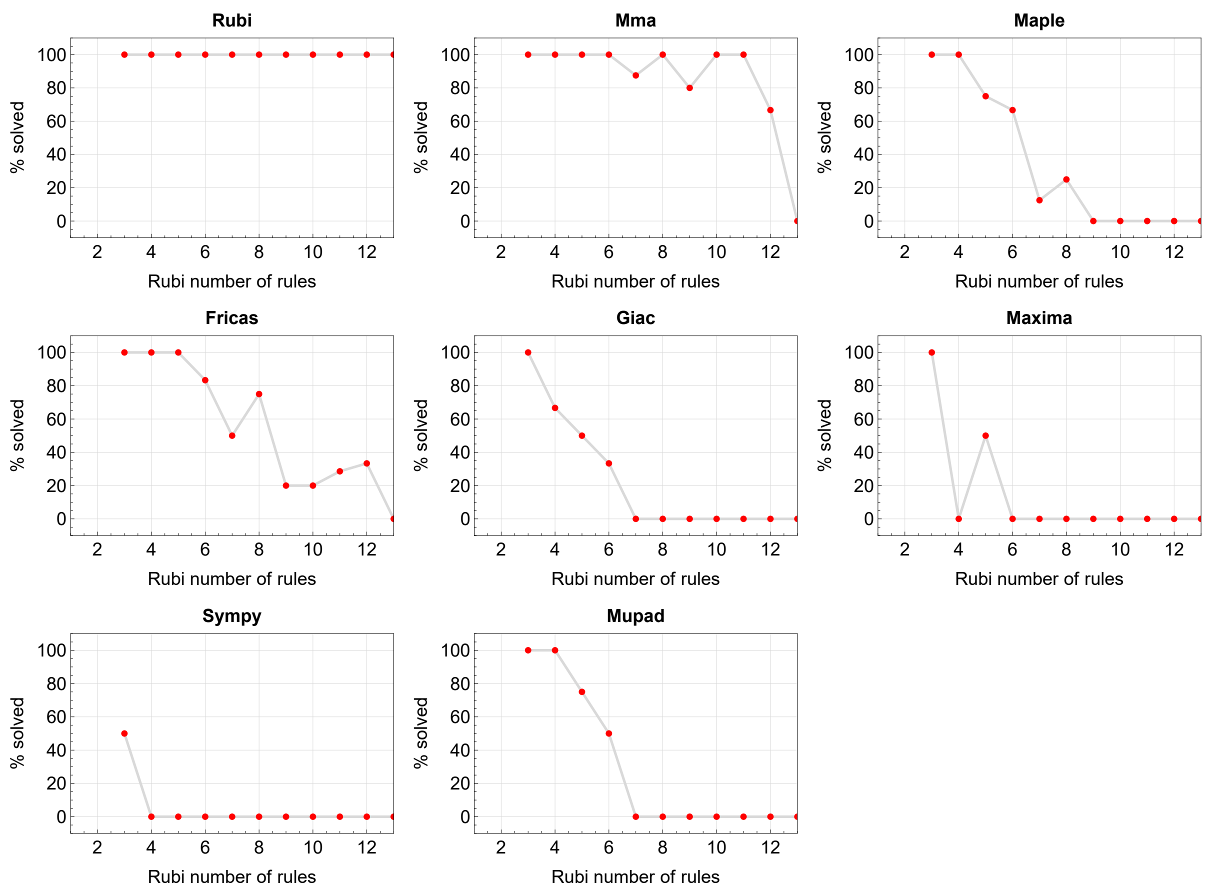


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

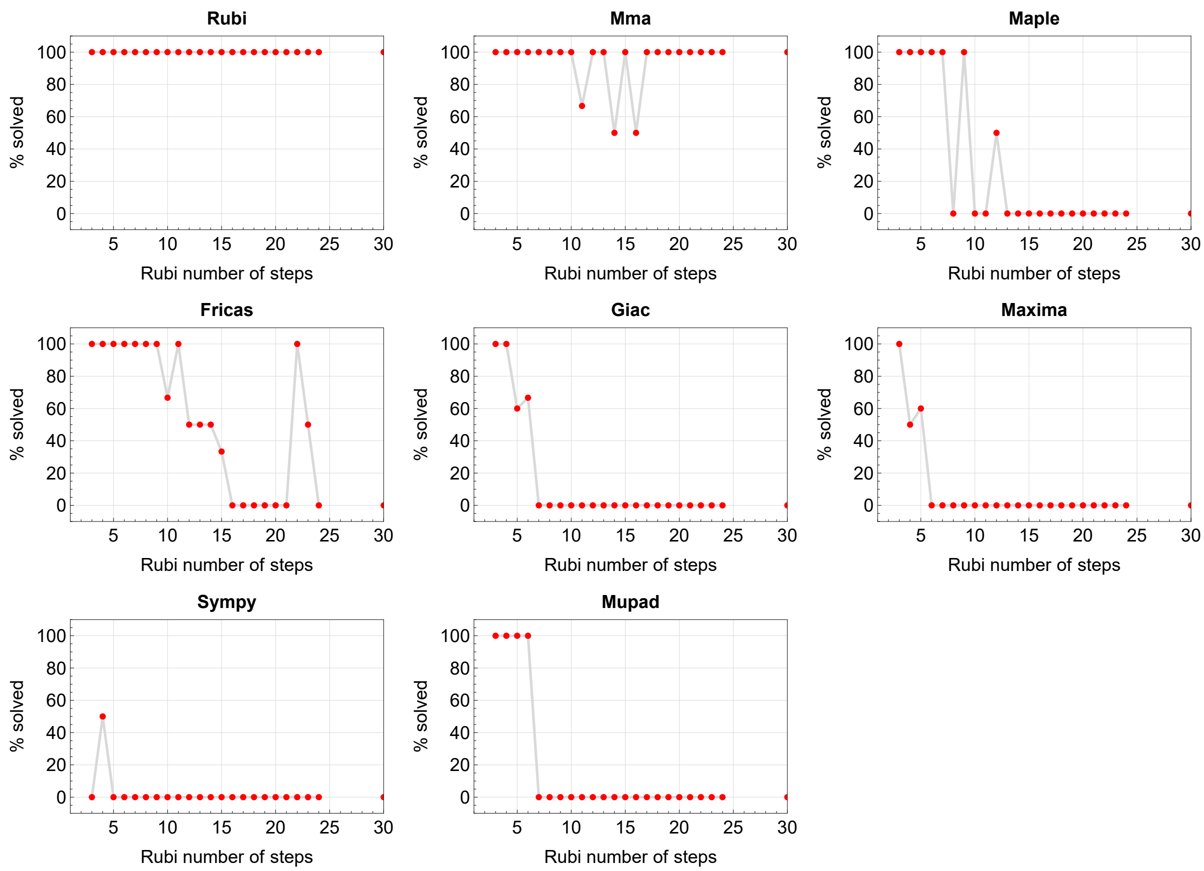


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

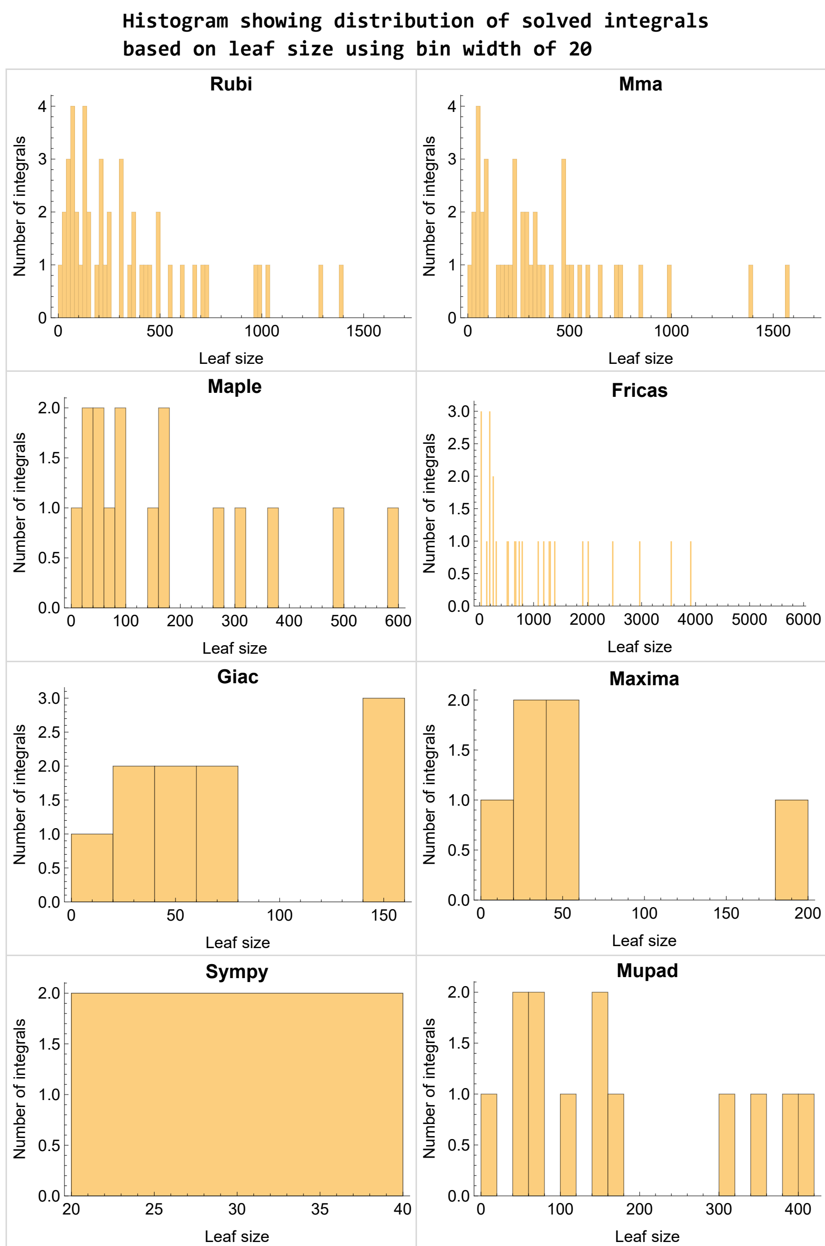


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

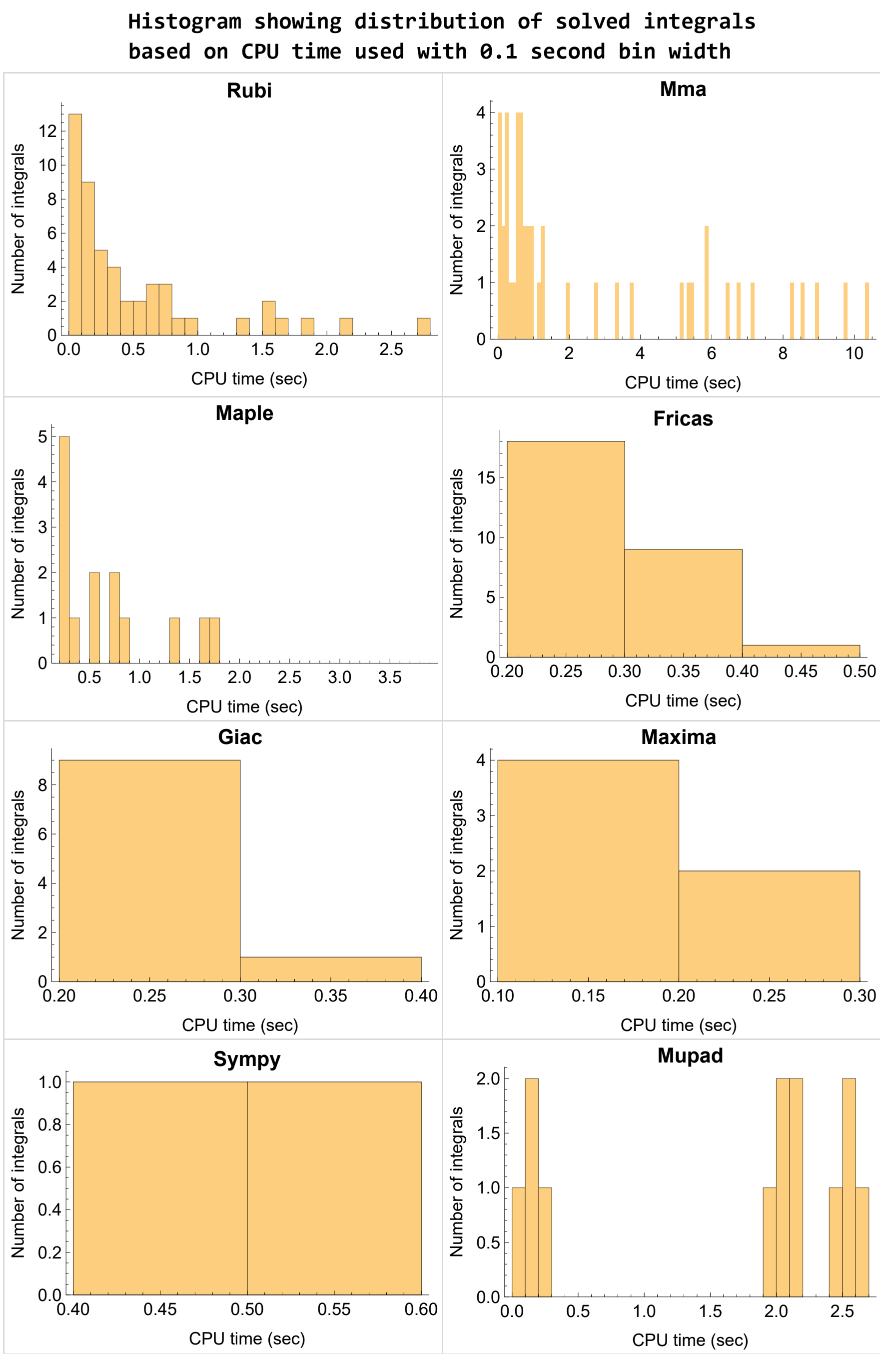


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

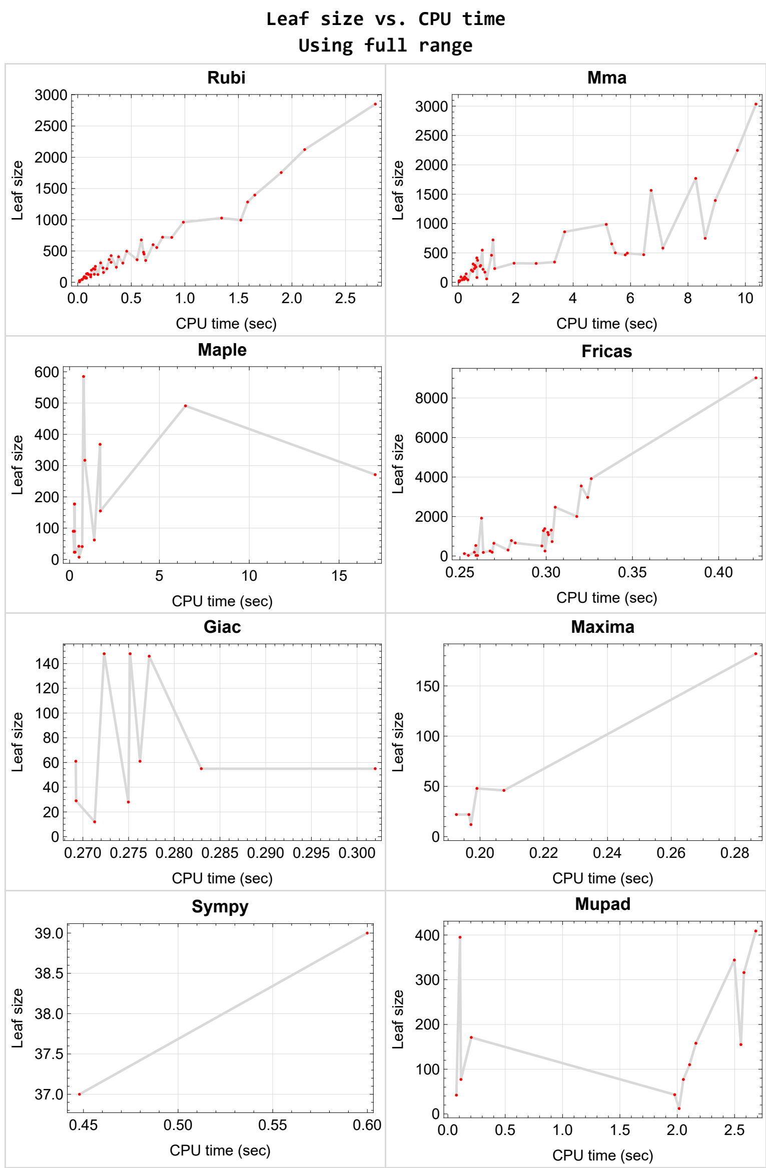


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{2, 4, 6, 7, 9, 11, 13, 14, 17, 19, 21, 22, 24, 26, 28, 29, 30, 35, 36, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61, 65, 66, 70, 71, 72\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {39}

Maple $\{73, 74, 76, 79, 80, 82\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

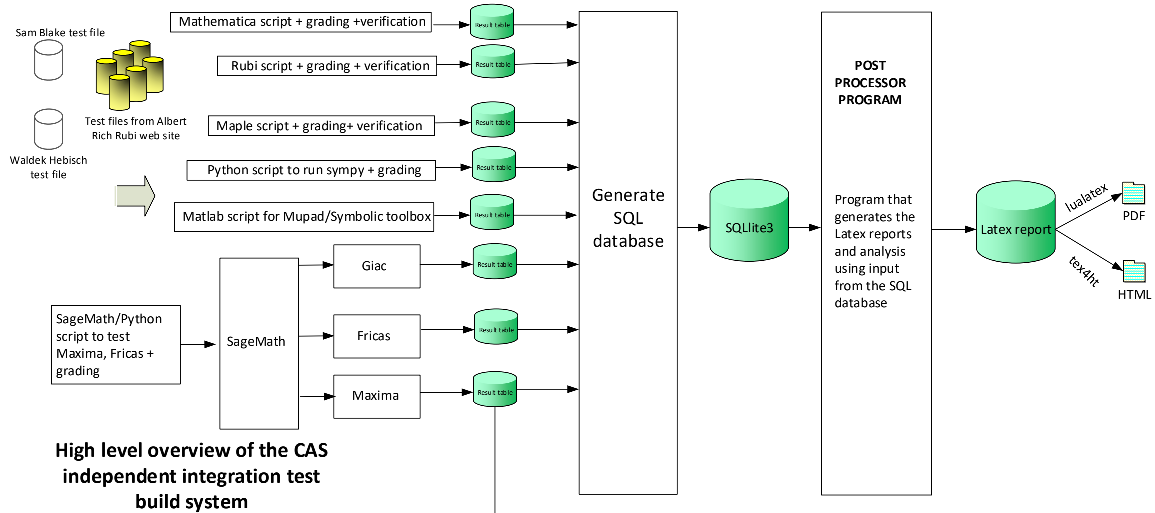
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

The following fields are present only in ***Rubi*** *Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 3, 5, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59, 62, 63, 64, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 3, 5, 8, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59, 62, 63, 64, 67, 68, 69, 73, 74, 76, 79, 82, 83 }

B grade { 10, 77 }

C grade { 80 }

F normal fail { 75, 78, 81, 84 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 5, 12, 15, 20, 27, 31, 54, 59, 64, 69 }

B grade { }

C grade { 73, 74, 76, 79, 80, 82 }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 25, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 57, 58, 62, 63, 67, 68, 75, 77, 78, 81, 83, 84 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 5, 20, 54 }

B grade { 1, 3, 8, 10, 12, 15, 16, 18, 23, 25, 27, 31, 59, 64, 69, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83 }

C grade { }

F normal fail { 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 57, 58, 62, 63, 67, 68 }

F(-1) timedout fail { 78, 84 }

F(-2) exception fail { }

Maxima

A grade { 5, 12, 31, 54, 59 }

B grade { 15 }

C grade { }

F normal fail { 1, 3, 8, 10, 32, 33, 34, 37, 38, 39, 52, 53, 57, 58, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

F(-1) timedout fail { 61 }

F(-2) exception fail { 16, 18, 20, 23, 25, 27, 42, 43, 44, 47, 48, 49, 62, 63, 64, 67, 68, 69 }

Giac

A grade { 5, 12, 15, 20, 27, 31, 54, 59, 64, 69 }

B grade { }

C grade { }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 25, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 57, 58, 62, 63, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 5, 12, 15, 20, 27, 31, 54, 59, 64, 69, 73, 76, 79 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 3, 8, 10, 16, 18, 23, 25, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 57, 58, 62, 63, 67, 68, 74, 75, 77, 78, 80, 81, 82, 83, 84 }

F(-2) exception fail { }

Sympy

A grade { 5, 54 }

B grade { }

C grade { }

F normal fail { 1, 3, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 57, 58, 59, 62, 63, 64, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	143	0	0	256	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	2.05	0.00	0.00	0.00
time (sec)	N/A	0.101	0.274	0.000	0.000	0.267	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	38	21	15	18	20
N.S.	1	1.00	1.12	1.00	2.38	1.31	0.94	1.12	1.25
time (sec)	N/A	0.014	5.382	0.047	0.312	0.252	1.179	0.301	1.978

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	92	0	0	184	0	0	0
N.S.	1	1.00	1.19	0.00	0.00	2.39	0.00	0.00	0.00
time (sec)	N/A	0.058	0.223	0.000	0.000	0.264	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	38	21	15	18	20
N.S.	1	1.00	1.12	1.00	2.38	1.31	0.94	1.12	1.25
time (sec)	N/A	0.013	4.694	0.058	0.300	0.247	0.708	0.269	1.987

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	33	39	28	42
N.S.	1	1.00	1.00	0.88	0.85	1.27	1.50	1.08	1.62
time (sec)	N/A	0.019	0.027	0.306	0.197	0.255	0.600	0.275	0.074

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	36	18	14	18	20
N.S.	1	1.00	1.12	1.00	2.25	1.12	0.88	1.12	1.25
time (sec)	N/A	0.013	4.548	0.048	0.306	0.262	1.951	0.288	2.111

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	38	18	15	18	20
N.S.	1	1.00	1.12	1.00	2.38	1.12	0.94	1.12	1.25
time (sec)	N/A	0.014	4.508	0.048	0.303	0.245	0.617	0.329	2.117

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	320	0	0	1198	0	0	0
N.S.	1	1.00	1.47	0.00	0.00	5.52	0.00	0.00	0.00
time (sec)	N/A	0.272	2.712	0.000	0.000	0.301	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	79	42	17	20	22
N.S.	1	1.00	1.11	1.00	4.39	2.33	0.94	1.11	1.22
time (sec)	N/A	0.017	11.216	0.077	0.413	0.245	1.556	0.670	2.003

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	324	0	0	782	0	0	0
N.S.	1	1.00	2.72	0.00	0.00	6.57	0.00	0.00	0.00
time (sec)	N/A	0.123	1.942	0.000	0.000	0.280	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	72	42	17	20	22
N.S.	1	1.00	1.11	1.00	4.00	2.33	0.94	1.11	1.22
time (sec)	N/A	0.017	9.295	0.072	0.404	0.256	1.009	0.672	1.992

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	41	46	194	0	55	77
N.S.	1	1.00	0.98	0.93	1.05	4.41	0.00	1.25	1.75
time (sec)	N/A	0.042	0.139	0.701	0.207	0.258	0.000	0.302	0.114

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	86	36	15	20	22
N.S.	1	1.00	1.11	1.00	4.78	2.00	0.83	1.11	1.22
time (sec)	N/A	0.017	27.465	0.074	0.399	0.255	6.611	0.310	2.163

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	87	36	17	20	22
N.S.	1	1.00	1.11	1.00	4.83	2.00	0.94	1.11	1.22
time (sec)	N/A	0.017	10.618	0.074	0.434	0.268	1.008	0.818	2.152

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	62	182	1918	0	146	395
N.S.	1	1.00	1.00	0.69	2.02	21.31	0.00	1.62	4.39
time (sec)	N/A	0.061	0.094	1.377	0.287	0.263	0.000	0.277	0.106

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	349	349	272	0	0	731	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	2.09	0.00	0.00	0.00
time (sec)	N/A	0.635	0.762	0.000	0.000	0.303	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	59	20	15	20	22
N.S.	1	1.00	1.11	1.00	3.28	1.11	0.83	1.11	1.22
time (sec)	N/A	0.021	3.845	0.053	0.302	0.265	0.580	0.301	1.975

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	241	241	187	0	0	536	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.360	0.503	0.000	0.000	0.259	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	59	20	15	20	22
N.S.	1	1.00	1.11	1.00	3.28	1.11	0.83	1.11	1.22
time (sec)	N/A	0.021	3.258	0.051	0.303	0.239	0.486	0.282	1.930

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	67	90	0	304	0	61	171
N.S.	1	1.00	1.02	1.36	0.00	4.61	0.00	0.92	2.59
time (sec)	N/A	0.082	0.267	0.204	0.000	0.278	0.000	0.269	0.204

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	58	19	15	20	22
N.S.	1	1.00	1.11	1.00	3.22	1.06	0.83	1.11	1.22
time (sec)	N/A	0.021	2.866	0.052	0.288	0.250	1.133	0.312	2.057

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	38	18	15	18	20
N.S.	1	1.00	1.12	1.00	2.38	1.12	0.94	1.12	1.25
time (sec)	N/A	0.012	0.086	0.010	0.296	0.246	0.703	0.305	0.002

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	994	994	1565	0	0	3918	0	0	0
N.S.	1	1.00	1.57	0.00	0.00	3.94	0.00	0.00	0.00
time (sec)	N/A	1.523	6.717	0.000	0.000	0.326	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	312	38	17	20	22
N.S.	1	1.00	1.11	1.00	17.33	2.11	0.94	1.11	1.22
time (sec)	N/A	0.019	20.983	0.057	0.418	0.266	1.002	0.402	2.063

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	555	555	654	0	0	2473	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	4.46	0.00	0.00	0.00
time (sec)	N/A	0.738	5.344	0.000	0.000	0.305	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	300	38	17	20	22
N.S.	1	1.00	1.11	1.00	16.67	2.11	0.94	1.11	1.22
time (sec)	N/A	0.019	19.202	0.054	0.432	0.254	0.808	0.401	2.044

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	220	177	0	1314	0	148	316
N.S.	1	1.00	1.79	1.44	0.00	10.68	0.00	1.20	2.57
time (sec)	N/A	0.189	0.862	0.275	0.000	0.303	0.000	0.275	2.582

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	249	38	17	20	22
N.S.	1	1.00	1.11	1.00	13.83	2.11	0.94	1.11	1.22
time (sec)	N/A	0.018	53.753	0.061	0.417	0.250	1.677	0.925	2.547

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	318	44	19	20	22
N.S.	1	1.00	1.11	1.00	17.67	2.44	1.06	1.11	1.22
time (sec)	N/A	0.018	26.111	0.057	0.422	0.266	1.442	0.374	2.439

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	319	44	19	3	22
N.S.	1	1.00	1.11	1.00	17.72	2.44	1.06	0.17	1.22
time (sec)	N/A	0.018	28.548	0.058	0.412	0.254	1.460	3.003	2.637

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	12	28	0	12	12
N.S.	1	1.00	1.00	1.17	2.00	4.67	0.00	2.00	2.00
time (sec)	N/A	0.016	0.027	0.528	0.197	0.260	0.000	0.271	2.016

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	426	426	415	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	0.646	0.000	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	310	310	311	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	0.516	0.000	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	207	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.128	0.456	0.000	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	36	18	15	18	20
N.S.	1	1.00	1.11	0.89	2.00	1.00	0.83	1.00	1.11
time (sec)	N/A	0.014	11.138	0.128	0.428	0.268	2.607	0.283	2.150

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	42	18	17	18	20
N.S.	1	1.00	1.11	0.89	2.33	1.00	0.94	1.00	1.11
time (sec)	N/A	0.014	10.482	0.144	0.429	0.252	1.485	0.301	2.200

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	677	677	748	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.594	8.600	0.000	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	497	497	582	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.457	7.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	319	319	466	0	0	0	0	0	0
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	5.815	0.000	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	86	36	17	20	22
N.S.	1	1.00	1.10	0.90	4.30	1.80	0.85	1.00	1.10
time (sec)	N/A	0.018	98.183	0.254	0.574	0.260	13.810	0.380	2.418

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	110	36	19	20	22
N.S.	1	1.00	1.10	0.90	5.50	1.80	0.95	1.00	1.10
time (sec)	N/A	0.017	37.837	0.261	0.570	0.282	2.464	0.491	2.375

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	961	961	721	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.986	1.209	0.000	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	721	721	547	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.792	0.835	0.000	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	481	481	373	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.615	0.673	0.000	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	58	19	17	20	22
N.S.	1	1.00	1.10	0.90	2.90	0.95	0.85	1.00	1.10
time (sec)	N/A	0.020	5.432	0.182	0.509	0.251	2.400	0.363	2.041

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	42	18	17	18	20
N.S.	1	1.00	1.11	0.89	2.33	1.00	0.94	1.00	1.11
time (sec)	N/A	0.012	0.105	0.009	0.443	0.261	1.464	0.324	0.002

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2851	2851	3035	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.778	10.368	0.000	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2123	2123	2247	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.119	9.722	0.000	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1395	1395	1393	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.653	8.951	0.000	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	253	38	19	20	22
N.S.	1	1.00	1.10	0.90	12.65	1.90	0.95	1.00	1.10
time (sec)	N/A	0.018	179.185	0.178	0.977	0.269	4.715	1.375	2.437

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	324	44	20	3	22
N.S.	1	1.00	1.10	0.90	16.20	2.20	1.00	0.15	1.10
time (sec)	N/A	0.019	91.963	0.181	1.158	0.274	9.896	3.100	2.477

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	254	254	288	0	0	0	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.165	0.576	0.000	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	172	0	0	0	0	0	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.092	0.926	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	33	37	29	43
N.S.	1	1.00	1.00	0.88	0.85	1.27	1.42	1.12	1.65
time (sec)	N/A	0.017	0.058	0.264	0.193	0.259	0.448	0.269	1.978

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	16	42	25	19	18	20
N.S.	1	1.00	1.10	0.80	2.10	1.25	0.95	0.90	1.00
time (sec)	N/A	0.012	13.992	0.132	0.442	0.264	1.083	0.294	2.195

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	16	42	25	19	18	20
N.S.	1	1.00	1.10	0.80	2.10	1.25	0.95	0.90	1.00
time (sec)	N/A	0.012	15.408	0.128	0.447	0.247	4.377	0.304	2.279

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	407	407	496	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.381	5.887	0.000	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	229	344	0	0	0	0	0	0
N.S.	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	3.352	0.000	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	42	48	194	0	55	77
N.S.	1	1.00	1.00	0.89	1.02	4.13	0.00	1.17	1.64
time (sec)	N/A	0.044	0.205	0.516	0.199	0.269	0.000	0.283	2.054

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	106	46	20	20	22
N.S.	1	1.00	1.09	0.82	4.82	2.09	0.91	0.91	1.00
time (sec)	N/A	0.017	36.169	0.258	0.551	0.259	2.050	0.381	2.375

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	0	46	20	20	22
N.S.	1	1.00	1.09	0.82	0.00	2.09	0.91	0.91	1.00
time (sec)	N/A	0.018	34.808	0.248	0.000	0.274	5.657	0.436	2.410

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	601	601	460	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.703	1.156	0.000	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	361	361	286	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.554	0.778	0.000	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	69	90	0	254	0	61	155
N.S.	1	1.00	1.01	1.32	0.00	3.74	0.00	0.90	2.28
time (sec)	N/A	0.071	0.187	0.268	0.000	0.299	0.000	0.276	2.555

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	66	27	20	20	22
N.S.	1	1.00	1.09	0.82	3.00	1.23	0.91	0.91	1.00
time (sec)	N/A	0.018	10.044	0.169	0.493	0.252	2.189	0.343	2.126

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	66	27	20	20	22
N.S.	1	1.00	1.09	0.82	3.00	1.23	0.91	0.91	1.00
time (sec)	N/A	0.020	10.413	0.166	0.550	0.267	6.603	0.428	2.168

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1755	1755	1769	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.899	8.270	0.000	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1027	1027	986	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.343	5.154	0.000	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	232	177	0	1387	0	148	344
N.S.	1	1.00	1.83	1.39	0.00	10.92	0.00	1.17	2.71
time (sec)	N/A	0.155	0.562	0.273	0.000	0.299	0.000	0.272	2.498

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	317	48	22	20	22
N.S.	1	1.00	1.09	0.82	14.41	2.18	1.00	0.91	1.00
time (sec)	N/A	0.018	81.808	0.176	1.022	0.282	5.812	1.384	2.450

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	324	48	22	3	22
N.S.	1	1.00	1.09	0.82	14.73	2.18	1.00	0.14	1.00
time (sec)	N/A	0.018	81.059	0.146	1.308	0.286	50.987	3.093	2.507

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	24
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.20
time (sec)	N/A	0.037	20.308	0.168	0.329	0.257	44.279	0.782	2.119

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	44	44	41	155	0	122	0	0	110
N.S.	1	1.00	0.93	3.52	0.00	2.77	0.00	0.00	2.50
time (sec)	N/A	0.039	0.335	1.716	0.000	0.253	0.000	0.000	2.108

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	135	135	260	368	0	664	0	0	0
N.S.	1	1.00	1.93	2.73	0.00	4.92	0.00	0.00	0.00
time (sec)	N/A	0.082	0.613	1.698	0.000	0.282	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	0	0	0	1082	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	4.99	0.00	0.00	0.00
time (sec)	N/A	0.146	0.000	0.000	0.000	0.301	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	79	79	57	271	0	646	0	0	158
N.S.	1	1.00	0.72	3.43	0.00	8.18	0.00	0.00	2.00
time (sec)	N/A	0.074	0.988	17.010	0.000	0.270	0.000	0.000	2.163

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	208	208	501	0	0	2972	0	0	0
N.S.	1	1.00	2.41	0.00	0.00	14.29	0.00	0.00	0.00
time (sec)	N/A	0.161	5.470	0.000	0.000	0.324	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	363	363	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	87	87	80	317	0	511	0	0	409
N.S.	1	1.00	0.92	3.64	0.00	5.87	0.00	0.00	4.70
time (sec)	N/A	0.121	0.648	0.850	0.000	0.298	0.000	0.000	2.685

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	307	307	859	585	0	1286	0	0	0
N.S.	1	1.00	2.80	1.91	0.00	4.19	0.00	0.00	0.00
time (sec)	N/A	0.421	3.704	0.778	0.000	0.298	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	452	452	0	0	0	2005	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	4.44	0.00	0.00	0.00
time (sec)	N/A	0.619	0.000	0.000	0.000	0.318	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	157	157	233	491	0	3547	0	0	0
N.S.	1	1.00	1.48	3.13	0.00	22.59	0.00	0.00	0.00
time (sec)	N/A	0.240	1.267	6.451	0.000	0.320	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	717	717	469	0	0	9020	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	12.58	0.00	0.00	0.00
time (sec)	N/A	0.878	6.458	0.000	0.000	0.422	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1284	1284	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.586	0.000	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [23] had the largest ratio of [.666699999999999959]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	10	6	1.00	16	0.375
2	N/A	0	0	1.00	16	0.000
3	A	8	5	1.00	16	0.312
4	N/A	0	0	1.00	16	0.000
5	A	4	3	1.00	14	0.214
6	N/A	0	0	1.00	16	0.000
7	N/A	0	0	1.00	16	0.000
8	A	15	11	1.00	18	0.611
9	N/A	0	0	1.00	18	0.000
10	A	10	7	1.00	18	0.389
11	N/A	0	0	1.00	18	0.000
12	A	5	5	1.00	16	0.312
13	N/A	0	0	1.00	18	0.000
14	N/A	0	0	1.00	18	0.000
15	A	5	3	1.00	12	0.250
16	A	13	8	1.00	18	0.444
17	N/A	0	0	1.00	18	0.000
18	A	11	7	1.00	18	0.389
19	N/A	0	0	1.00	18	0.000
20	A	4	4	1.00	16	0.250
21	N/A	0	0	1.00	18	0.000
22	N/A	0	0	1.00	16	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	31	12	1.00	18	0.667
24	N/A	0	0	1.00	18	0.000
25	A	22	10	1.00	18	0.556
26	N/A	0	0	1.00	18	0.000
27	A	6	6	1.00	16	0.375
28	N/A	0	0	1.00	18	0.000
29	N/A	0	0	1.00	18	0.000
30	N/A	0	0	1.00	18	0.000
31	A	3	3	1.00	10	0.300
32	A	20	7	1.00	18	0.389
33	A	16	7	1.00	18	0.389
34	A	12	7	1.00	16	0.438
35	N/A	0	0	1.00	18	0.000
36	N/A	0	0	1.00	18	0.000
37	A	30	10	1.00	20	0.500
38	A	24	10	1.00	20	0.500
39	A	18	10	1.00	18	0.556
40	N/A	0	0	1.00	20	0.000
41	N/A	0	0	1.00	20	0.000
42	A	23	9	1.00	20	0.450
43	A	19	9	1.00	20	0.450
44	A	15	9	1.00	18	0.500
45	N/A	0	0	1.00	20	0.000
46	N/A	0	0	1.00	18	0.000
47	A	61	11	1.00	20	0.550
48	A	49	11	1.00	20	0.550
49	A	37	11	1.00	18	0.611
50	N/A	0	0	1.00	20	0.000
51	N/A	0	0	1.00	20	0.000
52	A	14	7	1.00	20	0.350
53	A	10	6	1.00	20	0.300
54	A	4	3	1.00	20	0.150
55	N/A	0	0	1.00	20	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	N/A	0	0	1.00	20	0.000
57	A	21	10	1.00	22	0.454
58	A	15	11	1.00	22	0.500
59	A	5	5	1.00	22	0.227
60	N/A	0	0	1.00	22	0.000
61	N/A	0	0	1.00	22	0.000
62	A	17	9	1.00	22	0.409
63	A	13	8	1.00	22	0.364
64	A	4	4	1.00	22	0.182
65	N/A	0	0	1.00	22	0.000
66	N/A	0	0	1.00	22	0.000
67	A	43	11	1.00	22	0.500
68	A	31	12	1.00	22	0.546
69	A	6	6	1.00	22	0.273
70	N/A	0	0	1.00	22	0.000
71	N/A	0	0	1.00	22	0.000
72	N/A	0	0	1.00	20	0.000
73	A	5	4	1.00	20	0.200
74	A	9	6	1.00	22	0.273
75	A	11	7	1.00	22	0.318
76	A	6	6	1.00	22	0.273
77	A	11	8	1.00	24	0.333
78	A	16	12	1.00	24	0.500
79	A	5	5	1.00	22	0.227
80	A	12	8	1.00	24	0.333
81	A	14	9	1.00	24	0.375
82	A	7	7	1.00	22	0.318
83	A	23	11	1.00	24	0.458
84	A	32	13	1.00	24	0.542

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^5(a+b\operatorname{sech}(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int x^5(a+b\operatorname{sech}(c+dx^2))\,dx=\frac{ax^6}{6}+\frac{bx^4\arctan\left(e^{c+dx^2}\right)}{d}-\frac{ibx^2\operatorname{PolyLog}\left(2,-ie^{c+dx^2}\right)}{d^2}+\frac{ibx^2\operatorname{PolyLog}\left(2,ie^{c+dx^2}\right)}{d^2}+\frac{ib\operatorname{PolyLog}\left(3,-ie^{c+dx^2}\right)}{d^3}-\frac{ib\operatorname{PolyLog}\left(3,ie^{c+dx^2}\right)}{d^3}$$

```
[Out] 1/6*a*x^6+b*x^4*arctan(exp(d*x^2+c))/d-I*b*x^2*polylog(2,-I*exp(d*x^2+c))/d
^2+I*b*x^2*polylog(2,I*exp(d*x^2+c))/d^2+I*b*polylog(3,-I*exp(d*x^2+c))/d^3
-I*b*polylog(3,I*exp(d*x^2+c))/d^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {14, 5544, 4265, 2611, 2320, 6724}

$$\int x^5(a+b\operatorname{sech}(c+dx^2))\,dx=\frac{ax^6}{6}+\frac{bx^4\arctan\left(e^{c+dx^2}\right)}{d}+\frac{ib\operatorname{PolyLog}\left(3,-ie^{dx^2+c}\right)}{d^3}-\frac{ib\operatorname{PolyLog}\left(3,ie^{dx^2+c}\right)}{d^3}-\frac{ibx^2\operatorname{PolyLog}\left(2,-ie^{dx^2+c}\right)}{d^2}+\frac{ibx^2\operatorname{PolyLog}\left(2,ie^{dx^2+c}\right)}{d^2}$$

```
[In] Int[x^5*(a + b*Sech[c + d*x^2]),x]
```

```
[Out] (a*x^6)/6 + (b*x^4*ArcTan[E^(c + d*x^2)])/d - (I*b*x^2*PolyLog[2, (-I)*E^(c
 + d*x^2)])/d^2 + (I*b*x^2*PolyLog[2, I*E^(c + d*x^2)])/d^2 + (I*b*PolyLog[
3, (-I)*E^(c + d*x^2)])/d^3 - (I*b*PolyLog[3, I*E^(c + d*x^2)])/d^3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(ax^5 + bx^5\text{sech}\left(c+dx^2\right)\right)\,dx$$

$$= \frac{ax^6}{6} + b\int x^5\text{sech}\left(c+dx^2\right)\,dx$$

$$= \frac{ax^6}{6} + \frac{1}{2}b\text{Subst}\left(\int x^2\text{sech}(c+dx)\,dx, x, x^2\right)$$

$$= \frac{ax^6}{6} + \frac{bx^4\arctan\left(e^{c+dx^2}\right)}{d} - \frac{(ib)\text{Subst}\left(\int x\log\left(1-ie^{c+dx}\right)\,dx, x, x^2\right)}{d} + \frac{(ib)\text{Subst}\left(\int x\log\left(1+ie^{c+dx}\right)\,dx, x, x^2\right)}{d}$$

$$= \frac{ax^6}{6} + \frac{bx^4\arctan\left(e^{c+dx^2}\right)}{d} - \frac{ibx^2\,\text{PolyLog}\left(2,-ie^{c+dx^2}\right)}{d^2} + \frac{ibx^2\,\text{PolyLog}\left(2,ie^{c+dx^2}\right)}{d^2} + \frac{(ib)\text{Subst}\left(\int \text{PolyLog}\left(2,-ie^{c+dx}\right)\,dx, x, x^2\right)}{d^2} - \frac{(ib)\text{Subst}\left(\int \text{PolyLog}\left(2,ie^{c+dx}\right)\,dx, x, x^2\right)}{d^2}$$

$$= \frac{ax^6}{6} + \frac{bx^4\arctan\left(e^{c+dx^2}\right)}{d} - \frac{ibx^2\,\text{PolyLog}\left(2,-ie^{c+dx^2}\right)}{d^2} + \frac{ibx^2\,\text{PolyLog}\left(2,ie^{c+dx^2}\right)}{d^2} + \frac{(ib)\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x}\,dx, x, e^{c+dx^2}\right)}{d^3} - \frac{(ib)\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x}\,dx, x, e^{c+dx^2}\right)}{d^3}$$

$$= \frac{ax^6}{6} + \frac{bx^4\arctan\left(e^{c+dx^2}\right)}{d} - \frac{ibx^2\,\text{PolyLog}\left(2,-ie^{c+dx^2}\right)}{d^2} + \frac{ibx^2\,\text{PolyLog}\left(2,ie^{c+dx^2}\right)}{d^2} + \frac{ib\,\text{PolyLog}\left(3,-ie^{c+dx^2}\right)}{d^3} - \frac{ib\,\text{PolyLog}\left(3,ie^{c+dx^2}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.14

$$\int x^5 \left(a+b\text{sech}\left(c+dx^2\right)\right) \, dx = \frac{ax^6}{6} + \frac{ib\left(d^2x^4\log\left(1-ie^{c+dx^2}\right)-d^2x^4\log\left(1+ie^{c+dx^2}\right)-2dx^2\,\text{PolyLog}\left(2,-ie^{c+dx^2}\right)+2dx^2\,\text{PolyLog}\left(2,ie^{c\cdots}\right.}{2d^3}$$

```
[In] Integrate[x^5*(a + b*Sech[c + d*x^2]),x]
```

```
[Out] (a*x^6)/6 + ((I/2)*b*(d^2*x^4*Log[1 - I*E^(c + d*x^2)] - d^2*x^4*Log[1 + I*
E^(c + d*x^2)] - 2*d*x^2*PolyLog[2, (-I)*E^(c + d*x^2)] + 2*d*x^2*PolyLog[2
, I*E^(c + d*x^2)] + 2*PolyLog[3, (-I)*E^(c + d*x^2)] - 2*PolyLog[3, I*E^(c
 + d*x^2)]))/d^3
```

Maple [F]

$$\int x^5 \left(a+b\ \text{sech}\left(d\,x^2+c\right)\right) dx$$

```
[In] int(x^5*(a+b*sech(d*x^2+c)),x)
```

```
[Out] int(x^5*(a+b*sech(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(100) = 200.

Time = 0.27 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.05

$$\int x^5 \left(a+b\text{sech}\left(c+dx^2\right)\right) \, dx = \frac{ad^3x^6+6i\,bdx^2\text{Li}_2(i\,\cosh\left(dx^2+c\right)+i\,\sinh\left(dx^2+c\right))-6i\,bdx^2\text{Li}_2(-i\,\cosh\left(dx^2+c\right)-i\,\sinh\left(dx^2+c\right))\cdots}{}$$

```
[In] integrate(x^5*(a+b*sech(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(a*d^3*x^6 + 6*I*b*d*x^2*dilog(I*cosh(d*x^2 + c) + I*sinh(d*x^2 + c)) -
 6*I*b*d*x^2*dilog(-I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c)) + 3*I*b*c^2*log(
cosh(d*x^2 + c) + sinh(d*x^2 + c) + I) - 3*I*b*c^2*log(cosh(d*x^2 + c) + si
nh(d*x^2 + c) - I) - 3*(I*b*d^2*x^4 - I*b*c^2)*log(I*cosh(d*x^2 + c) + I*si
nh(d*x^2 + c) + 1) - 3*(-I*b*d^2*x^4 + I*b*c^2)*log(-I*cosh(d*x^2 + c) - I*
sinh(d*x^2 + c) + 1) - 6*I*b*polylog(3, I*cosh(d*x^2 + c) + I*sinh(d*x^2 +
c)) + 6*I*b*polylog(3, -I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c)))/d^3
```

Sympy [F]

$$\int x^5 \left(a + b\text{sech}\left(c + dx^2\right)\right)\,dx = \int x^5 \left(a + b\,\text{sech}\left(c + dx^2\right)\right)\,dx$$

```
[In] integrate(x**5*(a+b*sech(d*x**2+c)),x)
[Out] Integral(x**5*(a + b*sech(c + d*x**2)), x)
```

Maxima [F]

$$\int x^5 \left(a + b\text{sech}\left(c + dx^2\right)\right)\,dx = \int \left(b\,\text{sech}\left(dx^2 + c\right) + a\right)x^5\,dx$$

```
[In] integrate(x^5*(a+b*sech(d*x^2+c)),x, algorithm="maxima")
[Out] 1/6*a*x^6 + 2*b*integrate(x^5/(e^(d*x^2 + c) + e^(-d*x^2 - c)), x)
```

Giac [F]

$$\int x^5 \left(a + b\text{sech}\left(c + dx^2\right)\right)\,dx = \int \left(b\,\text{sech}\left(dx^2 + c\right) + a\right)x^5\,dx$$

```
[In] integrate(x^5*(a+b*sech(d*x^2+c)),x, algorithm="giac")
[Out] integrate((b*sech(d*x^2 + c) + a)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a + b\text{sech}\left(c + dx^2\right)\right)\,dx = \int x^5 \left(a + \frac{b}{\cosh\left(d\,x^2 + c\right)}\right)\,dx$$

```
[In] int(x^5*(a + b/cosh(c + d*x^2)),x)
[Out] int(x^5*(a + b/cosh(c + d*x^2)), x)
```

3.2 $\int x^4(a+b\,\mathrm{sech}(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^4(a+b\,\mathrm{sech}(c+dx^2))\,dx=\frac{ax^5}{5}+b\mathrm{Int}\left(x^4\mathrm{sech}\left(c+dx^2\right),x\right)$$

```
[Out] 1/5*a*x^5+b*Unintegrable(x^4*sech(d*x^2+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^4(a+b\,\mathrm{sech}(c+dx^2))\,dx=\int x^4(a+b\,\mathrm{sech}(c+dx^2))\,dx$$

```
[In] Int[x^4*(a + b*Sech[c + d*x^2]),x]
```

```
[Out] (a*x^5)/5 + b*Defer[Int][x^4*Sech[c + d*x^2], x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(ax^4+bx^4\mathrm{sech}\left(c+dx^2\right)\right)\,dx\\&=\frac{ax^5}{5}+b\int x^4\mathrm{sech}\left(c+dx^2\right)\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 5.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^4 (a + b\text{sech}(c + dx^2)) \, dx = \int x^4 (a + b\text{sech}(c + dx^2)) \, dx$$

```
[In] Integrate[x^4*(a + b*Sech[c + d*x^2]),x]
[Out] Integrate[x^4*(a + b*Sech[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^4 \left(a + b \ \text{sech}\left(d\,x^2 + c\right)\right) dx$$

```
[In] int(x^4*(a+b*sech(d*x^2+c)),x)
[Out] int(x^4*(a+b*sech(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^4 (a + b\text{sech}(c + dx^2)) \, dx = \int (b \,\text{sech}\left(dx^2 + c\right) + a) x^4 \, dx$$

```
[In] integrate(x^4*(a+b*sech(d*x^2+c)),x, algorithm="fricas")
[Out] integral(b*x^4*sech(d*x^2 + c) + a*x^4, x)
```

Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^4 (a + b\text{sech}(c + dx^2)) \, dx = \int x^4 \left(a + b \,\text{sech}\left(c + dx^2\right)\right) \, dx$$

```
[In] integrate(x**4*(a+b*sech(d*x**2+c)),x)
[Out] Integral(x**4*(a + b*sech(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int x^4 \left(a + b\text{sech}\left(c + dx^2\right)\right) \, dx = \int \left(b \,\text{sech}\left(dx^2 + c\right) + a\right)x^4 \, dx$$

```
[In] integrate(x^4*(a+b*sech(d*x^2+c)),x, algorithm="maxima")
[Out] 1/5*a*x^5 + 2*b*integrate(x^4/(e^(d*x^2 + c) + e^(-d*x^2 - c)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^4 \left(a + b\text{sech}\left(c + dx^2\right)\right) \, dx = \int \left(b \,\text{sech}\left(dx^2 + c\right) + a\right)x^4 \, dx$$

```
[In] integrate(x^4*(a+b*sech(d*x^2+c)),x, algorithm="giac")
[Out] integrate((b*sech(d*x^2 + c) + a)*x^4, x)
```

Mupad [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^4 \left(a + b\text{sech}\left(c + dx^2\right)\right) \, dx = \int x^4 \left(a + \frac{b}{\cosh\left(d\,x^2 + c\right)}\right) \, dx$$

```
[In] int(x^4*(a + b/cosh(c + d*x^2)),x)
[Out] int(x^4*(a + b/cosh(c + d*x^2)), x)
```

3.3 $\int x^3(a+b\text{sech}(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int x^3(a+b\text{sech}(c+dx^2))\,dx=\frac{ax^4}{4}+\frac{bx^2\arctan\left(e^{c+dx^2}\right)}{d}-\frac{ib\,\text{PolyLog}\left(2,-ie^{c+dx^2}\right)}{2d^2}+\frac{ib\,\text{PolyLog}\left(2,ie^{c+dx^2}\right)}{2d^2}$$

```
[Out] 1/4*a*x^4+b*x^2*arctan(exp(d*x^2+c))/d-1/2*I*b*polylog(2,-I*exp(d*x^2+c))/d
^2+1/2*I*b*polylog(2,I*exp(d*x^2+c))/d^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.312$, Rules used = {14, 5544, 4265, 2317, 2438}

$$\int x^3(a+b\text{sech}(c+dx^2))\,dx=\frac{ax^4}{4}+\frac{bx^2\arctan\left(e^{c+dx^2}\right)}{d}-\frac{ib\,\text{PolyLog}\left(2,-ie^{dx^2+c}\right)}{2d^2}+\frac{ib\,\text{PolyLog}\left(2,ie^{dx^2+c}\right)}{2d^2}$$

```
[In] Int[x^3*(a + b*Sech[c + d*x^2]),x]
[Out] (a*x^4)/4 + (b*x^2*ArcTan[E^(c + d*x^2)])/d - ((I/2)*b*PolyLog[2, (-I)*E^(c
 + d*x^2)])/d^2 + ((I/2)*b*PolyLog[2, I*E^(c + d*x^2)])/d^2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^3 + bx^3 \operatorname{sech}\left(c + dx^2\right)\right) \, dx \\
&= \frac{ax^4}{4} + b \int x^3 \operatorname{sech}\left(c + dx^2\right) \, dx \\
&= \frac{ax^4}{4} + \frac{1}{2} b \operatorname{Subst}\left(\int x \operatorname{sech}(c + dx) \, dx, x, x^2\right) \\
&= \frac{ax^4}{4} + \frac{bx^2 \arctan\left(e^{c+dx^2}\right)}{d} - \frac{(ib)\operatorname{Subst}\left(\int \log\left(1 - ie^{c+dx}\right) \, dx, x, x^2\right)}{2d} \\
&\quad + \frac{(ib)\operatorname{Subst}\left(\int \log\left(1 + ie^{c+dx}\right) \, dx, x, x^2\right)}{2d}
\end{aligned}$$

$$= \frac{ax^4}{4} + \frac{bx^2 \arctan\left(e^{c+dx^2}\right)}{d} - \frac{(ib)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{c+dx^2}\right)}{2d^2} + \frac{(ib)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{c+dx^2}\right)}{2d^2}$$

$$= \frac{ax^4}{4} + \frac{bx^2 \arctan\left(e^{c+dx^2}\right)}{d} - \frac{ib\,\text{PolyLog}\left(2, -ie^{c+dx^2}\right)}{2d^2} + \frac{ib\,\text{PolyLog}\left(2, ie^{c+dx^2}\right)}{2d^2}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int x^3\left(a+b\text{sech}\left(c+dx^2\right)\right)\,dx = \frac{ax^4}{4} + \frac{ib\left(dx^2\left(\log\left(1-ie^{c+dx^2}\right)-\log\left(1+ie^{c+dx^2}\right)\right)-\text{PolyLog}\left(2,-ie^{c+dx^2}\right)+\text{PolyLog}\left(2,ie^{c+dx^2}\right)\right)}{2d^2}$$

```
[In] Integrate[x^3*(a + b*Sech[c + d*x^2]),x]
[Out] (a*x^4)/4 + ((I/2)*b*(d*x^2*(Log[1 - I*E^(c + d*x^2)] - Log[1 + I*E^(c + d*
x^2)]) - PolyLog[2, (-I)*E^(c + d*x^2)] + PolyLog[2, I*E^(c + d*x^2)]))/d^2
```

Maple [F]

$$\int x^3\left(a+b\,\text{sech}\left(d\,x^2+c\right)\right)dx$$

```
[In] int(x^3*(a+b*sech(d*x^2+c)),x)
[Out] int(x^3*(a+b*sech(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(58) = 116.

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.39

$$\int x^3\left(a+b\text{sech}\left(c+dx^2\right)\right)\,dx = \frac{ad^2x^4 - 2i\,bc\log\left(\cosh\left(dx^2+c\right)+\sinh\left(dx^2+c\right)+i\right)+2i\,bc\log\left(\cosh\left(dx^2+c\right)+\sinh\left(dx^2+c\right)-i\right)+2}{}$$

```
[In] integrate(x^3*(a+b*sech(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^2*x^4 - 2*I*b*c*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) + I) + 2*I*b
*c*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) - I) + 2*I*b*dilog(I*cosh(d*x^2 +
c) + I*sinh(d*x^2 + c)) - 2*I*b*dilog(-I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c
)) - 2*(I*b*d*x^2 + I*b*c)*log(I*cosh(d*x^2 + c) + I*sinh(d*x^2 + c) + 1) -
 2*(-I*b*d*x^2 - I*b*c)*log(-I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c) + 1))/d^
2
```

Sympy [F]

$$\int x^3\left(a+b\text{sech}\left(c+dx^2\right)\right)\,dx=\int x^3\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)\,dx$$

```
[In] integrate(x**3*(a+b*sech(d*x**2+c)),x)
```

```
[Out] Integral(x**3*(a + b*sech(c + d*x**2)), x)
```

Maxima [F]

$$\int x^3\left(a+b\text{sech}\left(c+dx^2\right)\right)\,dx=\int \left(b\operatorname{sech}\left(dx^2+c\right)+a\right)x^3\,dx$$

```
[In] integrate(x^3*(a+b*sech(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 2*b*integrate(x^3/(e^(d*x^2 + c) + e^(-d*x^2 - c)), x)
```

Giac [F]

$$\int x^3\left(a+b\text{sech}\left(c+dx^2\right)\right)\,dx=\int \left(b\operatorname{sech}\left(dx^2+c\right)+a\right)x^3\,dx$$

```
[In] integrate(x^3*(a+b*sech(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x^2 + c) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\operatorname{sech}(c+dx^2))\,dx=\int x^3\left(a+\frac{b}{\cosh\left(d\,x^2+c\right)}\right)\,dx$$

```
[In] int(x^3*(a + b/cosh(c + d*x^2)),x)
[Out] int(x^3*(a + b/cosh(c + d*x^2)), x)
```

3.4 $\int x^2(a+b\,\mathrm{sech}(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^2\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)\,dx=\frac{ax^3}{3}+b\mathrm{Int}\left(x^2\mathrm{sech}\left(c+dx^2\right),x\right)$$

```
[Out] 1/3*a*x^3+b*Unintegrable(x^2*sech(d*x^2+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)\,dx=\int x^2\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)\,dx$$

```
[In] Int[x^2*(a + b*Sech[c + d*x^2]),x]
```

```
[Out] (a*x^3)/3 + b*Defer[Int][x^2*Sech[c + d*x^2], x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(ax^2+bx^2\mathrm{sech}\left(c+dx^2\right)\right)\,dx\\&=\frac{ax^3}{3}+b\int x^2\mathrm{sech}\left(c+dx^2\right)\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 4.69 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^2\left(a+b\text{sech}\left(c+dx^2\right)\right)\,dx=\int x^2\left(a+b\text{sech}\left(c+dx^2\right)\right)\,dx$$

```
[In] Integrate[x^2*(a + b*Sech[c + d*x^2]),x]
[Out] Integrate[x^2*(a + b*Sech[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^{2}\left(a+b\,\operatorname{sech}\left(d\,x^{2}+c\right)\right)dx$$

```
[In] int(x^2*(a+b*sech(d*x^2+c)),x)
[Out] int(x^2*(a+b*sech(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^2\left(a+b\text{sech}\left(c+dx^2\right)\right)\,dx=\int \left(b\operatorname{sech}\left(dx^2+c\right)+a\right)x^2\,dx$$

```
[In] integrate(x^2*(a+b*sech(d*x^2+c)),x, algorithm="fricas")
[Out] integral(b*x^2*sech(d*x^2 + c) + a*x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^2\left(a+b\text{sech}\left(c+dx^2\right)\right)\,dx=\int x^2\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)\,dx$$

```
[In] integrate(x**2*(a+b*sech(d*x**2+c)),x)
[Out] Integral(x**2*(a + b*sech(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int x^2 (a + b\text{sech}(c + dx^2)) \, dx = \int (b \,\text{sech} \left(dx^2 + c\right) + a)x^2 \, dx$$

```
[In] integrate(x^2*(a+b*sech(d*x^2+c)),x, algorithm="maxima")
[Out] 1/3*a*x^3 + 2*b*integrate(x^2/(e^(d*x^2 + c) + e^(-d*x^2 - c)), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^2 (a + b\text{sech}(c + dx^2)) \, dx = \int (b \,\text{sech} \left(dx^2 + c\right) + a)x^2 \, dx$$

```
[In] integrate(x^2*(a+b*sech(d*x^2+c)),x, algorithm="giac")
[Out] integrate((b*sech(d*x^2 + c) + a)*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^2 (a + b\text{sech}(c + dx^2)) \, dx = \int x^2 \left(a + \frac{b}{\cosh\left(d\,x^2 + c\right)}\right) \, dx$$

```
[In] int(x^2*(a + b/cosh(c + d*x^2)),x)
[Out] int(x^2*(a + b/cosh(c + d*x^2)), x)
```

3.5 $\int x(a+b\,\mathrm{sech}(c+dx^2))\,dx$

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int x\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)\,dx=\frac{ax^2}{2}+\frac{b\arctan\left(\sinh\left(c+dx^2\right)\right)}{2d}$$

```
[Out] 1/2*a*x^2+1/2*b*arctan(sinh(d*x^2+c))/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {14, 5544, 3855}

$$\int x\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)\,dx=\frac{ax^2}{2}+\frac{b\arctan\left(\sinh\left(c+dx^2\right)\right)}{2d}$$

```
[In] Int[x*(a + b*Sech[c + d*x^2]),x]
```

```
[Out] (a*x^2)/2 + (b*ArcTan[Sinh[c + d*x^2]])/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(ax + bx\text{sech}\left(c + dx^2\right)\right)\,dx \\ &= \frac{ax^2}{2} + b\int x\text{sech}\left(c + dx^2\right)\,dx \\ &= \frac{ax^2}{2} + \frac{1}{2}b\text{Subst}\left(\int \text{sech}(c + dx)\,dx, x, x^2\right) \\ &= \frac{ax^2}{2} + \frac{b\arctan\left(\sinh\left(c + dx^2\right)\right)}{2d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x\left(a + b\text{sech}\left(c + dx^2\right)\right)\,dx = \frac{ax^2}{2} + \frac{b\arctan\left(\sinh\left(c + dx^2\right)\right)}{2d}$$

```
[In] Integrate[x*(a + b*Sech[c + d*x^2]),x]
[Out] (a*x^2)/2 + (b*ArcTan[Sinh[c + d*x^2]])/(2*d)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
parts	$\frac{a\,x^{2}}{2}+\frac{b\arctan\left(\sinh\left(d\,x^{2}+c\right)\right)}{2d}$	23
derivativedivides	$\frac{\left(d\,x^{2}+c\right)a+b\arctan\left(\sinh\left(d\,x^{2}+c\right)\right)}{2d}$	27
default	$\frac{\left(d\,x^{2}+c\right)a+b\arctan\left(\sinh\left(d\,x^{2}+c\right)\right)}{2d}$	27
risch	$\frac{a\,x^{2}}{2}+\frac{ib\ln\left(\mathrm{e}^{d\,x^{2}+c}+i\right)}{2d}-\frac{ib\ln\left(\mathrm{e}^{d\,x^{2}+c}-i\right)}{2d}$	46
parallelrisch	$\frac{a d\,x^{2}-ib\ln\left(\tanh\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)-i\right)+ib\ln\left(\tanh\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)+i\right)}{2d}$	51

```
[In] int(x*(a+b*sech(d*x^2+c)),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x^2+1/2*b*arctan(sinh(d*x^2+c))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int x\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)\,dx=\frac{adx^2+2\,b\arctan\left(\cosh\left(dx^2+c\right)+\sinh\left(dx^2+c\right)\right)}{2\,d}$$

```
[In] integrate(x*(a+b*sech(d*x^2+c)),x, algorithm="fricas")
[Out] 1/2*(a*d*x^2 + 2*b*arctan(cosh(d*x^2 + c) + sinh(d*x^2 + c)))/d
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int x\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)\,dx=\begin{cases}\frac{a\left(c+dx^2\right)+2b\operatorname{atan}\left(\tanh\left(\frac{c}{2}+\frac{dx^2}{2}\right)\right)}{2d} & \text{for } d\neq 0\\ \frac{x^2\left(a+b\operatorname{sech}\left(c\right)\right)}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(a+b*sech(d*x**2+c)),x)
[Out] Piecewise(((a*(c + d*x**2) + 2*b*atan(tanh(c/2 + d*x**2/2)))/(2*d), Ne(d, 0
)), (x**2*(a + b*sech(c))/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)\,dx=\frac{1}{2}\,ax^2+\frac{b\arctan\left(\sinh\left(dx^2+c\right)\right)}{2\,d}$$

```
[In] integrate(x*(a+b*sech(d*x^2+c)),x, algorithm="maxima")
[Out] 1/2*a*x^2 + 1/2*b*arctan(sinh(d*x^2 + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)\,dx=\frac{(dx^2+c)a}{2\,d}+\frac{b\arctan\left(e^{(dx^2+c)}\right)}{d}$$

```
[In] integrate(x*(a+b*sech(d*x^2+c)),x, algorithm="giac")
```

```
[Out] 1/2*(d*x^2 + c)*a/d + b*arctan(e^(d*x^2 + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int x\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)\,dx=\frac{a\,x^2}{2}+\frac{\operatorname{atan}\left(\frac{b\,e^{d\,x^2}\,e^c\,\sqrt{d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{d^2}}$$

```
[In] int(x*(a + b/cosh(c + d*x^2)),x)
```

```
[Out] (a*x^2)/2 + (atan((b*exp(d*x^2)*exp(c)*(d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^
(1/2))/(d^2)^(1/2)
```

3.6 $\int \frac{a+b\operatorname{sech}(c+dx^2)}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a+b\operatorname{sech}(c+dx^2)}{x} \, dx = a\log(x) + b\text{Int}\left(\frac{\operatorname{sech}(c+dx^2)}{x}, x\right)$$

```
[Out] a*ln(x)+b*Unintegrable(sech(d*x^2+c)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\operatorname{sech}(c+dx^2)}{x} \, dx = \int \frac{a+b\operatorname{sech}(c+dx^2)}{x} \, dx$$

```
[In] Int[(a + b*Sech[c + d*x^2])/x,x]
[Out] a*Log[x] + b*Defer[Int][Sech[c + d*x^2]/x, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{a}{x} + \frac{b\operatorname{sech}(c+dx^2)}{x}\right) \, dx \\ &= a\log(x) + b\int \frac{\operatorname{sech}(c+dx^2)}{x} \, dx \end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 4.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{sech}(c+dx^2)}{x}\,dx=\int \frac{a+b\text{sech}(c+dx^2)}{x}\,dx$$

```
[In] Integrate[(a + b*Sech[c + d*x^2])/x,x]
[Out] Integrate[(a + b*Sech[c + d*x^2])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a+b\,\operatorname{sech}\left(d\,x^{2}+c\right)}{x}dx$$

```
[In] int((a+b*sech(d*x^2+c))/x,x)
[Out] int((a+b*sech(d*x^2+c))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{sech}(c+dx^2)}{x}\,dx=\int \frac{b\,\text{sech}\left(dx^2+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*sech(d*x^2+c))/x,x, algorithm="fricas")
[Out] integral((b*sech(d*x^2 + c) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{a+b\text{sech}(c+dx^2)}{x}\,dx=\int \frac{a+b\,\text{sech}\left(c+dx^2\right)}{x}\,dx$$

```
[In] integrate((a+b*sech(d*x**2+c))/x,x)
[Out] Integral((a + b*sech(c + d*x**2))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{a + b\text{sech}(c + dx^2)}{x} dx = \int \frac{b\,\text{sech}\left(dx^2 + c\right) + a}{x} dx$$

```
[In] integrate((a+b*sech(d*x^2+c))/x,x, algorithm="maxima")
[Out] 2*b*integrate(1/(x*(e^(d*x^2 + c) + e^(-d*x^2 - c))), x) + a*log(x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\text{sech}(c + dx^2)}{x} dx = \int \frac{b\,\text{sech}\left(dx^2 + c\right) + a}{x} dx$$

```
[In] integrate((a+b*sech(d*x^2+c))/x,x, algorithm="giac")
[Out] integrate((b*sech(d*x^2 + c) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a + b\text{sech}(c + dx^2)}{x} dx = \int \frac{a + \frac{b}{\cosh(d\,x^2+c)}}{x} dx$$

```
[In] int((a + b/cosh(c + d*x^2))/x,x)
[Out] int((a + b/cosh(c + d*x^2))/x, x)
```

3.7 $\int \frac{a+b\,\mathrm{sech}(c+dx^2)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a+b\,\mathrm{sech}(c+dx^2)}{x^2}\,dx=-\frac{a}{x}+b\mathrm{Int}\left(\frac{\mathrm{sech}(c+dx^2)}{x^2},x\right)$$

```
[Out] -a/x+b*Unintegrable(sech(d*x^2+c)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\,\mathrm{sech}(c+dx^2)}{x^2}\,dx=\int \frac{a+b\,\mathrm{sech}(c+dx^2)}{x^2}\,dx$$

```
[In] Int[(a + b*Sech[c + d*x^2])/x^2,x]
```

```
[Out] -(a/x) + b*Defer[Int][Sech[c + d*x^2]/x^2, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{a}{x^2}+\frac{b\,\mathrm{sech}(c+dx^2)}{x^2}\right)dx\\&=-\frac{a}{x}+b\int \frac{\mathrm{sech}(c+dx^2)}{x^2}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 4.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{sech}(c+dx^2)}{x^2}\,dx=\int \frac{a+b\text{sech}(c+dx^2)}{x^2}\,dx$$

```
[In] Integrate[(a + b*Sech[c + d*x^2])/x^2,x]
[Out] Integrate[(a + b*Sech[c + d*x^2])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a+b\ \text{sech}\left(d\,x^2+c\right)}{x^2}dx$$

```
[In] int((a+b*sech(d*x^2+c))/x^2,x)
[Out] int((a+b*sech(d*x^2+c))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{sech}(c+dx^2)}{x^2}\,dx=\int \frac{b\,\text{sech}\left(dx^2+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sech(d*x^2+c))/x^2,x, algorithm="fricas")
[Out] integral((b*sech(d*x^2 + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a+b\text{sech}(c+dx^2)}{x^2}\,dx=\int \frac{a+b\,\text{sech}\left(c+dx^2\right)}{x^2}\,dx$$

```
[In] integrate((a+b*sech(d*x**2+c))/x**2,x)
[Out] Integral((a + b*sech(c + d*x**2))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int \frac{a + b\text{sech}(c + dx^2)}{x^2}\,dx = \int \frac{b\,\text{sech}\,(dx^2 + c) + a}{x^2}\,dx$$

```
[In] integrate((a+b*sech(d*x^2+c))/x^2,x, algorithm="maxima")
[Out] 2*b*integrate(1/(x^2*(e^(d*x^2 + c) + e^(-d*x^2 - c))), x) - a/x
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\text{sech}(c + dx^2)}{x^2}\,dx = \int \frac{b\,\text{sech}\,(dx^2 + c) + a}{x^2}\,dx$$

```
[In] integrate((a+b*sech(d*x^2+c))/x^2,x, algorithm="giac")
[Out] integrate((b*sech(d*x^2 + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a + b\text{sech}(c + dx^2)}{x^2}\,dx = \int \frac{a + \frac{b}{\cosh(d\,x^2+c)}}{x^2}\,dx$$

```
[In] int((a + b/cosh(c + d*x^2))/x^2,x)
[Out] int((a + b/cosh(c + d*x^2))/x^2, x)
```

3.8 $\int x^5(a+b\,\mathrm{sech}(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 217

$$\int x^5\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2\,dx=\frac{b^2x^4}{2d}+\frac{a^2x^6}{6}+\frac{2abx^4\arctan\left(e^{c+dx^2}\right)}{d}-\frac{b^2x^2\log\left(1+e^{2(c+dx^2)}\right)}{d^2}-\frac{2iabx^2\,\mathrm{PolyLog}\left(2,-ie^{c+dx^2}\right)}{d^2}+\frac{2iabx^2\,\mathrm{PolyLog}\left(2,ie^{c+dx^2}\right)}{d^2}-\frac{b^2\,\mathrm{PolyLog}\left(2,-e^{2(c+dx^2)}\right)}{2d^3}+\frac{2iab\,\mathrm{PolyLog}\left(3,-ie^{c+dx^2}\right)}{d^3}-\frac{2iab\,\mathrm{PolyLog}\left(3,ie^{c+dx^2}\right)}{d^3}+\frac{b^2x^4\tanh\left(c+dx^2\right)}{2d}$$

```
[Out] 1/2*b^2*x^4/d+1/6*a^2*x^6+2*a*b*x^4*arctan(exp(d*x^2+c))/d-b^2*x^2*ln(1+exp
(2*d*x^2+2*c))/d^2-2*I*a*b*x^2*polylog(2,-I*exp(d*x^2+c))/d^2+2*I*a*b*x^2*p
olylog(2,I*exp(d*x^2+c))/d^2-1/2*b^2*polylog(2,-exp(2*d*x^2+2*c))/d^3+2*I*a
*b*polylog(3,-I*exp(d*x^2+c))/d^3-2*I*a*b*polylog(3,I*exp(d*x^2+c))/d^3+1/2
*b^2*x^4*tanh(d*x^2+c)/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {5544, 4275, 4265, 2611, 2320, 6724, 4269, 3799, 2221, 2317, 2438}

$$\int x^5\left(a+b\,\text{sech}\left(c+dx^2\right)\right)^2\,dx=\frac{a^2x^6}{6}+\frac{2abx^4\arctan\left(e^{c+dx^2}\right)}{d}+\frac{2iab\,\text{PolyLog}\left(3,-ie^{dx^2+c}\right)}{d^3}-\frac{2iab\,\text{PolyLog}\left(3,ie^{dx^2+c}\right)}{d^3}-\frac{2iabx^2\,\text{PolyLog}\left(2,-ie^{dx^2+c}\right)}{d^2}+\frac{2iabx^2\,\text{PolyLog}\left(2,ie^{dx^2+c}\right)}{d^2}-\frac{b^2\,\text{PolyLog}\left(2,-e^{2(dx^2+c)}\right)}{2d^3}-\frac{b^2x^2\log\left(e^{2(c+dx^2)}+1\right)}{d^2}+\frac{b^2x^4\tanh\left(c+dx^2\right)}{2d}+\frac{b^2x^4}{2d}$$

```
[In] Int[x^5*(a + b*Sech[c + d*x^2])^2,x]
```

```
[Out] (b^2*x^4)/(2*d) + (a^2*x^6)/6 + (2*a*b*x^4*ArcTan[E^(c + d*x^2)])/d - (b^2*
x^2*Log[1 + E^(2*(c + d*x^2))])/d^2 - ((2*I)*a*b*x^2*PolyLog[2, (-I)*E^(c +
 d*x^2)])/d^2 + ((2*I)*a*b*x^2*PolyLog[2, I*E^(c + d*x^2)])/d^2 - (b^2*Poly
Log[2, -E^(2*(c + d*x^2))])/(2*d^3) + ((2*I)*a*b*PolyLog[3, (-I)*E^(c + d*x
^2)])/d^3 - ((2*I)*a*b*PolyLog[3, I*E^(c + d*x^2)])/d^3 + (b^2*x^4*Tanh[c +
 d*x^2])/(2*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^2(a+b\text{sech}(c+dx))^2\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2x^2+2abx^2\text{sech}(c+dx)+b^2x^2\text{sech}^2(c+dx)\right)\,dx, x, x^2\right)\\
&= \frac{a^2x^6}{6}+(ab)\text{Subst}\left(\int x^2\text{sech}(c+dx)\,dx, x, x^2\right)+\frac{1}{2}b^2\text{Subst}\left(\int x^2\text{sech}^2(c+dx)\,dx, x, x^2\right)\\
&= \frac{a^2x^6}{6}+\frac{2abx^4\arctan\left(e^{c+dx^2}\right)}{d}+\frac{b^2x^4\tanh\left(c+dx^2\right)}{2d}\\
&\quad-\frac{(2iab)\text{Subst}\left(\int x\log\left(1-ie^{c+dx}\right)\,dx, x, x^2\right)}{d}\\
&\quad+\frac{(2iab)\text{Subst}\left(\int x\log\left(1+ie^{c+dx}\right)\,dx, x, x^2\right)}{d}\\
&\quad-\frac{b^2\text{Subst}\left(\int x\tanh(c+dx)\,dx, x, x^2\right)}{d}\\
&= \frac{b^2x^4}{2d}+\frac{a^2x^6}{6}+\frac{2abx^4\arctan\left(e^{c+dx^2}\right)}{d}-\frac{2iabx^2\,\text{PolyLog}\left(2,-ie^{c+dx^2}\right)}{d^2}\\
&\quad+\frac{2iabx^2\,\text{PolyLog}\left(2,ie^{c+dx^2}\right)}{d^2}+\frac{b^2x^4\tanh\left(c+dx^2\right)}{2d}\\
&\quad+\frac{(2iab)\text{Subst}\left(\int \text{PolyLog}\left(2,-ie^{c+dx}\right)\,dx, x, x^2\right)}{d^2}\\
&\quad-\frac{(2iab)\text{Subst}\left(\int \text{PolyLog}\left(2,ie^{c+dx}\right)\,dx, x, x^2\right)}{d^2}\\
&\quad-\frac{(2b^2)\,\text{Subst}\left(\int \frac{e^{2(c+dx)}x}{1+e^{2(c+dx)}}\,dx, x, x^2\right)}{d}
\end{aligned}$$

$$= \frac{b^2x^4}{2d} + \frac{a^2x^6}{6} + \frac{2abx^4\arctan\left(e^{c+dx^2}\right)}{d} - \frac{b^2x^2\log\left(1+e^{2(c+dx^2)}\right)}{d^2} - \frac{2iabx^2\,\text{PolyLog}\left(2,-ie^{c+dx^2}\right)}{d^2} + \frac{2iabx^2\,\text{PolyLog}\left(2,ie^{c+dx^2}\right)}{d^2} + \frac{b^2x^4\tanh\left(c+dx^2\right)}{2d} + \frac{(2iab)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{c+dx^2}\right)}{d^3} - \frac{(2iab)\text{Subst}\left(\int\frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{c+dx^2}\right)}{d^3} + \frac{b^2\text{Subst}\left(\int\log\left(1+e^{2(c+dx)}\right)\,dx,x,x^2\right)}{d^2}$$

$$= \frac{b^2x^4}{2d} + \frac{a^2x^6}{6} + \frac{2abx^4\arctan\left(e^{c+dx^2}\right)}{d} - \frac{b^2x^2\log\left(1+e^{2(c+dx^2)}\right)}{d^2} - \frac{2iabx^2\,\text{PolyLog}\left(2,-ie^{c+dx^2}\right)}{d^2} + \frac{2iabx^2\,\text{PolyLog}\left(2,ie^{c+dx^2}\right)}{d^2} + \frac{2iab\,\text{PolyLog}\left(3,-ie^{c+dx^2}\right)}{d^3} - \frac{2iab\,\text{PolyLog}\left(3,ie^{c+dx^2}\right)}{d^3} + \frac{b^2x^4\tanh\left(c+dx^2\right)}{2d} + \frac{b^2\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2(c+dx^2)}\right)}{2d^3}$$

$$= \frac{b^2x^4}{2d} + \frac{a^2x^6}{6} + \frac{2abx^4\arctan\left(e^{c+dx^2}\right)}{d} - \frac{b^2x^2\log\left(1+e^{2(c+dx^2)}\right)}{d^2} - \frac{2iabx^2\,\text{PolyLog}\left(2,-ie^{c+dx^2}\right)}{d^2} + \frac{2iabx^2\,\text{PolyLog}\left(2,ie^{c+dx^2}\right)}{d^2} - \frac{b^2\,\text{PolyLog}\left(2,-e^{2(c+dx^2)}\right)}{2d^3} + \frac{2iab\,\text{PolyLog}\left(3,-ie^{c+dx^2}\right)}{d^3} - \frac{2iab\,\text{PolyLog}\left(3,ie^{c+dx^2}\right)}{d^3} + \frac{b^2x^4\tanh\left(c+dx^2\right)}{2d}$$

Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.47

$$\int x^5\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx$$

$$= \frac{\cosh\left(c+dx^2\right)\left(a+b\text{sech}(c+dx^2)\right)^2\left(a^2x^6\cosh\left(c+dx^2\right)+\frac{3b\cosh(c+dx^2)\left(2bd^2e^{2c}x^4-2bd^2(1+e^{2c})x^4+b(1+e^{2c})\left(2dx\right.\right.}{}\right.}{}$$

```
[In] Integrate[x^5*(a + b*Sech[c + d*x^2])^2,x]
```

```
[Out] (Cosh[c + d*x^2]*(a + b*Sech[c + d*x^2])^2*(a^2*x^6*Cosh[c + d*x^2] + (3*b*
Cosh[c + d*x^2]*(2*b*d^2*E^(2*c)*x^4 - 2*b*d^2*(1 + E^(2*c))*x^4 + b*(1 + E
^(2*c))*(2*d*x^2*(d*x^2 - Log[1 + E^(2*(c + d*x^2))]) - PolyLog[2, -E^(2*(c
 + d*x^2))]) + (2*I)*a*(1 + E^(2*c))*(d^2*x^4*Log[1 - I*E^(c + d*x^2)] - d^
2*x^4*Log[1 + I*E^(c + d*x^2)] - 2*d*x^2*PolyLog[2, (-I)*E^(c + d*x^2)] + 2
*d*x^2*PolyLog[2, I*E^(c + d*x^2)] + 2*PolyLog[3, (-I)*E^(c + d*x^2)] - 2*P
olyLog[3, I*E^(c + d*x^2)])))/(d^3*(1 + E^(2*c))) + (3*b^2*x^4*Sech[c]*Sinh
[d*x^2])/d))/(6*(b + a*Cosh[c + d*x^2])^2)
```

Maple [F]

$$\int x^5 \left(a + b\ \mathrm{sech}\left(d\, x^2 + c\right)\right)^2 dx$$

```
[In] int(x^5*(a+b*sech(d*x^2+c))^2,x)
```

```
[Out] int(x^5*(a+b*sech(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1198 vs. $2(185) = 370$.

Time = 0.30 (sec) , antiderivative size = 1198, normalized size of antiderivative = 5.52

$$\int x^5 \left(a + b\mathrm{sech}\left(c + dx^2\right)\right)^2 \, dx = \text{Too large to display}$$

```
[In] integrate(x^5*(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(a^2*d^3*x^6 - 6*b^2*c^2 + (a^2*d^3*x^6 + 6*b^2*d^2*x^4 - 6*b^2*c^2)*co
sh(d*x^2 + c)^2 + 2*(a^2*d^3*x^6 + 6*b^2*d^2*x^4 - 6*b^2*c^2)*cosh(d*x^2 +
c)*sinh(d*x^2 + c) + (a^2*d^3*x^6 + 6*b^2*d^2*x^4 - 6*b^2*c^2)*sinh(d*x^2 +
 c)^2 - 6*(-2*I*a*b*d*x^2 + (-2*I*a*b*d*x^2 + b^2)*cosh(d*x^2 + c)^2 + 2*(-
2*I*a*b*d*x^2 + b^2)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (-2*I*a*b*d*x^2 + b^
2)*sinh(d*x^2 + c)^2 + b^2)*dilog(I*cosh(d*x^2 + c) + I*sinh(d*x^2 + c)) -
6*(2*I*a*b*d*x^2 + (2*I*a*b*d*x^2 + b^2)*cosh(d*x^2 + c)^2 + 2*(2*I*a*b*d*x
^2 + b^2)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (2*I*a*b*d*x^2 + b^2)*sinh(d*x^
2 + c)^2 + b^2)*dilog(-I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c)) - 6*(-I*a*b*c
^2 - b^2*c + (-I*a*b*c^2 - b^2*c)*cosh(d*x^2 + c)^2 + 2*(-I*a*b*c^2 - b^2*c
)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (-I*a*b*c^2 - b^2*c)*sinh(d*x^2 + c)^2)
*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) + I) - 6*(I*a*b*c^2 - b^2*c + (I*a*b
*c^2 - b^2*c)*cosh(d*x^2 + c)^2 + 2*(I*a*b*c^2 - b^2*c)*cosh(d*x^2 + c)*sin
h(d*x^2 + c) + (I*a*b*c^2 - b^2*c)*sinh(d*x^2 + c)^2)*log(cosh(d*x^2 + c) +
 sinh(d*x^2 + c) - I) - 6*(I*a*b*d^2*x^4 + b^2*d*x^2 - I*a*b*c^2 + b^2*c +
```

```
(I*a*b*d^2*x^4 + b^2*d*x^2 - I*a*b*c^2 + b^2*c)*cosh(d*x^2 + c)^2 + 2*(I*a*
b*d^2*x^4 + b^2*d*x^2 - I*a*b*c^2 + b^2*c)*cosh(d*x^2 + c)*sinh(d*x^2 + c)
+ (I*a*b*d^2*x^4 + b^2*d*x^2 - I*a*b*c^2 + b^2*c)*sinh(d*x^2 + c)^2)*log(I*
cosh(d*x^2 + c) + I*sinh(d*x^2 + c) + 1) - 6*(-I*a*b*d^2*x^4 + b^2*d*x^2 +
I*a*b*c^2 + b^2*c + (-I*a*b*d^2*x^4 + b^2*d*x^2 + I*a*b*c^2 + b^2*c)*cosh(d
*x^2 + c)^2 + 2*(-I*a*b*d^2*x^4 + b^2*d*x^2 + I*a*b*c^2 + b^2*c)*cosh(d*x^2
 + c)*sinh(d*x^2 + c) + (-I*a*b*d^2*x^4 + b^2*d*x^2 + I*a*b*c^2 + b^2*c)*si
nh(d*x^2 + c)^2)*log(-I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c) + 1) - 12*(I*a*
b*cosh(d*x^2 + c)^2 + 2*I*a*b*cosh(d*x^2 + c)*sinh(d*x^2 + c) + I*a*b*sinh(
d*x^2 + c)^2 + I*a*b)*polylog(3, I*cosh(d*x^2 + c) + I*sinh(d*x^2 + c)) - 1
2*(-I*a*b*cosh(d*x^2 + c)^2 - 2*I*a*b*cosh(d*x^2 + c)*sinh(d*x^2 + c) - I*a
*b*sinh(d*x^2 + c)^2 - I*a*b)*polylog(3, -I*cosh(d*x^2 + c) - I*sinh(d*x^2
+ c)))/(d^3*cosh(d*x^2 + c)^2 + 2*d^3*cosh(d*x^2 + c)*sinh(d*x^2 + c) + d^3
*sinh(d*x^2 + c)^2 + d^3)
```

Sympy [F]

$$\int x^5(a+b\text{sech}(c+dx^2))^2\,dx=\int x^5\left(a+b\,\text{sech}\left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate(x**5*(a+b*sech(d*x**2+c))**2,x)
[Out] Integral(x**5*(a + b*sech(c + d*x**2))**2, x)
```

Maxima [F]

$$\int x^5(a+b\text{sech}(c+dx^2))^2\,dx=\int\left(b\,\text{sech}\left(dx^2+c\right)+a\right)^2x^5\,dx$$

```
[In] integrate(x^5*(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
[Out] 1/6*a^2*x^6 - b^2*x^4/(d*e^(2*d*x^2 + 2*c) + d) + integrate(4*(a*b*d*x^5*e^
(d*x^2 + c) + b^2*x^3)/(d*e^(2*d*x^2 + 2*c) + d), x)
```

Giac [F]

$$\int x^5(a+b\text{sech}(c+dx^2))^2\,dx=\int\left(b\,\text{sech}\left(dx^2+c\right)+a\right)^2x^5\,dx$$

```
[In] integrate(x^5*(a+b*sech(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate((b*sech(d*x^2 + c) + a)^2*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 (a + b\operatorname{sech}(c + dx^2))^2 \, dx = \int x^5 \left(a + \frac{b}{\cosh(d x^2 + c)}\right)^2 \, dx$$

```
[In] int(x^5*(a + b/cosh(c + d*x^2))^2,x)
[Out] int(x^5*(a + b/cosh(c + d*x^2))^2, x)
```

3.9 $\int x^4 (a + b\,\text{sech}(c + dx^2))^2 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^4 (a + b\text{sech}(c + dx^2))^2 \, dx = \text{Int}\left(x^4 (a + b\text{sech}(c + dx^2))^2, x\right)$$

```
[Out] Unintegrable(x^4*(a+b*sech(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^4 (a + b\text{sech}(c + dx^2))^2 \, dx = \int x^4 (a + b\text{sech}(c + dx^2))^2 \, dx$$

```
[In] Int[x^4*(a + b*Sech[c + d*x^2])^2,x]
[Out] Defer[Int][x^4*(a + b*Sech[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral} = \int x^4 (a + b\text{sech}(c + dx^2))^2 \, dx$$

Mathematica [N/A]

Not integrable

Time = 11.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^4\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx=\int x^4\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx$$

```
[In] Integrate[x^4*(a + b*Sech[c + d*x^2])^2,x]
[Out] Integrate[x^4*(a + b*Sech[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^4\left(a+b\,\operatorname{sech}\left(d\,x^2+c\right)\right)^2dx$$

```
[In] int(x^4*(a+b*sech(d*x^2+c))^2,x)
[Out] int(x^4*(a+b*sech(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x^4\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx=\int\left(b\,\operatorname{sech}\left(dx^2+c\right)+a\right)^2x^4\,dx$$

```
[In] integrate(x^4*(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(b^2*x^4*sech(d*x^2 + c)^2 + 2*a*b*x^4*sech(d*x^2 + c) + a^2*x^4, x
)
```

Sympy [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int x^4\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx=\int x^4\left(a+b\,\operatorname{sech}\left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate(x**4*(a+b*sech(d*x**2+c))**2,x)
[Out] Integral(x**4*(a + b*sech(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.39

$$\int x^4 \left(a + b\operatorname{sech}\left(c + dx^2\right)\right)^2 \, dx = \int \left(b \operatorname{sech}\left(dx^2 + c\right) + a\right)^2 x^4 \, dx$$

```
[In] integrate(x^4*(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 - b^2*x^3/(d*e^(2*d*x^2 + 2*c) + d) + integrate((4*a*b*d*x^4*e^
(d*x^2 + c) + 3*b^2*x^2)/(d*e^(2*d*x^2 + 2*c) + d), x)
```

Giac [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^4 \left(a + b\operatorname{sech}\left(c + dx^2\right)\right)^2 \, dx = \int \left(b \operatorname{sech}\left(dx^2 + c\right) + a\right)^2 x^4 \, dx$$

```
[In] integrate(x^4*(a+b*sech(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x^2 + c) + a)^2*x^4, x)
```

Mupad [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^4 \left(a + b\operatorname{sech}\left(c + dx^2\right)\right)^2 \, dx = \int x^4 \left(a + \frac{b}{\cosh\left(d\,x^2 + c\right)}\right)^2 \, dx$$

```
[In] int(x^4*(a + b/cosh(c + d*x^2))^2,x)
```

```
[Out] int(x^4*(a + b/cosh(c + d*x^2))^2, x)
```

3.10 $\int x^3(a+b\text{sech}(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 119

$$\int x^3\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx=\frac{a^2x^4}{4}+\frac{2abx^2\arctan\left(e^{c+dx^2}\right)}{d}-\frac{b^2\log\left(\cosh\left(c+dx^2\right)\right)}{2d^2}-\frac{iab\,\text{PolyLog}\left(2,-ie^{c+dx^2}\right)}{d^2}+\frac{iab\,\text{PolyLog}\left(2,ie^{c+dx^2}\right)}{d^2}+\frac{b^2x^2\tanh\left(c+dx^2\right)}{2d}$$

```
[Out] 1/4*a^2*x^4+2*a*b*x^2*arctan(exp(d*x^2+c))/d-1/2*b^2*ln(cosh(d*x^2+c))/d^2-
I*a*b*polylog(2,-I*exp(d*x^2+c))/d^2+I*a*b*polylog(2,I*exp(d*x^2+c))/d^2+1/
2*b^2*x^2*tanh(d*x^2+c)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5544, 4275, 4265, 2317, 2438, 4269, 3556}

$$\int x^3\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx=\frac{a^2x^4}{4}+\frac{2abx^2\arctan\left(e^{c+dx^2}\right)}{d}-\frac{iab\,\text{PolyLog}\left(2,-ie^{dx^2+c}\right)}{d^2}+\frac{iab\,\text{PolyLog}\left(2,ie^{dx^2+c}\right)}{d^2}-\frac{b^2\log\left(\cosh\left(c+dx^2\right)\right)}{2d^2}+\frac{b^2x^2\tanh\left(c+dx^2\right)}{2d}$$

```
[In] Int[x^3*(a + b*Sech[c + d*x^2])^2,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^2*ArcTan[E^(c + d*x^2)])/d - (b^2*Log[Cosh[c + d*x^2
]])/(2*d^2) - (I*a*b*PolyLog[2, (-I)*E^(c + d*x^2)])/d^2 + (I*a*b*PolyLog[2
, I*E^(c + d*x^2)])/d^2 + (b^2*x^2*Tanh[c + d*x^2])/(2*d)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
```

```
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x(a+b\text{sech}(c+dx))^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2x+2abx\text{sech}(c+dx)+b^2x\text{sech}^2(c+dx)\right)\,dx,x,x^2\right)\\
&= \frac{a^2x^4}{4}+(ab)\text{Subst}\left(\int x\text{sech}(c+dx)\,dx,x,x^2\right)+\frac{1}{2}b^2\text{Subst}\left(\int x\text{sech}^2(c+dx)\,dx,x,x^2\right)\\
&= \frac{a^2x^4}{4}+\frac{2abx^2\arctan\left(e^{c+dx^2}\right)}{d}+\frac{b^2x^2\tanh\left(c+dx^2\right)}{2d}\\
&\quad-\frac{(iab)\text{Subst}\left(\int\log\left(1-ie^{c+dx}\right)\,dx,x,x^2\right)}{d}\\
&\quad+\frac{(iab)\text{Subst}\left(\int\log\left(1+ie^{c+dx}\right)\,dx,x,x^2\right)}{d}-\frac{b^2\text{Subst}\left(\int\tanh(c+dx)\,dx,x,x^2\right)}{2d}\\
&= \frac{a^2x^4}{4}+\frac{2abx^2\arctan\left(e^{c+dx^2}\right)}{d}-\frac{b^2\log\left(\cosh\left(c+dx^2\right)\right)}{2d^2}+\frac{b^2x^2\tanh\left(c+dx^2\right)}{2d}\\
&\quad-\frac{(iab)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{c+dx^2}\right)}{d^2}+\frac{(iab)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{c+dx^2}\right)}{d^2}\\
&= \frac{a^2x^4}{4}+\frac{2abx^2\arctan\left(e^{c+dx^2}\right)}{d}-\frac{b^2\log\left(\cosh\left(c+dx^2\right)\right)}{2d^2}\\
&\quad-\frac{iab\,\text{PolyLog}\left(2,-ie^{c+dx^2}\right)}{d^2}+\frac{iab\,\text{PolyLog}\left(2,ie^{c+dx^2}\right)}{d^2}+\frac{b^2x^2\tanh\left(c+dx^2\right)}{2d}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 324 vs. $2(119) = 238$.

Time = 1.94 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.72

$$
\int x^3\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx
$$
$$
=\frac{4b^2de^{2c}x^2+a^2d^2x^4+a^2d^2e^{2c}x^4+4iabdx^2\log\left(1-ie^{c+dx^2}\right)+4iabde^{2c}x^2\log\left(1-ie^{c+dx^2}\right)-4iabdx^2\log\left(\right.}{}
$$

```
[In] Integrate[x^3*(a + b*Sech[c + d*x^2])^2,x]
```

```
[Out] (4*b^2*d*E^(2*c)*x^2 + a^2*d^2*x^4 + a^2*d^2*E^(2*c)*x^4 + (4*I)*a*b*d*x^2*
Log[1 - I*E^(c + d*x^2)] + (4*I)*a*b*d*E^(2*c)*x^2*Log[1 - I*E^(c + d*x^2)]
 - (4*I)*a*b*d*x^2*Log[1 + I*E^(c + d*x^2)] - (4*I)*a*b*d*E^(2*c)*x^2*Log[1
 + I*E^(c + d*x^2)] - 2*b^2*Log[1 + E^(2*(c + d*x^2))] - 2*b^2*E^(2*c)*Log[
1 + E^(2*(c + d*x^2))] - (4*I)*a*b*(1 + E^(2*c))*PolyLog[2, (-I)*E^(c + d*x
^2)] + (4*I)*a*b*(1 + E^(2*c))*PolyLog[2, I*E^(c + d*x^2)] + 2*b^2*d*x^2*Se
ch[c]*Sech[c + d*x^2]*Sinh[d*x^2] + 2*b^2*d*E^(2*c)*x^2*Sech[c]*Sech[c + d*
x^2]*Sinh[d*x^2])/(4*d^2*(1 + E^(2*c)))
```

Maple [F]

$$\int x^3\left(a+b\ \mathrm{sech}\left(d\,x^2+c\right)\right)^2dx$$

```
[In] int(x^3*(a+b*sech(d*x^2+c))^2,x)
```

```
[Out] int(x^3*(a+b*sech(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 782 vs. 2(100) = 200.

Time = 0.28 (sec) , antiderivative size = 782, normalized size of antiderivative = 6.57

$$\int x^3\left(a+b\mathrm{sech}\left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{a^2d^2x^4+4\,b^2c+\left(a^2d^2x^4+4\,b^2dx^2+4\,b^2c\right)\cosh\left(dx^2+c\right)^2+2\left(a^2d^2x^4+4\,b^2dx^2+4\,b^2c\right)\cosh\left(dx^2+c\right)\mathrm{s}}{}$$

```
[In] integrate(x^3*(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*d^2*x^4 + 4*b^2*c + (a^2*d^2*x^4 + 4*b^2*d*x^2 + 4*b^2*c)*cosh(d*x
^2 + c)^2 + 2*(a^2*d^2*x^4 + 4*b^2*d*x^2 + 4*b^2*c)*cosh(d*x^2 + c)*sinh(d*
x^2 + c) + (a^2*d^2*x^4 + 4*b^2*d*x^2 + 4*b^2*c)*sinh(d*x^2 + c)^2 - 4*(-I*
a*b*cosh(d*x^2 + c)^2 - 2*I*a*b*cosh(d*x^2 + c)*sinh(d*x^2 + c) - I*a*b*sin
h(d*x^2 + c)^2 - I*a*b)*dilog(I*cosh(d*x^2 + c) + I*sinh(d*x^2 + c)) - 4*(I
*a*b*cosh(d*x^2 + c)^2 + 2*I*a*b*cosh(d*x^2 + c)*sinh(d*x^2 + c) + I*a*b*si
nh(d*x^2 + c)^2 + I*a*b)*dilog(-I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c)) - 2*
(2*I*a*b*c + (2*I*a*b*c + b^2)*cosh(d*x^2 + c)^2 + 2*(2*I*a*b*c + b^2)*cosh
(d*x^2 + c)*sinh(d*x^2 + c) + (2*I*a*b*c + b^2)*sinh(d*x^2 + c)^2 + b^2)*lo
g(cosh(d*x^2 + c) + sinh(d*x^2 + c) + I) - 2*(-2*I*a*b*c + (-2*I*a*b*c + b^
2)*cosh(d*x^2 + c)^2 + 2*(-2*I*a*b*c + b^2)*cosh(d*x^2 + c)*sinh(d*x^2 + c)
 + (-2*I*a*b*c + b^2)*sinh(d*x^2 + c)^2 + b^2)*log(cosh(d*x^2 + c) + sinh(d
*x^2 + c) - I) - 4*(I*a*b*d*x^2 + I*a*b*c + (I*a*b*d*x^2 + I*a*b*c)*cosh(d*
```

```
x^2 + c)^2 + 2*(I*a*b*d*x^2 + I*a*b*c)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (I
*a*b*d*x^2 + I*a*b*c)*sinh(d*x^2 + c)^2)*log(I*cosh(d*x^2 + c) + I*sinh(d*x
^2 + c) + 1) - 4*(-I*a*b*d*x^2 - I*a*b*c + (-I*a*b*d*x^2 - I*a*b*c)*cosh(d*
x^2 + c)^2 + 2*(-I*a*b*d*x^2 - I*a*b*c)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (
-I*a*b*d*x^2 - I*a*b*c)*sinh(d*x^2 + c)^2)*log(-I*cosh(d*x^2 + c) - I*sinh(
d*x^2 + c) + 1))/(d^2*cosh(d*x^2 + c)^2 + 2*d^2*cosh(d*x^2 + c)*sinh(d*x^2
+ c) + d^2*sinh(d*x^2 + c)^2 + d^2)
```

Sympy [F]

$$\int x^3 (a + b\mathrm{sech}(c + dx^2))^2\, dx = \int x^3 \left(a + b\,\mathrm{sech}\,(c + dx^2)\right)^2\, dx$$

```
[In] integrate(x**3*(a+b*sech(d*x**2+c))**2,x)
```

```
[Out] Integral(x**3*(a + b*sech(c + d*x**2))**2, x)
```

Maxima [F]

$$\int x^3 (a + b\mathrm{sech}(c + dx^2))^2\, dx = \int \left(b\,\mathrm{sech}\,(dx^2 + c) + a\right)^2 x^3\, dx$$

```
[In] integrate(x^3*(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*x^4 + 1/2*(2*x^2*e^(2*d*x^2 + 2*c)/(d*e^(2*d*x^2 + 2*c) + d) - log(
(e^(2*d*x^2 + 2*c) + 1)*e^(-2*c))/d^2)*b^2 + 4*a*b*integrate(x^3*e^(d*x^2 +
 c)/(e^(2*d*x^2 + 2*c) + 1), x)
```

Giac [F]

$$\int x^3 (a + b\mathrm{sech}(c + dx^2))^2\, dx = \int \left(b\,\mathrm{sech}\,(dx^2 + c) + a\right)^2 x^3\, dx$$

```
[In] integrate(x^3*(a+b*sech(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x^2 + c) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b\operatorname{sech}\left(c + dx^2\right)\right)^2 \, dx = \int x^3 \left(a + \frac{b}{\cosh\left(d\,x^2 + c\right)}\right)^2 \, dx$$

```
[In] int(x^3*(a + b/cosh(c + d*x^2))^2,x)
[Out] int(x^3*(a + b/cosh(c + d*x^2))^2, x)
```

3.11 $\int x^2(a+b\text{sech}(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^2\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx=\text{Int}\left(x^2\left(a+b\text{sech}\left(c+dx^2\right)\right)^2,x\right)$$

`[Out] Unintegrable(x^2*(a+b*sech(d*x^2+c))^2,x)`

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int x^2\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx=\int x^2\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx$$

`[In] Int[x^2*(a + b*Sech[c + d*x^2])^2,x]`

`[Out] Defer[Int][x^2*(a + b*Sech[c + d*x^2])^2, x]`

Rubi steps

$$\text{integral}=\int x^2\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx$$

Mathematica [N/A]

Not integrable

Time = 9.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx=\int x^2\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx$$

```
[In] Integrate[x^2*(a + b*Sech[c + d*x^2])^2,x]
[Out] Integrate[x^2*(a + b*Sech[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\,\operatorname{sech}\left(d\,x^2+c\right)\right)^2dx$$

```
[In] int(x^2*(a+b*sech(d*x^2+c))^2,x)
[Out] int(x^2*(a+b*sech(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x^2\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx=\int\left(b\operatorname{sech}\left(dx^2+c\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*sech(d*x^2 + c)^2 + 2*a*b*x^2*sech(d*x^2 + c) + a^2*x^2, x
)
```

Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int x^2\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx=\int x^2\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate(x**2*(a+b*sech(d*x**2+c))**2,x)
[Out] Integral(x**2*(a + b*sech(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 4.00

$$\int x^2\left(a+b\mathrm{sech}\left(c+dx^2\right)\right)^2\,dx=\int\left(b\,\mathrm{sech}\left(dx^2+c\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
[Out] 1/3*a^2*x^3 - b^2*x/(d*e^(2*d*x^2 + 2*c) + d) + integrate((4*a*b*d*x^2*e^(d
*x^2 + c) + b^2)/(d*e^(2*d*x^2 + 2*c) + d), x)
```

Giac [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2\left(a+b\mathrm{sech}\left(c+dx^2\right)\right)^2\,dx=\int\left(b\,\mathrm{sech}\left(dx^2+c\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*sech(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate((b*sech(d*x^2 + c) + a)^2*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^2\left(a+b\mathrm{sech}\left(c+dx^2\right)\right)^2\,dx=\int x^2\left(a+\frac{b}{\cosh\left(d\,x^2+c\right)}\right)^2\,dx$$

```
[In] int(x^2*(a + b/cosh(c + d*x^2))^2,x)
[Out] int(x^2*(a + b/cosh(c + d*x^2))^2, x)
```

3.12 $\int x(a+b\text{sech}(c+dx^2))^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int x\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx=\frac{a^2x^2}{2}+\frac{ab\arctan\left(\sinh\left(c+dx^2\right)\right)}{d}+\frac{b^2\tanh\left(c+dx^2\right)}{2d}$$

```
[Out] 1/2*a^2*x^2+a*b*arctan(sinh(d*x^2+c))/d+1/2*b^2*tanh(d*x^2+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5544, 3858, 3855, 3852, 8}

$$\int x\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx=\frac{a^2x^2}{2}+\frac{ab\arctan\left(\sinh\left(c+dx^2\right)\right)}{d}+\frac{b^2\tanh\left(c+dx^2\right)}{2d}$$

```
[In] Int[x*(a + b*Sech[c + d*x^2])^2,x]
[Out] (a^2*x^2)/2 + (a*b*ArcTan[Sinh[c + d*x^2]])/d + (b^2*Tanh[c + d*x^2])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (a+b\operatorname{sech}(c+dx))^2\,dx, x, x^2\right)\\
&= \frac{a^2x^2}{2}+(ab)\text{Subst}\left(\int \operatorname{sech}(c+dx)\,dx, x, x^2\right)+\frac{1}{2}b^2\text{Subst}\left(\int \operatorname{sech}^2(c+dx)\,dx, x, x^2\right)\\
&= \frac{a^2x^2}{2}+\frac{ab\arctan\left(\sinh\left(c+dx^2\right)\right)}{d}+\frac{(ib^2)\,\text{Subst}\left(\int 1\,dx, x, -i\tanh\left(c+dx^2\right)\right)}{2d}\\
&= \frac{a^2x^2}{2}+\frac{ab\arctan\left(\sinh\left(c+dx^2\right)\right)}{d}+\frac{b^2\tanh\left(c+dx^2\right)}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$
\int x\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)^2\,dx=\frac{1}{2}\left(a^2x^2+\frac{2ab\arctan\left(\sinh\left(c+dx^2\right)\right)}{d}+\frac{b^2\tanh\left(c+dx^2\right)}{d}\right)
$$

```
[In] Integrate[x*(a + b*Sech[c + d*x^2])^2,x]
[Out] (a^2*x^2 + (2*a*b*ArcTan[Sinh[c + d*x^2]])/d + (b^2*Tanh[c + d*x^2])/d)/2
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result
parts	$\frac{a^2x^2}{2}+\frac{ab\arctan(\sinh(d\,x^2+c))}{d}+\frac{b^2\tanh(d\,x^2+c)}{2d}$
derivativedivides	$\frac{a^2(d\,x^2+c)+4ab\arctan\left(\mathrm{e}^{d\,x^2+c}\right)+b^2\tanh(d\,x^2+c)}{2d}$
default	$\frac{a^2(d\,x^2+c)+4ab\arctan\left(\mathrm{e}^{d\,x^2+c}\right)+b^2\tanh(d\,x^2+c)}{2d}$
risch	$\frac{a^2x^2}{2}-\frac{b^2}{d\left(1+\mathrm{e}^{2d\,x^2+2c}\right)}+\frac{iba\ln\left(\mathrm{e}^{d\,x^2+c}+i\right)}{d}-\frac{iba\ln\left(\mathrm{e}^{d\,x^2+c}-i\right)}{d}$
parallelrisch	$\frac{a^2d\,x^2\cosh(d\,x^2+c)-2i\cosh(d\,x^2+c)\ln\left(\tanh\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)-i\right)ab+2i\cosh(d\,x^2+c)\ln\left(\tanh\left(\frac{d\,x^2}{2}+\frac{c}{2}\right)+i\right)ab+b^2\sinh(d\,x}{2d\cosh(d\,x^2+c)}$

```
[In] int(x*(a+b*sech(d*x^2+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*a^2*x^2+a*b*arctan(sinh(d*x^2+c))/d+1/2*b^2*tanh(d*x^2+c)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(40) = 80.

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 4.41

$$\int x\left(a+b\mathrm{sech}\left(c+dx^2\right)\right)^2\,dx$$

$$=\frac{a^2dx^2\cosh\left(dx^2+c\right)^2+2\,a^2dx^2\cosh\left(dx^2+c\right)\sinh\left(dx^2+c\right)+a^2dx^2\sinh\left(dx^2+c\right)^2+a^2dx^2-2\,b^2+4\,(}{2\left(d\cosh\left(dx^2+c\right)^2+2\,d\cosh\left(\right.\right.}$$

```
[In] integrate(x*(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")
[Out] 1/2*(a^2*d*x^2*cosh(d*x^2 + c)^2 + 2*a^2*d*x^2*cosh(d*x^2 + c)*sinh(d*x^2 +
 c) + a^2*d*x^2*sinh(d*x^2 + c)^2 + a^2*d*x^2 - 2*b^2 + 4*(a*b*cosh(d*x^2 +
 c)^2 + 2*a*b*cosh(d*x^2 + c)*sinh(d*x^2 + c) + a*b*sinh(d*x^2 + c)^2 + a*b
)*arctan(cosh(d*x^2 + c) + sinh(d*x^2 + c)))/(d*cosh(d*x^2 + c)^2 + 2*d*cos
h(d*x^2 + c)*sinh(d*x^2 + c) + d*sinh(d*x^2 + c)^2 + d)
```

Sympy [F]

$$\int x\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx=\int x\left(a+b\,\text{sech}\left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate(x*(a+b*sech(d*x**2+c))**2,x)
```

```
[Out] Integral(x*(a + b*sech(c + d*x**2))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int x\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx=\frac{1}{2}a^2x^2+\frac{ab\arctan\left(\sinh\left(dx^2+c\right)\right)}{d}+\frac{b^2}{d(e^{(-2\,dx^2-2\,c)}+1)}$$

```
[In] integrate(x*(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + a*b*arctan(sinh(d*x^2 + c))/d + b^2/(d*(e^(-2*d*x^2 - 2*c) +
1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int x\left(a+b\text{sech}\left(c+dx^2\right)\right)^2\,dx=\frac{(dx^2+c)a^2}{2\,d}+\frac{2\,ab\arctan\left(e^{(dx^2+c)}\right)}{d}-\frac{b^2}{d(e^{(2\,dx^2+2\,c)}+1)}$$

```
[In] integrate(x*(a+b*sech(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(d*x^2 + c)*a^2/d + 2*a*b*arctan(e^(d*x^2 + c))/d - b^2/(d*(e^(2*d*x^2
+ 2*c) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.75

$$\int x\left(a+b\mathrm{sech}\left(c+dx^2\right)\right)^2\,dx=\frac{a^2\,x^2}{2}+\frac{2\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{d\,x^2}\,\mathrm{e}^c\,\sqrt{d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{d^2}}-\frac{b^2}{d\,\left(\mathrm{e}^{2\,d\,x^2+2\,c}+1\right)}$$

```
[In] int(x*(a + b/cosh(c + d*x^2))^2,x)
```

```
[Out] (a^2*x^2)/2 + (2*atan((a*b*exp(d*x^2)*exp(c)*(d^2)^(1/2))/(d*(a^2*b^2)^(1/2
)))*(a^2*b^2)^(1/2))/(d^2)^(1/2) - b^2/(d*(exp(2*c + 2*d*x^2) + 1))
```

3.13 $\int \frac{\left(a+b\,\mathbf{sech}\left(c+dx^2\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\,\mathrm{sech}(c+dx^2))^2}{x}\,dx = \mathrm{Int}\left(\frac{(a+b\,\mathrm{sech}(c+dx^2))^2}{x},x\right)$$

```
[Out] Unintegrable((a+b*sech(d*x^2+c))^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\,\mathrm{sech}(c+dx^2))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{sech}(c+dx^2))^2}{x}\,dx$$

```
[In] Int[(a + b*Sech[c + d*x^2])^2/x,x]
```

```
[Out] Defer[Int][(a + b*Sech[c + d*x^2])^2/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\,\mathrm{sech}(c+dx^2))^2}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 27.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\text{sech}(c+dx^2))^2}{x}\,dx=\int \frac{(a+b\text{sech}(c+dx^2))^2}{x}\,dx$$

```
[In] Integrate[(a + b*Sech[c + d*x^2])^2/x,x]
[Out] Integrate[(a + b*Sech[c + d*x^2])^2/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\,\operatorname{sech}\left(d\,x^{2}+c\right)\right)^{2}}{x}dx$$

```
[In] int((a+b*sech(d*x^2+c))^2/x,x)
[Out] int((a+b*sech(d*x^2+c))^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(a+b\text{sech}(c+dx^2))^2}{x}\,dx=\int \frac{(b\,\operatorname{sech}\left(dx^2+c\right)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*sech(d*x^2+c))^2/x,x, algorithm="fricas")
[Out] integral((b^2*sech(d*x^2 + c)^2 + 2*a*b*sech(d*x^2 + c) + a^2)/x, x)
```

Sympy [N/A]

Not integrable

Time = 6.61 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\text{sech}(c+dx^2))^2}{x}\,dx=\int \frac{(a+b\,\operatorname{sech}\left(c+dx^2\right))^2}{x}\,dx$$

```
[In] integrate((a+b*sech(d*x**2+c))**2/x,x)
[Out] Integral((a + b*sech(c + d*x**2))**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 4.78

$$\int \frac{(a+b\mathrm{sech}(c+dx^2))^2}{x}\,dx = \int \frac{(b\,\mathrm{sech}\,(dx^2+c)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*sech(d*x^2+c))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) - b^2/(d*x^2*e^(2*d*x^2 + 2*c) + d*x^2) + integrate(2*(2*a*b*d*x
^2*e^(d*x^2 + c) - b^2)/(d*x^3*e^(2*d*x^2 + 2*c) + d*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\mathrm{sech}(c+dx^2))^2}{x}\,dx = \int \frac{(b\,\mathrm{sech}\,(dx^2+c)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*sech(d*x^2+c))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x^2 + c) + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{(a+b\mathrm{sech}(c+dx^2))^2}{x}\,dx = \int \frac{\left(a+\frac{b}{\cosh(d\,x^2+c)}\right)^2}{x}\,dx$$

```
[In] int((a + b/cosh(c + d*x^2))^2/x,x)
```

```
[Out] int((a + b/cosh(c + d*x^2))^2/x, x)
```

3.14 $\int \frac{\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\,\mathrm{sech}(c+dx^2))^2}{x^2}\,dx = \mathrm{Int}\left(\frac{(a+b\,\mathrm{sech}(c+dx^2))^2}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*sech(d*x^2+c))^2/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\,\mathrm{sech}(c+dx^2))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{sech}(c+dx^2))^2}{x^2}\,dx$$

```
[In] Int[(a + b*Sech[c + d*x^2])^2/x^2,x]
```

```
[Out] Defer[Int][(a + b*Sech[c + d*x^2])^2/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\,\mathrm{sech}(c+dx^2))^2}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\text{sech}(c+dx^2))^2}{x^2}\,dx = \int \frac{(a+b\text{sech}(c+dx^2))^2}{x^2}\,dx$$

```
[In] Integrate[(a + b*Sech[c + d*x^2])^2/x^2,x]
[Out] Integrate[(a + b*Sech[c + d*x^2])^2/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\ \text{sech}\left(d\,x^{2}+c\right)\right)^{2}}{x^{2}}dx$$

```
[In] int((a+b*sech(d*x^2+c))^2/x^2,x)
[Out] int((a+b*sech(d*x^2+c))^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(a+b\text{sech}(c+dx^2))^2}{x^2}\,dx = \int \frac{\left(b\,\text{sech}\left(dx^2+c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sech(d*x^2+c))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*sech(d*x^2 + c)^2 + 2*a*b*sech(d*x^2 + c) + a^2)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\text{sech}(c+dx^2))^2}{x^2}\,dx = \int \frac{\left(a+b\,\text{sech}\left(c+dx^2\right)\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sech(d*x**2+c))**2/x**2,x)
[Out] Integral((a + b*sech(c + d*x**2))**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.83

$$\int \frac{(a+b\mathrm{sech}(c+dx^2))^2}{x^2}\,dx = \int \frac{\left(b\,\mathrm{sech}\left(dx^2+c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sech(d*x^2+c))^2/x^2,x, algorithm="maxima")
```

```
[Out] -b^2/(d*x^3*e^(2*d*x^2 + 2*c) + d*x^3) - a^2/x + integrate((4*a*b*d*x^2*e^(
d*x^2 + c) - 3*b^2)/(d*x^4*e^(2*d*x^2 + 2*c) + d*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\mathrm{sech}(c+dx^2))^2}{x^2}\,dx = \int \frac{\left(b\,\mathrm{sech}\left(dx^2+c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sech(d*x^2+c))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x^2 + c) + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{(a+b\mathrm{sech}(c+dx^2))^2}{x^2}\,dx = \int \frac{\left(a+\frac{b}{\cosh\left(d\,x^2+c\right)}\right)^2}{x^2}\,dx$$

```
[In] int((a + b/cosh(c + d*x^2))^2/x^2,x)
```

```
[Out] int((a + b/cosh(c + d*x^2))^2/x^2, x)
```

3.15 $\int x\text{sech}^7(a+bx^2)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int x\text{sech}^7(a+bx^2)\,dx=\frac{5\arctan(\sinh(a+bx^2))}{32b}+\frac{5\text{sech}(a+bx^2)\tanh(a+bx^2)}{32b}+\frac{5\text{sech}^3(a+bx^2)\tanh(a+bx^2)}{48b}+\frac{\text{sech}^5(a+bx^2)\tanh(a+bx^2)}{12b}$$

```
[Out] 5/32*arctan(sinh(b*x^2+a))/b+5/32*sech(b*x^2+a)*tanh(b*x^2+a)/b+5/48*sech(b
*x^2+a)^3*tanh(b*x^2+a)/b+1/12*sech(b*x^2+a)^5*tanh(b*x^2+a)/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5544, 3853, 3855}

$$\int x\text{sech}^7(a+bx^2)\,dx=\frac{5\arctan(\sinh(a+bx^2))}{32b}+\frac{\tanh(a+bx^2)\text{sech}^5(a+bx^2)}{12b}+\frac{5\tanh(a+bx^2)\text{sech}^3(a+bx^2)}{48b}+\frac{5\tanh(a+bx^2)\text{sech}(a+bx^2)}{32b}$$

```
[In] Int[x*Sech[a + b*x^2]^7,x]
```

```
[Out] (5*ArcTan[Sinh[a + b*x^2]])/(32*b) + (5*Sech[a + b*x^2]*Tanh[a + b*x^2])/(3
2*b) + (5*Sech[a + b*x^2]^3*Tanh[a + b*x^2])/(48*b) + (Sech[a + b*x^2]^5*Ta
nh[a + b*x^2])/(12*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \text{sech}^7(a+bx)\,dx, x, x^2\right)\\
&= \frac{\text{sech}^5(a+bx^2)\tanh(a+bx^2)}{12b} + \frac{5}{12}\text{Subst}\left(\int \text{sech}^5(a+bx)\,dx, x, x^2\right)\\
&= \frac{5\text{sech}^3(a+bx^2)\tanh(a+bx^2)}{48b} + \frac{\text{sech}^5(a+bx^2)\tanh(a+bx^2)}{12b}\\
&\quad + \frac{5}{16}\text{Subst}\left(\int \text{sech}^3(a+bx)\,dx, x, x^2\right)\\
&= \frac{5\text{sech}(a+bx^2)\tanh(a+bx^2)}{32b} + \frac{5\text{sech}^3(a+bx^2)\tanh(a+bx^2)}{48b}\\
&\quad + \frac{\text{sech}^5(a+bx^2)\tanh(a+bx^2)}{12b} + \frac{5}{32}\text{Subst}\left(\int \text{sech}(a+bx)\,dx, x, x^2\right)\\
&= \frac{5\arctan(\sinh(a+bx^2))}{32b} + \frac{5\text{sech}(a+bx^2)\tanh(a+bx^2)}{32b}\\
&\quad + \frac{5\text{sech}^3(a+bx^2)\tanh(a+bx^2)}{48b} + \frac{\text{sech}^5(a+bx^2)\tanh(a+bx^2)}{12b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int x\text{sech}^7\left(a+bx^2\right)\,dx=\frac{5\arctan\left(\sinh\left(a+bx^2\right)\right)}{32b}+\frac{5\text{sech}(a+bx^2)\tanh\left(a+bx^2\right)}{32b}+\frac{5\text{sech}^3(a+bx^2)\tanh\left(a+bx^2\right)}{48b}+\frac{\text{sech}^5(a+bx^2)\tanh\left(a+bx^2\right)}{12b}$$

```
[In] Integrate[x*Sech[a + b*x^2]^7,x]
```

```
[Out] (5*ArcTan[Sinh[a + b*x^2]])/(32*b) + (5*Sech[a + b*x^2]*Tanh[a + b*x^2])/(3
2*b) + (5*Sech[a + b*x^2]^3*Tanh[a + b*x^2])/(48*b) + (Sech[a + b*x^2]^5*Ta
nh[a + b*x^2])/(12*b)
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{\left(\frac{\operatorname{sech}\left(b\,x^{2}+a\right)^{5}}{6}+\frac{5\operatorname{sech}\left(b\,x^{2}+a\right)^{3}}{24}+\frac{5\operatorname{sech}\left(b\,x^{2}+a\right)}{16}\right)\tanh\left(b\,x^{2}+a\right)+\frac{5\arctan\left({\mathrm e}^{b\,x^{2}+a}\right)}{8}}{2b}$
default	$\frac{\left(\frac{\operatorname{sech}\left(b\,x^{2}+a\right)^{5}}{6}+\frac{5\operatorname{sech}\left(b\,x^{2}+a\right)^{3}}{24}+\frac{5\operatorname{sech}\left(b\,x^{2}+a\right)}{16}\right)\tanh\left(b\,x^{2}+a\right)+\frac{5\arctan\left({\mathrm e}^{b\,x^{2}+a}\right)}{8}}{2b}$
risch	$\frac{{\mathrm e}^{b\,x^{2}+a}\left(15\,{\mathrm e}^{10b\,x^{2}+10a}+85\,{\mathrm e}^{8b\,x^{2}+8a}+198\,{\mathrm e}^{6b\,x^{2}+6a}-198\,{\mathrm e}^{4b\,x^{2}+4a}-85\,{\mathrm e}^{2b\,x^{2}+2a}-15\right)}{48b\left({\mathrm e}^{2b\,x^{2}+2a}+1\right)^{6}}+\frac{5i\ln\left({\mathrm e}^{b\,x^{2}+a}+i\right)}{32b}-\frac{5i\ln\left({\mathrm e}\right.}{}$
parallelrisch	$\frac{15i\left(-10-\cosh\left(6b\,x^{2}+6a\right)-6\cosh\left(4b\,x^{2}+4a\right)-15\cosh\left(2b\,x^{2}+2a\right)\right)\ln\left(\tanh\left(\frac{b\,x^{2}}{2}+\frac{a}{2}\right)-i\right)+15i\left(10+\cosh\left(6b\,x^{2}+6a\right)+6\,\mathrm{c}\right.}{96b\left(10+\cosh\left(6b\,x^{2}+6a\right)+6\cosh\right.}$

```
[In] int(x*sech(b*x^2+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*((1/6*sech(b*x^2+a)^5+5/24*sech(b*x^2+a)^3+5/16*sech(b*x^2+a))*tanh(b
*x^2+a)+5/8*arctan(exp(b*x^2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1918 vs. $2(82) = 164$.

Time = 0.26 (sec) , antiderivative size = 1918, normalized size of antiderivative = 21.31

$$\int x\text{sech}^7\left(a + bx^2\right) \, dx = \text{Too large to display}$$

```
[In] integrate(x*sech(b*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/48*(15*cosh(b*x^2 + a)^11 + 165*cosh(b*x^2 + a)*sinh(b*x^2 + a)^10 + 15*s
inh(b*x^2 + a)^11 + 5*(165*cosh(b*x^2 + a)^2 + 17)*sinh(b*x^2 + a)^9 + 85*c
osh(b*x^2 + a)^9 + 45*(55*cosh(b*x^2 + a)^3 + 17*cosh(b*x^2 + a))*sinh(b*x^
2 + a)^8 + 18*(275*cosh(b*x^2 + a)^4 + 170*cosh(b*x^2 + a)^2 + 11)*sinh(b*x
^2 + a)^7 + 198*cosh(b*x^2 + a)^7 + 42*(165*cosh(b*x^2 + a)^5 + 170*cosh(b*
x^2 + a)^3 + 33*cosh(b*x^2 + a))*sinh(b*x^2 + a)^6 + 18*(385*cosh(b*x^2 + a
)^6 + 595*cosh(b*x^2 + a)^4 + 231*cosh(b*x^2 + a)^2 - 11)*sinh(b*x^2 + a)^5
 - 198*cosh(b*x^2 + a)^5 + 90*(55*cosh(b*x^2 + a)^7 + 119*cosh(b*x^2 + a)^5
 + 77*cosh(b*x^2 + a)^3 - 11*cosh(b*x^2 + a))*sinh(b*x^2 + a)^4 + 5*(495*co
sh(b*x^2 + a)^8 + 1428*cosh(b*x^2 + a)^6 + 1386*cosh(b*x^2 + a)^4 - 396*cos
h(b*x^2 + a)^2 - 17)*sinh(b*x^2 + a)^3 - 85*cosh(b*x^2 + a)^3 + 3*(275*cosh
(b*x^2 + a)^9 + 1020*cosh(b*x^2 + a)^7 + 1386*cosh(b*x^2 + a)^5 - 660*cosh(
b*x^2 + a)^3 - 85*cosh(b*x^2 + a))*sinh(b*x^2 + a)^2 + 15*(cosh(b*x^2 + a)^
12 + 12*cosh(b*x^2 + a)*sinh(b*x^2 + a)^11 + sinh(b*x^2 + a)^12 + 6*(11*cos
h(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)^10 + 6*cosh(b*x^2 + a)^10 + 20*(11*cosh
(b*x^2 + a)^3 + 3*cosh(b*x^2 + a))*sinh(b*x^2 + a)^9 + 15*(33*cosh(b*x^2 +
a)^4 + 18*cosh(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)^8 + 15*cosh(b*x^2 + a)^8 +
 24*(33*cosh(b*x^2 + a)^5 + 30*cosh(b*x^2 + a)^3 + 5*cosh(b*x^2 + a))*sinh(
b*x^2 + a)^7 + 4*(231*cosh(b*x^2 + a)^6 + 315*cosh(b*x^2 + a)^4 + 105*cosh(
b*x^2 + a)^2 + 5)*sinh(b*x^2 + a)^6 + 20*cosh(b*x^2 + a)^6 + 24*(33*cosh(b*
x^2 + a)^7 + 63*cosh(b*x^2 + a)^5 + 35*cosh(b*x^2 + a)^3 + 5*cosh(b*x^2 + a
))*sinh(b*x^2 + a)^5 + 15*(33*cosh(b*x^2 + a)^8 + 84*cosh(b*x^2 + a)^6 + 70
*cosh(b*x^2 + a)^4 + 20*cosh(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)^4 + 15*cosh(
b*x^2 + a)^4 + 20*(11*cosh(b*x^2 + a)^9 + 36*cosh(b*x^2 + a)^7 + 42*cosh(b*
x^2 + a)^5 + 20*cosh(b*x^2 + a)^3 + 3*cosh(b*x^2 + a))*sinh(b*x^2 + a)^3 +
6*(11*cosh(b*x^2 + a)^10 + 45*cosh(b*x^2 + a)^8 + 70*cosh(b*x^2 + a)^6 + 50
*cosh(b*x^2 + a)^4 + 15*cosh(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)^2 + 6*cosh(b
*x^2 + a)^2 + 12*(cosh(b*x^2 + a)^11 + 5*cosh(b*x^2 + a)^9 + 10*cosh(b*x^2
+ a)^7 + 10*cosh(b*x^2 + a)^5 + 5*cosh(b*x^2 + a)^3 + cosh(b*x^2 + a))*sinh
(b*x^2 + a) + 1)*arctan(cosh(b*x^2 + a) + sinh(b*x^2 + a)) + 3*(55*cosh(b*x
^2 + a)^10 + 255*cosh(b*x^2 + a)^8 + 462*cosh(b*x^2 + a)^6 - 330*cosh(b*x^2
 + a)^4 - 85*cosh(b*x^2 + a)^2 - 5)*sinh(b*x^2 + a) - 15*cosh(b*x^2 + a))/(
b*cosh(b*x^2 + a)^12 + 12*b*cosh(b*x^2 + a)*sinh(b*x^2 + a)^11 + b*sinh(b*x
^2 + a)^12 + 6*b*cosh(b*x^2 + a)^10 + 6*(11*b*cosh(b*x^2 + a)^2 + b)*sinh(b
*x^2 + a)^10 + 20*(11*b*cosh(b*x^2 + a)^3 + 3*b*cosh(b*x^2 + a))*sinh(b*x^2
```

```
 + a)^9 + 15*b*cosh(b*x^2 + a)^8 + 15*(33*b*cosh(b*x^2 + a)^4 + 18*b*cosh(b
*x^2 + a)^2 + b)*sinh(b*x^2 + a)^8 + 24*(33*b*cosh(b*x^2 + a)^5 + 30*b*cosh
(b*x^2 + a)^3 + 5*b*cosh(b*x^2 + a))*sinh(b*x^2 + a)^7 + 20*b*cosh(b*x^2 +
a)^6 + 4*(231*b*cosh(b*x^2 + a)^6 + 315*b*cosh(b*x^2 + a)^4 + 105*b*cosh(b*
x^2 + a)^2 + 5*b)*sinh(b*x^2 + a)^6 + 24*(33*b*cosh(b*x^2 + a)^7 + 63*b*cos
h(b*x^2 + a)^5 + 35*b*cosh(b*x^2 + a)^3 + 5*b*cosh(b*x^2 + a))*sinh(b*x^2 +
 a)^5 + 15*b*cosh(b*x^2 + a)^4 + 15*(33*b*cosh(b*x^2 + a)^8 + 84*b*cosh(b*x
^2 + a)^6 + 70*b*cosh(b*x^2 + a)^4 + 20*b*cosh(b*x^2 + a)^2 + b)*sinh(b*x^2
 + a)^4 + 20*(11*b*cosh(b*x^2 + a)^9 + 36*b*cosh(b*x^2 + a)^7 + 42*b*cosh(b
*x^2 + a)^5 + 20*b*cosh(b*x^2 + a)^3 + 3*b*cosh(b*x^2 + a))*sinh(b*x^2 + a)
^3 + 6*b*cosh(b*x^2 + a)^2 + 6*(11*b*cosh(b*x^2 + a)^10 + 45*b*cosh(b*x^2 +
 a)^8 + 70*b*cosh(b*x^2 + a)^6 + 50*b*cosh(b*x^2 + a)^4 + 15*b*cosh(b*x^2 +
 a)^2 + b)*sinh(b*x^2 + a)^2 + 12*(b*cosh(b*x^2 + a)^11 + 5*b*cosh(b*x^2 +
a)^9 + 10*b*cosh(b*x^2 + a)^7 + 10*b*cosh(b*x^2 + a)^5 + 5*b*cosh(b*x^2 + a
)^3 + b*cosh(b*x^2 + a))*sinh(b*x^2 + a) + b)
```

Sympy [F]

$$\int x\text{sech}^7(a+bx^2)\,dx=\int x\,\text{sech}^7\left(a+bx^2\right)dx$$

```
[In] integrate(x*sech(b*x**2+a)**7,x)
[Out] Integral(x*sech(a + b*x**2)**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(82) = 164.

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.02

$$\int x\text{sech}^7(a+bx^2)\,dx=-\frac{5\arctan\left(e^{(-bx^2-a)}\right)}{16\,b}+\frac{15\,e^{(-bx^2-a)}+85\,e^{(-3\,bx^2-3\,a)}+198\,e^{(-5\,bx^2-5\,a)}-198\,e^{(-7\,bx^2-7\,a)}-85\,e^{(-9\,bx^2-9\,a)}-15\,e^{(-11\,bx^2-11\,a)}}{48\,b(6\,e^{(-2\,bx^2-2\,a)}+15\,e^{(-4\,bx^2-4\,a)}+20\,e^{(-6\,bx^2-6\,a)}+15\,e^{(-8\,bx^2-8\,a)}+6\,e^{(-10\,bx^2-10\,a)}+e^{(-12\,bx^2-12\,a)}+1)}$$

```
[In] integrate(x*sech(b*x^2+a)^7,x, algorithm="maxima")
[Out] -5/16*arctan(e^(-b*x^2 - a))/b + 1/48*(15*e^(-b*x^2 - a) + 85*e^(-3*b*x^2 -
 3*a) + 198*e^(-5*b*x^2 - 5*a) - 198*e^(-7*b*x^2 - 7*a) - 85*e^(-9*b*x^2 -
9*a) - 15*e^(-11*b*x^2 - 11*a))/(b*(6*e^(-2*b*x^2 - 2*a) + 15*e^(-4*b*x^2 -
 4*a) + 20*e^(-6*b*x^2 - 6*a) + 15*e^(-8*b*x^2 - 8*a) + 6*e^(-10*b*x^2 - 10
*a) + e^(-12*b*x^2 - 12*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62

$$\int x\operatorname{sech}^7(a+bx^2)\,dx$$
$$=\frac{5\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2bx^2+2a)}-1\right)e^{(-bx^2-a)}\right)\right)}{64b}$$
$$+\frac{15\left(e^{(bx^2+a)}-e^{(-bx^2-a)}\right)^5+160\left(e^{(bx^2+a)}-e^{(-bx^2-a)}\right)^3+528e^{(bx^2+a)}-528e^{(-bx^2-a)}}{48\left(\left(e^{(bx^2+a)}-e^{(-bx^2-a)}\right)^2+4\right)^3b}$$

```
[In] integrate(x*sech(b*x^2+a)^7,x, algorithm="giac")
```

```
[Out] 5/64*(pi + 2*arctan(1/2*(e^(2*b*x^2 + 2*a) - 1)*e^(-b*x^2 - a)))/b + 1/48*(
15*(e^(b*x^2 + a) - e^(-b*x^2 - a))^5 + 160*(e^(b*x^2 + a) - e^(-b*x^2 - a)
)^3 + 528*e^(b*x^2 + a) - 528*e^(-b*x^2 - a))/(((e^(b*x^2 + a) - e^(-b*x^2
- a))^2 + 4)^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 395, normalized size of antiderivative = 4.39

$$\int x\operatorname{sech}^7(a+bx^2)\,dx=\frac{5\,\mathrm{atan}\left(\frac{\mathrm{e}^a\,\mathrm{e}^{b\,x^2}\,\sqrt{b^2}}{b}\right)}{16\,\sqrt{b^2}}$$
$$-\frac{8\,\mathrm{e}^{3\,b\,x^2+3\,a}}{3\,b\,\left(5\,\mathrm{e}^{2\,b\,x^2+2\,a}+10\,\mathrm{e}^{4\,b\,x^2+4\,a}+10\,\mathrm{e}^{6\,b\,x^2+6\,a}+5\,\mathrm{e}^{8\,b\,x^2+8\,a}+\mathrm{e}^{10\,b\,x^2+10\,a}+1\right)}$$
$$-\frac{\mathrm{e}^{b\,x^2+a}}{b\,\left(4\,\mathrm{e}^{2\,b\,x^2+2\,a}+6\,\mathrm{e}^{4\,b\,x^2+4\,a}+4\,\mathrm{e}^{6\,b\,x^2+6\,a}+\mathrm{e}^{8\,b\,x^2+8\,a}+1\right)}$$
$$+\frac{5\,\mathrm{e}^{b\,x^2+a}}{24\,b\,\left(2\,\mathrm{e}^{2\,b\,x^2+2\,a}+\mathrm{e}^{4\,b\,x^2+4\,a}+1\right)}$$
$$-\frac{16\,\mathrm{e}^{5\,b\,x^2+5\,a}}{3\,b\,\left(6\,\mathrm{e}^{2\,b\,x^2+2\,a}+15\,\mathrm{e}^{4\,b\,x^2+4\,a}+20\,\mathrm{e}^{6\,b\,x^2+6\,a}+15\,\mathrm{e}^{8\,b\,x^2+8\,a}+6\,\mathrm{e}^{10\,b\,x^2+10\,a}+\mathrm{e}^{12\,b\,x^2+12\,a}+1\right)}$$
$$+\frac{\mathrm{e}^{b\,x^2+a}}{6\,b\,\left(3\,\mathrm{e}^{2\,b\,x^2+2\,a}+3\,\mathrm{e}^{4\,b\,x^2+4\,a}+\mathrm{e}^{6\,b\,x^2+6\,a}+1\right)}+\frac{5\,\mathrm{e}^{b\,x^2+a}}{16\,b\,\left(\mathrm{e}^{2\,b\,x^2+2\,a}+1\right)}$$

```
[In] int(x/cosh(a + b*x^2)^7,x)
```

```
[Out] (5*atan((exp(a)*exp(b*x^2)*(b^2)^(1/2))/b))/(16*(b^2)^(1/2)) - (8*exp(3*a +
 3*b*x^2))/(3*b*(5*exp(2*a + 2*b*x^2) + 10*exp(4*a + 4*b*x^2) + 10*exp(6*a
```

```
+ 6*b*x^2) + 5*exp(8*a + 8*b*x^2) + exp(10*a + 10*b*x^2) + 1)) - exp(a + b*
x^2)/(b*(4*exp(2*a + 2*b*x^2) + 6*exp(4*a + 4*b*x^2) + 4*exp(6*a + 6*b*x^2)
 + exp(8*a + 8*b*x^2) + 1)) + (5*exp(a + b*x^2))/(24*b*(2*exp(2*a + 2*b*x^2
) + exp(4*a + 4*b*x^2) + 1)) - (16*exp(5*a + 5*b*x^2))/(3*b*(6*exp(2*a + 2*
b*x^2) + 15*exp(4*a + 4*b*x^2) + 20*exp(6*a + 6*b*x^2) + 15*exp(8*a + 8*b*x
^2) + 6*exp(10*a + 10*b*x^2) + exp(12*a + 12*b*x^2) + 1)) + exp(a + b*x^2)/
(6*b*(3*exp(2*a + 2*b*x^2) + 3*exp(4*a + 4*b*x^2) + exp(6*a + 6*b*x^2) + 1)
) + (5*exp(a + b*x^2))/(16*b*(exp(2*a + 2*b*x^2) + 1))
```

3.16 $\int \frac{x^5}{a+b\,\text{sech}(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 349

$$\int \frac{x^5}{a+b\,\text{sech}(c+dx^2)}\,dx = \frac{x^6}{6a} - \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{b\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{b\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$

```
[Out] 1/6*x^6/a-1/2*b*x^4*ln(1+a*exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a/d/(-a^2+b^2
)^(1/2)+1/2*b*x^4*ln(1+a*exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a/d/(-a^2+b^2)^
(1/2)-b*x^2*polylog(2,-a*exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2
)^(1/2)+b*x^2*polylog(2,-a*exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b
^2)^(1/2)+b*polylog(3,-a*exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2
)^(1/2)-b*polylog(3,-a*exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used

$= \{5544, 4276, 3401, 2296, 2221, 2611, 2320, 6724\}$

$$\int \frac{x^5}{a+b\text{sech}\left(c+dx^2\right)}\,dx = \frac{b\,\text{PolyLog}\left(3,-\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{b\,\text{PolyLog}\left(3,-\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{bx^4\log\left(\frac{ae^{c+dx^2}}{b-\sqrt{b^2-a^2}}+1\right)}{2ad\sqrt{b^2-a^2}} + \frac{bx^4\log\left(\frac{ae^{c+dx^2}}{\sqrt{b^2-a^2}+b}+1\right)}{2ad\sqrt{b^2-a^2}} + \frac{x^6}{6a}$$

```
[In] Int[x^5/(a + b*Sech[c + d*x^2]),x]
```

```
[Out] x^6/(6*a) - (b*x^4*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2])])/(2*a*
Sqrt[-a^2 + b^2]*d) + (b*x^4*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2
])])/(2*a*Sqrt[-a^2 + b^2]*d) - (b*x^2*PolyLog[2, -((a*E^(c + d*x^2))/(b -
Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (b*x^2*PolyLog[2, -((a*E^(c
 + d*x^2))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (b*PolyLog[
3, -((a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) -
 (b*PolyLog[3, -((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 +
 b^2]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+b\text{sech}(c+dx)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{x^2}{a}-\frac{bx^2}{a(b+a\cosh(c+dx))}\right)\,dx, x, x^2\right) \\
&= \frac{x^6}{6a}-\frac{b\text{Subst}\left(\int \frac{x^2}{b+a\cosh(c+dx)}\,dx, x, x^2\right)}{2a}
\end{aligned}$$

$$= \frac{x^6}{6a} - \frac{b\text{Subst}\left(\int \frac{e^{c+dx}x^2}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^2\right)}{a}$$

$$= \frac{x^6}{6a} - \frac{b\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{\sqrt{-a^2+b^2}} + \frac{b\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{\sqrt{-a^2+b^2}}$$

$$= \frac{x^6}{6a} - \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{b\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, x^2\right)}{a\sqrt{-a^2+b^2}d} - \frac{b\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, x^2\right)}{a\sqrt{-a^2+b^2}d}$$

$$= \frac{x^6}{6a} - \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{b\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, x^2\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{b\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, x^2\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$= \frac{x^6}{6a} - \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{b\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{c+dx^2}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{b\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{c+dx^2}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$= \frac{x^6}{6a} - \frac{bx^4 \log\left(1 + \frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{bx^4 \log\left(1 + \frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{b \operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{b \operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{a + b\text{sech}\,(c + dx^2)}\,dx = \frac{\sqrt{-a^2+b^2}d^3x^6 - 3bd^2x^4 \log\left(1 + \frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right) + 3bd^2x^4 \log\left(1 + \frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right) - 6bdx^2 \operatorname{PolyLog}\left(2, \frac{ae^{c+dx^2}}{-b+\sqrt{-a^2+b^2}}\right) + 6bdx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right) + 6b \operatorname{PolyLog}\left(3, \frac{ae^{c+dx^2}}{-b+\sqrt{-a^2+b^2}}\right) - 6b \operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{6a\sqrt{-a^2+b^2}d^3}$$

```
[In] Integrate[x^5/(a + b*Sech[c + d*x^2]),x]
```

```
[Out] (Sqrt[-a^2 + b^2]*d^3*x^6 - 3*b*d^2*x^4*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt
[-a^2 + b^2])] + 3*b*d^2*x^4*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2
])] - 6*b*d*x^2*PolyLog[2, (a*E^(c + d*x^2))/(-b + Sqrt[-a^2 + b^2])] + 6*b
*d*x^2*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]))] + 6*b*PolyLo
g[3, (a*E^(c + d*x^2))/(-b + Sqrt[-a^2 + b^2])] - 6*b*PolyLog[3, -((a*E^(c
+ d*x^2))/(b + Sqrt[-a^2 + b^2]))])/(6*a*Sqrt[-a^2 + b^2]*d^3)
```

Maple [F]

$$\int \frac{x^5}{a + b\,\text{sech}\,(d\,x^2 + c)}dx$$

```
[In] int(x^5/(a+b*sech(d*x^2+c)),x)
```

```
[Out] int(x^5/(a+b*sech(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. 2(311) = 622.

Time = 0.30 (sec) , antiderivative size = 731, normalized size of antiderivative = 2.09

$$\int \frac{x^5}{a+b\text{sech}(c+dx^2)}\,dx = \frac{(a^2-b^2)d^3x^6 + 6\,abdx^2\sqrt{-\frac{a^2-b^2}{a^2}}\text{Li}_2\left(-\frac{b\cosh(dx^2+c)+b\sinh(dx^2+c)+(a\cosh(dx^2+c)+a\sinh(dx^2+c))\sqrt{-\frac{a^2-b^2}{a^2}}+a}{a}+1\right) -}{}$$

```
[In] integrate(x^5/(a+b*sech(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/6*((a^2 - b^2)*d^3*x^6 + 6*a*b*d*x^2*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b*cos
h(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*
sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) - 6*a*b*d*x^2*sqrt(-(a^2 - b^2)/a^2)*dil
og(-(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*sinh(d*
x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) - 3*a*b*c^2*sqrt(-(a^2 - b^2)/
a^2)*log(2*a*cosh(d*x^2 + c) + 2*a*sinh(d*x^2 + c) + 2*a*sqrt(-(a^2 - b^2)/
a^2) + 2*b) + 3*a*b*c^2*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cosh(d*x^2 + c) + 2*
a*sinh(d*x^2 + c) - 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) - 6*a*b*sqrt(-(a^2 -
b^2)/a^2)*polylog(3, -(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^
2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))/a) + 6*a*b*sqrt(-(a^2 -
 b^2)/a^2)*polylog(3, -(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) - (a*cosh(d*x
^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))/a) + 3*(a*b*d^2*x^4 -
a*b*c^2)*sqrt(-(a^2 - b^2)/a^2)*log((b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c)
+ (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) -
3*(a*b*d^2*x^4 - a*b*c^2)*sqrt(-(a^2 - b^2)/a^2)*log((b*cosh(d*x^2 + c) + b
*sinh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2
)/a^2) + a)/a))/((a^3 - a*b^2)*d^3)
```

Sympy [F]

$$\int \frac{x^5}{a+b\text{sech}(c+dx^2)}\,dx = \int \frac{x^5}{a+b\,\text{sech}(c+dx^2)}\,dx$$

```
[In] integrate(x**5/(a+b*sech(d*x**2+c)),x)
```

```
[Out] Integral(x**5/(a + b*sech(c + d*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{a + b\text{sech}\left(c + dx^2\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(a+b*sech(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{x^5}{a + b\text{sech}\left(c + dx^2\right)}\,dx = \int \frac{x^5}{b\,\text{sech}\left(dx^2 + c\right) + a}\,dx$$

```
[In] integrate(x^5/(a+b*sech(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(x^5/(b*sech(d*x^2 + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{a + b\text{sech}\left(c + dx^2\right)}\,dx = \int \frac{x^5}{a + \frac{b}{\cosh(d\,x^2+c)}}\,dx$$

```
[In] int(x^5/(a + b/cosh(c + d*x^2)),x)
```

```
[Out] int(x^5/(a + b/cosh(c + d*x^2)), x)
```

3.17 $\int \frac{x^4}{a+b\,\mathbf{sech}(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^4}{a+b\,\mathrm{sech}\left(c+dx^2\right)}\,dx=\mathrm{Int}\left(\frac{x^4}{a+b\,\mathrm{sech}\left(c+dx^2\right)},x\right)$$

```
[Out] Unintegrable(x^4/(a+b*sech(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4}{a+b\,\mathrm{sech}\left(c+dx^2\right)}\,dx=\int \frac{x^4}{a+b\,\mathrm{sech}\left(c+dx^2\right)}\,dx$$

```
[In] Int[x^4/(a + b*Sech[c + d*x^2]),x]
```

```
[Out] Defer[Int][x^4/(a + b*Sech[c + d*x^2]), x]
```

Rubi steps

$$\text{integral}=\int \frac{x^4}{a+b\,\mathrm{sech}\left(c+dx^2\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a+b\text{sech}\left(c+dx^2\right)}\,dx=\int \frac{x^4}{a+b\text{sech}\left(c+dx^2\right)}\,dx$$

```
[In] Integrate[x^4/(a + b*Sech[c + d*x^2]),x]
[Out] Integrate[x^4/(a + b*Sech[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^{4}}{a+b\,\operatorname{sech}\left(d\,x^{2}+c\right)}dx$$

```
[In] int(x^4/(a+b*sech(d*x^2+c)),x)
[Out] int(x^4/(a+b*sech(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a+b\text{sech}\left(c+dx^2\right)}\,dx=\int \frac{x^{4}}{b\operatorname{sech}\left(dx^{2}+c\right)+a}\,dx$$

```
[In] integrate(x^4/(a+b*sech(d*x^2+c)),x, algorithm="fricas")
[Out] integral(x^4/(b*sech(d*x^2 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a+b\text{sech}\left(c+dx^2\right)}\,dx=\int \frac{x^{4}}{a+b\operatorname{sech}\left(c+dx^{2}\right)}\,dx$$

```
[In] integrate(x**4/(a+b*sech(d*x**2+c)),x)
[Out] Integral(x**4/(a + b*sech(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.28

$$\int \frac{x^4}{a + b\text{sech}\left(c + dx^2\right)} dx = \int \frac{x^4}{b\,\text{sech}\left(dx^2 + c\right) + a} dx$$

```
[In] integrate(x^4/(a+b*sech(d*x^2+c)),x, algorithm="maxima")
[Out] 1/5*x^5/a - 2*b*integrate(x^4*e^(d*x^2 + c)/(a^2*e^(2*d*x^2 + 2*c) + 2*a*b*
e^(d*x^2 + c) + a^2), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a + b\text{sech}\left(c + dx^2\right)} dx = \int \frac{x^4}{b\,\text{sech}\left(dx^2 + c\right) + a} dx$$

```
[In] integrate(x^4/(a+b*sech(d*x^2+c)),x, algorithm="giac")
[Out] integrate(x^4/(b*sech(d*x^2 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{a + b\text{sech}\left(c + dx^2\right)} dx = \int \frac{x^4}{a + \frac{b}{\cosh(d\,x^2+c)}} dx$$

```
[In] int(x^4/(a + b/cosh(c + d*x^2)),x)
[Out] int(x^4/(a + b/cosh(c + d*x^2)), x)
```

3.18 $\int \frac{x^3}{a+b\operatorname{sech}(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 241

$$\int \frac{x^3}{a+b\operatorname{sech}(c+dx^2)}\,dx = \frac{x^4}{4a} - \frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{b\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d^2} + \frac{b\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d^2}$$

```
[Out] 1/4*x^4/a-1/2*b*x^2*ln(1+a*exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a/d/(-a^2+b^2
)^(1/2)+1/2*b*x^2*ln(1+a*exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a/d/(-a^2+b^2)^
(1/2)-1/2*b*polylog(2,-a*exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2
)^(1/2)+1/2*b*polylog(2,-a*exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5544, 4276, 3401, 2296, 2221, 2317, 2438}

$$\int \frac{x^3}{a+b\operatorname{sech}(c+dx^2)}\,dx = -\frac{b\operatorname{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)}{2ad^2\sqrt{b^2-a^2}} + \frac{b\operatorname{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)}{2ad^2\sqrt{b^2-a^2}} - \frac{bx^2\log\left(\frac{ae^{c+dx^2}}{b-\sqrt{b^2-a^2}}+1\right)}{2ad\sqrt{b^2-a^2}} + \frac{bx^2\log\left(\frac{ae^{c+dx^2}}{\sqrt{b^2-a^2}+b}+1\right)}{2ad\sqrt{b^2-a^2}} + \frac{x^4}{4a}$$

```
[In] Int[x^3/(a + b*Sech[c + d*x^2]),x]
```

```
[Out] x^4/(4*a) - (b*x^2*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2])])/(2*a*
Sqrt[-a^2 + b^2]*d) + (b*x^2*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2
])])/(2*a*Sqrt[-a^2 + b^2]*d) - (b*PolyLog[2, -((a*E^(c + d*x^2))/(b - Sqrt
[-a^2 + b^2]))])/(2*a*Sqrt[-a^2 + b^2]*d^2) + (b*PolyLog[2, -((a*E^(c + d*x
^2))/(b + Sqrt[-a^2 + b^2]))])/(2*a*Sqrt[-a^2 + b^2]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+b\text{sech}(c+dx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{x}{a}-\frac{bx}{a(b+a\cosh(c+dx))}\right)\,dx, x, x^2\right)\\
&= \frac{x^4}{4a}-\frac{b\text{Subst}\left(\int \frac{x}{b+a\cosh(c+dx)}\,dx, x, x^2\right)}{2a}\\
&= \frac{x^4}{4a}-\frac{b\text{Subst}\left(\int \frac{e^{c+dx}x}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^2\right)}{a}\\
&= \frac{x^4}{4a}-\frac{b\text{Subst}\left(\int \frac{e^{c+dx}x}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{\sqrt{-a^2+b^2}}+\frac{b\text{Subst}\left(\int \frac{e^{c+dx}x}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{\sqrt{-a^2+b^2}}\\
&= \frac{x^4}{4a}-\frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}+\frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}\\
&\quad+\frac{b\text{Subst}\left(\int \log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{2a\sqrt{-a^2+b^2}d}\\
&\quad-\frac{b\text{Subst}\left(\int \log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{2a\sqrt{-a^2+b^2}d}\\
&= \frac{x^4}{4a}-\frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}+\frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}\\
&\quad+\frac{b\text{Subst}\left(\int \frac{\log\left(1+\frac{2ax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{c+dx^2}\right)}{2a\sqrt{-a^2+b^2}d^2}\\
&\quad-\frac{b\text{Subst}\left(\int \frac{\log\left(1+\frac{2ax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{c+dx^2}\right)}{2a\sqrt{-a^2+b^2}d^2}
\end{aligned}$$

$$= \frac{x^4}{4a} - \frac{bx^2 \log\left(1 + \frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{bx^2 \log\left(1 + \frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{b\,\text{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d^2} + \frac{b\,\text{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d^2}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{a + b\text{sech}\,(c + dx^2)}\,dx = \frac{dx^2\left(\sqrt{-a^2+b^2}dx^2 - 2b\log\left(1 + \frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right) + 2b\log\left(1 + \frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)\right) - 2b\,\text{PolyLog}\left(2, \frac{ae^{c+dx^2}}{-b+\sqrt{-a^2+b^2}}\right) +}{4a\sqrt{-a^2+b^2}d^2}$$

```
[In] Integrate[x^3/(a + b*Sech[c + d*x^2]),x]
```

```
[Out] (d*x^2*(Sqrt[-a^2 + b^2]*d*x^2 - 2*b*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[-a
^2 + b^2])] + 2*b*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2])]) - 2*b*
PolyLog[2, (a*E^(c + d*x^2))/(-b + Sqrt[-a^2 + b^2])] + 2*b*PolyLog[2, -((a
*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]))])/(4*a*Sqrt[-a^2 + b^2]*d^2)
```

Maple [F]

$$\int \frac{x^3}{a + b\,\text{sech}\,(d\,x^2 + c)}dx$$

```
[In] int(x^3/(a+b*sech(d*x^2+c)),x)
```

```
[Out] int(x^3/(a+b*sech(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. 2(209) = 418.

Time = 0.26 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.22

$$\int \frac{x^3}{a + b\text{sech}\,(c + dx^2)}\,dx = \frac{(a^2 - b^2)d^2x^4 + 2\,abc\sqrt{-\frac{a^2-b^2}{a^2}}\log\left(2\,a\cosh\left(dx^2 + c\right) + 2\,a\sinh\left(dx^2 + c\right) + 2\,a\sqrt{-\frac{a^2-b^2}{a^2}} + 2\,b\right) - 2\,abc\sqrt{}}{}$$

```
[In] integrate(x^3/(a+b*sech(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 - b^2)*d^2*x^4 + 2*a*b*c*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cosh(d*x^
2 + c) + 2*a*sinh(d*x^2 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) - 2*a*b*c*
sqrt(-(a^2 - b^2)/a^2)*log(2*a*cosh(d*x^2 + c) + 2*a*sinh(d*x^2 + c) - 2*a*
sqrt(-(a^2 - b^2)/a^2) + 2*b) + 2*a*b*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b*cosh
(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*s
qrt(-(a^2 - b^2)/a^2) + a)/a + 1) - 2*a*b*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b*
cosh(d*x^2 + c) + b*sinh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c
))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + 2*(a*b*d*x^2 + a*b*c)*sqrt(-(a^2 -
b^2)/a^2)*log((b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c) +
 a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) - 2*(a*b*d*x^2 + a*b*c)*
sqrt(-(a^2 - b^2)/a^2)*log((b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) - (a*cosh
(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a))/((a^3 - a*
b^2)*d^2)
```

Sympy [F]

$$\int \frac{x^3}{a + b\mathrm{sech}\left(c + dx^2\right)}\,dx = \int \frac{x^3}{a + b\,\mathrm{sech}\left(c + dx^2\right)}\,dx$$

```
[In] integrate(x**3/(a+b*sech(d*x**2+c)),x)
```

```
[Out] Integral(x**3/(a + b*sech(c + d*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{a + b\mathrm{sech}\left(c + dx^2\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(a+b*sech(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{x^3}{a + b\text{sech}\left(c + dx^2\right)}\,dx = \int \frac{x^3}{b\,\text{sech}\left(dx^2 + c\right) + a}\,dx$$

```
[In] integrate(x^3/(a+b*sech(d*x^2+c)),x, algorithm="giac")
[Out] integrate(x^3/(b*sech(d*x^2 + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a + b\text{sech}\left(c + dx^2\right)}\,dx = \int \frac{x^3}{a + \frac{b}{\cosh(d\,x^2+c)}}\,dx$$

```
[In] int(x^3/(a + b/cosh(c + d*x^2)),x)
[Out] int(x^3/(a + b/cosh(c + d*x^2)), x)
```

3.19 $\int \frac{x^2}{a+b\,\text{sech}(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{a+b\text{sech}\left(c+dx^2\right)}\,dx=\text{Int}\left(\frac{x^2}{a+b\text{sech}\left(c+dx^2\right)},x\right)$$

```
[Out] Unintegrable(x^2/(a+b*sech(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{a+b\text{sech}\left(c+dx^2\right)}\,dx=\int \frac{x^2}{a+b\text{sech}\left(c+dx^2\right)}\,dx$$

```
[In] Int[x^2/(a + b*Sech[c + d*x^2]),x]
```

```
[Out] Defer[Int][x^2/(a + b*Sech[c + d*x^2]), x]
```

Rubi steps

$$\text{integral}=\int \frac{x^2}{a+b\text{sech}\left(c+dx^2\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\text{sech}\left(c+dx^2\right)}\,dx=\int \frac{x^2}{a+b\text{sech}\left(c+dx^2\right)}\,dx$$

```
[In] Integrate[x^2/(a + b*Sech[c + d*x^2]),x]
[Out] Integrate[x^2/(a + b*Sech[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^{2}}{a+b\,\operatorname{sech}\left(d\,x^{2}+c\right)}dx$$

```
[In] int(x^2/(a+b*sech(d*x^2+c)),x)
[Out] int(x^2/(a+b*sech(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\text{sech}\left(c+dx^2\right)}\,dx=\int \frac{x^{2}}{b\operatorname{sech}\left(dx^{2}+c\right)+a}\,dx$$

```
[In] integrate(x^2/(a+b*sech(d*x^2+c)),x, algorithm="fricas")
[Out] integral(x^2/(b*sech(d*x^2 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a+b\text{sech}\left(c+dx^2\right)}\,dx=\int \frac{x^{2}}{a+b\operatorname{sech}\left(c+dx^{2}\right)}\,dx$$

```
[In] integrate(x**2/(a+b*sech(d*x**2+c)),x)
[Out] Integral(x**2/(a + b*sech(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.28

$$\int \frac{x^2}{a + b\text{sech}\,(c + dx^2)}\,dx = \int \frac{x^2}{b\,\text{sech}\,(dx^2 + c) + a}\,dx$$

```
[In] integrate(x^2/(a+b*sech(d*x^2+c)),x, algorithm="maxima")
[Out] 1/3*x^3/a - 2*b*integrate(x^2*e^(d*x^2 + c)/(a^2*e^(2*d*x^2 + 2*c) + 2*a*b*
e^(d*x^2 + c) + a^2), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a + b\text{sech}\,(c + dx^2)}\,dx = \int \frac{x^2}{b\,\text{sech}\,(dx^2 + c) + a}\,dx$$

```
[In] integrate(x^2/(a+b*sech(d*x^2+c)),x, algorithm="giac")
[Out] integrate(x^2/(b*sech(d*x^2 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{a + b\text{sech}\,(c + dx^2)}\,dx = \int \frac{x^2}{a + \frac{b}{\cosh(d\,x^2+c)}}\,dx$$

```
[In] int(x^2/(a + b/cosh(c + d*x^2)),x)
[Out] int(x^2/(a + b/cosh(c + d*x^2)), x)
```

3.20 $\int \frac{x}{a+b\,\mathrm{sech}(c+dx^2)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int \frac{x}{a+b\,\mathrm{sech}\left(c+dx^2\right)}\,dx=\frac{x^2}{2a}-\frac{b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}\left(c+dx^2\right)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}\,d}$$

```
[Out] 1/2*x^2/a-b*arctan((a-b)^(1/2)*tanh(1/2*d*x^2+1/2*c)/(a+b)^(1/2))/a/d/(a-b)
^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5544, 3868, 2738, 214}

$$\int \frac{x}{a+b\,\mathrm{sech}\left(c+dx^2\right)}\,dx=\frac{x^2}{2a}-\frac{b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}\left(c+dx^2\right)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

```
[In] Int[x/(a + b*Sech[c + d*x^2]),x]
```

```
[Out] x^2/(2*a) - (b*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x^2)/2])/Sqrt[a + b]])/(a*Sq
rt[a - b]*Sqrt[a + b]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b\text{sech}(c+dx)}\,dx,x,x^2\right)\\
&= \frac{x^2}{2a}-\frac{\text{Subst}\left(\int \frac{1}{1+\frac{a\cosh(c+dx)}{b}}\,dx,x,x^2\right)}{2a}\\
&= \frac{x^2}{2a}+\frac{i\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,i\tanh\left(\frac{1}{2}(c+dx^2)\right)\right)}{ad}\\
&= \frac{x^2}{2a}-\frac{b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{x}{a+b\text{sech}\left(c+dx^2\right)}\,dx=\frac{\frac{c}{d}+x^2+\frac{2b\arctan\left(\frac{(-a+b)\tanh\left(\frac{1}{2}\left(c+dx^2\right)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}}{2a}$$

```
[In] Integrate[x/(a + b*Sech[c + d*x^2]),x]
```

```
[Out] (c/d + x^2 + (2*b*ArcTan[((-a + b)*Tanh[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])/(
Sqrt[a^2 - b^2]*d))/(2*a)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36

method	result	size
derivativedivides	$\frac{-\frac{2b\arctan\left(\frac{(a-b)\tanh\left(\frac{d x^2}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}+\frac{\ln\left(1+\tanh\left(\frac{d x^2}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\ln\left(\tanh\left(\frac{d x^2}{2}+\frac{c}{2}\right)-1\right)}{a}}{2d}$	90
default	$\frac{-\frac{2b\arctan\left(\frac{(a-b)\tanh\left(\frac{d x^2}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}+\frac{\ln\left(1+\tanh\left(\frac{d x^2}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\ln\left(\tanh\left(\frac{d x^2}{2}+\frac{c}{2}\right)-1\right)}{a}}{2d}$	90
risch	$\frac{x^2}{2a}-\frac{b\ln\left(e^{d x^2+c}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{2\sqrt{-a^2+b^2}\,da}+\frac{b\ln\left(e^{d x^2+c}+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{2\sqrt{-a^2+b^2}\,da}$	144

```
[In] int(x/(a+b*sech(d*x^2+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(-2*b/a/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x^2+1/2*c)/((a+b)
*(a-b))^(1/2))+1/a*ln(1+tanh(1/2*d*x^2+1/2*c))-1/a*ln(tanh(1/2*d*x^2+1/2*c)
-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 304, normalized size of antiderivative = 4.61

$$\int \frac{x}{a+b\text{sech}\left(c+dx^2\right)}\,dx$$
$$=\left[\frac{(a^2-b^2)dx^2-\sqrt{-a^2+b^2}b\log\left(\frac{a^2\cosh\left(dx^2+c\right)^2+a^2\sinh\left(dx^2+c\right)^2+2\,ab\cosh\left(dx^2+c\right)-a^2+2\,b^2+2\,\left(a^2\cosh\left(dx^2+c\right)+ab\right)\sinh\left(dx^2\right.}{a\cosh\left(dx^2+c\right)^2+a\sinh\left(dx^2+c\right)^2+2\,b\cosh\left(dx^2+c\right)+2\,\left(a\cosh\left(dx^2+\right.\right.}\right.}{2\left(a^3-ab^2\right)d}\right.$$

```
[In] integrate(x/(a+b*sech(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2 - b^2)*d*x^2 - sqrt(-a^2 + b^2)*b*log((a^2*cosh(d*x^2 + c)^2 + a
^2*sinh(d*x^2 + c)^2 + 2*a*b*cosh(d*x^2 + c) - a^2 + 2*b^2 + 2*(a^2*cosh(d*
x^2 + c) + a*b)*sinh(d*x^2 + c) + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x^2 + c) + a
*sinh(d*x^2 + c) + b))/(a*cosh(d*x^2 + c)^2 + a*sinh(d*x^2 + c)^2 + 2*b*cos
h(d*x^2 + c) + 2*(a*cosh(d*x^2 + c) + b)*sinh(d*x^2 + c) + a)))/((a^3 - a*b
^2)*d), 1/2*((a^2 - b^2)*d*x^2 + 2*sqrt(a^2 - b^2)*b*arctan(-(a*cosh(d*x^2
+ c) + a*sinh(d*x^2 + c) + b)/sqrt(a^2 - b^2)))/((a^3 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{x}{a + b\text{sech}\left(c + dx^2\right)}\,dx = \int \frac{x}{a + b\,\text{sech}\left(c + dx^2\right)}\,dx$$

```
[In] integrate(x/(a+b*sech(d*x**2+c)),x)
```

```
[Out] Integral(x/(a + b*sech(c + d*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a + b\text{sech}\left(c + dx^2\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(a+b*sech(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{x}{a + b\text{sech}\left(c + dx^2\right)}\,dx = -\frac{b\arctan\left(\frac{ae^{\left(dx^2+c\right)}+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}ad} + \frac{dx^2+c}{2\,ad}$$

```
[In] integrate(x/(a+b*sech(d*x^2+c)),x, algorithm="giac")
```

```
[Out] -b*arctan((a*e^(d*x^2 + c) + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a*d) + 1/
2*(d*x^2 + c)/(a*d)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.59

$$\int \frac{x}{a+b\text{sech}\,(c+dx^2)}\,dx$$
$$=\frac{x^2}{2\,a}-\frac{\text{atan}\left(\frac{a\,d\,\sqrt{b^2}}{\sqrt{a^4\,d^2-a^2\,b^2\,d^2}}+\frac{b\,\mathrm{e}^{d\,x^2}\,\mathrm{e}^c\,\sqrt{a^4\,d^2-a^2\,b^2\,d^2}}{a^2\,d\,\sqrt{b^2}}+\frac{a^2\,b\,d\,\mathrm{e}^{d\,x^2}\,\mathrm{e}^c\,\sqrt{b^2}\,\sqrt{a^4\,d^2-a^2\,b^2\,d^2}}{a^6\,d^2-a^4\,b^2\,d^2}\right)\,\sqrt{b^2}}{\sqrt{a^4\,d^2-a^2\,b^2\,d^2}}$$

```
[In] int(x/(a + b/cosh(c + d*x^2)),x)
```

```
[Out] x^2/(2*a) - (atan((a*d*(b^2)^(1/2))/(a^4*d^2 - a^2*b^2*d^2)^(1/2) + (b*exp(
d*x^2)*exp(c)*(a^4*d^2 - a^2*b^2*d^2)^(1/2))/(a^2*d*(b^2)^(1/2)) + (a^2*b*d
*exp(d*x^2)*exp(c)*(b^2)^(1/2)*(a^4*d^2 - a^2*b^2*d^2)^(1/2))/(a^6*d^2 - a^
4*b^2*d^2))*(b^2)^(1/2))/(a^4*d^2 - a^2*b^2*d^2)^(1/2)
```

3.21 $\int \frac{1}{x\left(a+b\,\text{sech}\left(c+dx^2\right)\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\,\text{sech}\left(c+dx^2\right)\right)}\,dx=\text{Int}\left(\frac{1}{x\left(a+b\,\text{sech}\left(c+dx^2\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sech(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\,\text{sech}\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\,\text{sech}\left(c+dx^2\right)\right)}\,dx$$

```
[In] Int[1/(x*(a + b*Sech[c + d*x^2])),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Sech[c + d*x^2])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\,\text{sech}\left(c+dx^2\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\text{sech}\left(c+dx^2\right)\right)}\,dx$$

```
[In] Integrate[1/(x*(a + b*Sech[c + d*x^2])),x]
[Out] Integrate[1/(x*(a + b*Sech[c + d*x^2])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\,\text{sech}\left(d\,x^{2}+c\right)\right)}dx$$

```
[In] int(1/x/(a+b*sech(d*x^2+c)),x)
[Out] int(1/x/(a+b*sech(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{(b\,\text{sech}\left(dx^2+c\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*sech(d*x^2+c)),x, algorithm="fricas")
[Out] integral(1/(b*x*sech(d*x^2 + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\,\text{sech}\left(c+dx^2\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*sech(d*x**2+c)),x)
[Out] Integral(1/(x*(a + b*sech(c + d*x**2))), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.22

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{(b\,\text{sech}\left(dx^2+c\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*sech(d*x^2+c)),x, algorithm="maxima")
[Out] -2*b*integrate(e^(d*x^2 + c)/(a^2*x*e^(2*d*x^2 + 2*c) + 2*a*b*x*e^(d*x^2 +
c) + a^2*x), x) + log(x)/a
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{(b\,\text{sech}\left(dx^2+c\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*sech(d*x^2+c)),x, algorithm="giac")
[Out] integrate(1/((b*sech(d*x^2 + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\,\left(a+\frac{b}{\cosh\left(d\,x^2+c\right)}\right)}\,dx$$

```
[In] int(1/(x*(a + b/cosh(c + d*x^2))),x)
[Out] int(1/(x*(a + b/cosh(c + d*x^2))), x)
```

3.22 $\int \frac{a+b\,\mathrm{sech}(c+dx^2)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a+b\,\mathrm{sech}(c+dx^2)}{x^2}\,dx = -\frac{a}{x}+b\,\mathrm{Int}\left(\frac{\mathrm{sech}(c+dx^2)}{x^2},x\right)$$

```
[Out] -a/x+b*Unintegrable(sech(d*x^2+c)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\,\mathrm{sech}(c+dx^2)}{x^2}\,dx = \int \frac{a+b\,\mathrm{sech}(c+dx^2)}{x^2}\,dx$$

```
[In] Int[(a + b*Sech[c + d*x^2])/x^2,x]
[Out] -(a/x) + b*Defer[Int][Sech[c + d*x^2]/x^2, x]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a}{x^2}+\frac{b\,\mathrm{sech}(c+dx^2)}{x^2}\right)dx$$

$$= -\frac{a}{x}+b\int \frac{\mathrm{sech}(c+dx^2)}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{sech}(c+dx^2)}{x^2}\,dx=\int \frac{a+b\text{sech}(c+dx^2)}{x^2}\,dx$$

```
[In] Integrate[(a + b*Sech[c + d*x^2])/x^2,x]
[Out] Integrate[(a + b*Sech[c + d*x^2])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a+b\,\operatorname{sech}\left(d\,x^{2}+c\right)}{x^{2}}dx$$

```
[In] int((a+b*sech(d*x^2+c))/x^2,x)
[Out] int((a+b*sech(d*x^2+c))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{sech}(c+dx^2)}{x^2}\,dx=\int \frac{b\,\operatorname{sech}\left(dx^2+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sech(d*x^2+c))/x^2,x, algorithm="fricas")
[Out] integral((b*sech(d*x^2 + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a+b\text{sech}(c+dx^2)}{x^2}\,dx=\int \frac{a+b\,\operatorname{sech}\left(c+dx^2\right)}{x^2}\,dx$$

```
[In] integrate((a+b*sech(d*x**2+c))/x**2,x)
[Out] Integral((a + b*sech(c + d*x**2))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int \frac{a + b\text{sech}(c + dx^2)}{x^2} \, dx = \int \frac{b \,\text{sech}\,(dx^2 + c) + a}{x^2} \, dx$$

```
[In] integrate((a+b*sech(d*x^2+c))/x^2,x, algorithm="maxima")
[Out] 2*b*integrate(1/(x^2*(e^(d*x^2 + c) + e^(-d*x^2 - c))), x) - a/x
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\text{sech}(c + dx^2)}{x^2} \, dx = \int \frac{b \,\text{sech}\,(dx^2 + c) + a}{x^2} \, dx$$

```
[In] integrate((a+b*sech(d*x^2+c))/x^2,x, algorithm="giac")
[Out] integrate((b*sech(d*x^2 + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a + b\text{sech}(c + dx^2)}{x^2} \, dx = \int \frac{a + \frac{b}{\cosh(d\,x^2+c)}}{x^2} \, dx$$

```
[In] int((a + b/cosh(c + d*x^2))/x^2,x)
[Out] int((a + b/cosh(c + d*x^2))/x^2, x)
```

3.23 $\int \frac{x^5}{\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 994

$$\int \frac{x^5}{(a+b\operatorname{sech}(c+dx^2))^2}\,dx = \frac{b^2x^4}{2a^2(a^2-b^2)d}+\frac{x^6}{6a^2}-\frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+\frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}-\frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$-\frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2}-\frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$
$$+\frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}-\frac{b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$+\frac{b^3x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$-\frac{2bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$-\frac{b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$-\frac{b^3x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$+\frac{2bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$-\frac{b^3\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$+\frac{2b\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$+\frac{b^3\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$-\frac{2b\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$+\frac{b^2x^4\sinh(c+dx^2)}{2a(a^2-b^2)d(b+a\cosh(c+dx^2))}$$

```
[Out] 1/2*b^2*x^4/a^2/(a^2-b^2)/d+1/6*x^6/a^2-b^2*x^2*ln(1+a*exp(d*x^2+c)/(b-(-a^
2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^2+1/2*b^3*x^4*ln(1+a*exp(d*x^2+c)/(b-(-a^2+b
^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-b^2*x^2*ln(1+a*exp(d*x^2+c)/(b+(-a^2+b^2
)^(1/2)))/a^2/(a^2-b^2)/d^2-1/2*b^3*x^4*ln(1+a*exp(d*x^2+c)/(b+(-a^2+b^2)^(
1/2)))/a^2/(-a^2+b^2)^(3/2)/d-b^2*polylog(2,-a*exp(d*x^2+c)/(b-(-a^2+b^2)^(
1/2)))/a^2/(a^2-b^2)/d^3+b^3*x^2*polylog(2,-a*exp(d*x^2+c)/(b-(-a^2+b^2)^(1
/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-b^2*polylog(2,-a*exp(d*x^2+c)/(b+(-a^2+b^2)^
(1/2)))/a^2/(a^2-b^2)/d^3-b^3*x^2*polylog(2,-a*exp(d*x^2+c)/(b+(-a^2+b^2)^(
1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-b^3*polylog(3,-a*exp(d*x^2+c)/(b-(-a^2+b^2)
^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+b^3*polylog(3,-a*exp(d*x^2+c)/(b+(-a^2+b^
2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+1/2*b^2*x^4*sinh(d*x^2+c)/a/(a^2-b^2)/d
/(b+a*cosh(d*x^2+c))-b*x^4*ln(1+a*exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a^2/d/
(-a^2+b^2)^(1/2)+b*x^4*ln(1+a*exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a^2/d/(-a^
2+b^2)^(1/2)-2*b*x^2*polylog(2,-a*exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a^2/d^
2/(-a^2+b^2)^(1/2)+2*b*x^2*polylog(2,-a*exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/
a^2/d^2/(-a^2+b^2)^(1/2)+2*b*polylog(3,-a*exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2))
)/a^2/d^3/(-a^2+b^2)^(1/2)-2*b*polylog(3,-a*exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2
)))/a^2/d^3/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 994, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules

used = {5544, 4276, 3405, 3401, 2296, 2221, 2611, 2320, 6724, 5681, 2317, 2438}

$$\int \frac{x^5}{(a+b\text{sech}\,(c+dx^2))^2}\,dx = \frac{x^6}{6a^2} - \frac{b\log\left(\frac{e^{dx^2+c}a}{b-\sqrt{b^2-a^2}}+1\right)x^4}{a^2\sqrt{b^2-a^2}d} + \frac{b^3\log\left(\frac{e^{dx^2+c}a}{b-\sqrt{b^2-a^2}}+1\right)x^4}{2a^2(b^2-a^2)^{3/2}d} + \frac{b\log\left(\frac{e^{dx^2+c}a}{b+\sqrt{b^2-a^2}}+1\right)x^4}{a^2\sqrt{b^2-a^2}d} - \frac{b^3\log\left(\frac{e^{dx^2+c}a}{b+\sqrt{b^2-a^2}}+1\right)x^4}{2a^2(b^2-a^2)^{3/2}d} + \frac{b^2\sinh(dx^2+c)x^4}{2a(a^2-b^2)d(b+a\cosh(dx^2+c))} + \frac{b^2x^4}{2a^2(a^2-b^2)d} - \frac{b^2\log\left(\frac{e^{dx^2+c}a}{b-\sqrt{b^2-a^2}}+1\right)x^2}{a^2(a^2-b^2)d^2} - \frac{b^2\log\left(\frac{e^{dx^2+c}a}{b+\sqrt{b^2-a^2}}+1\right)x^2}{a^2(a^2-b^2)d^2} - \frac{2b\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)x^2}{a^2\sqrt{b^2-a^2}d^2} + \frac{b^3\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)x^2}{a^2(b^2-a^2)^{3/2}d^2} + \frac{2b\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)x^2}{a^2\sqrt{b^2-a^2}d^2} - \frac{b^3\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)x^2}{a^2(b^2-a^2)^{3/2}d^2} - \frac{b^2\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{b^2\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)}{a^2(a^2-b^2)d^3} + \frac{2b\,\text{PolyLog}\left(3,-\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)}{a^2\sqrt{b^2-a^2}d^3} - \frac{b^3\,\text{PolyLog}\left(3,-\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)}{a^2(b^2-a^2)^{3/2}d^3} - \frac{2b\,\text{PolyLog}\left(3,-\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)}{a^2\sqrt{b^2-a^2}d^3} + \frac{b^3\,\text{PolyLog}\left(3,-\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)}{a^2(b^2-a^2)^{3/2}d^3}$$

```
[In] Int[x^5/(a + b*Sech[c + d*x^2])^2,x]
```

```
[Out] (b^2*x^4)/(2*a^2*(a^2 - b^2)*d) + x^6/(6*a^2) - (b^2*x^2*Log[1 + (a*E^(c +
d*x^2))/(b - Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) + (b^3*x^4*Log[1 + (
a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2])])/(2*a^2*(-a^2 + b^2)^(3/2)*d) - (b
*x^4*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^
2]*d) - (b^2*x^2*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2])])/(a^2*(a
^2 - b^2)*d^2) - (b^3*x^4*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2])]
)/(2*a^2*(-a^2 + b^2)^(3/2)*d) + (b*x^4*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt
[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - (b^2*PolyLog[2, -((a*E^(c + d*x^
2))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^3) + (b^3*x^2*PolyLog[2, -
```

```
((a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) -
 (2*b*x^2*PolyLog[2, -((a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqr
t[-a^2 + b^2]*d^2) - (b^2*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b
^2]))])/(a^2*(a^2 - b^2)*d^3) - (b^3*x^2*PolyLog[2, -((a*E^(c + d*x^2))/(b
+ Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (2*b*x^2*PolyLog[2, -
((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2) - (
b^3*PolyLog[3, -((a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b
^2)^(3/2)*d^3) + (2*b*PolyLog[3, -((a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2])
)])/(a^2*Sqrt[-a^2 + b^2]*d^3) + (b^3*PolyLog[3, -((a*E^(c + d*x^2))/(b + S
qrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3) - (2*b*PolyLog[3, -((a*E^(
c + d*x^2))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^3) + (b^2*x^4
*Sinh[c + d*x^2])/(2*a*(a^2 - b^2)*d*(b + a*Cosh[c + d*x^2]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+b\text{sech}(c+dx))^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{x^2}{a^2} + \frac{b^2x^2}{a^2(b+a\cosh(c+dx))^2} - \frac{2bx^2}{a^2(b+a\cosh(c+dx))}\right)\,dx, x, x^2\right)$$

$$= \frac{x^6}{6a^2} - \frac{b\text{Subst}\left(\int \frac{x^2}{b+a\cosh(c+dx)}\,dx, x, x^2\right)}{a^2} + \frac{b^2\text{Subst}\left(\int \frac{x^2}{(b+a\cosh(c+dx))^2}\,dx, x, x^2\right)}{2a^2}$$

$$= \frac{x^6}{6a^2} + \frac{b^2x^4\sinh(c+dx^2)}{2a(a^2-b^2)\,d\,(b+a\cosh(c+dx^2))} - \frac{(2b)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^2\right)}{a^2} - \frac{b^3\text{Subst}\left(\int \frac{x^2}{b+a\cosh(c+dx)}\,dx, x, x^2\right)}{2a^2(a^2-b^2)} - \frac{b^2\text{Subst}\left(\int \frac{x\sinh(c+dx)}{b+a\cosh(c+dx)}\,dx, x, x^2\right)}{a(a^2-b^2)\,d}$$

$$= \frac{b^2x^4}{2a^2(a^2-b^2)\,d} + \frac{x^6}{6a^2} + \frac{b^2x^4\sinh(c+dx^2)}{2a(a^2-b^2)\,d\,(b+a\cosh(c+dx^2))} - \frac{b^3\text{Subst}\left(\int \frac{e^{c+dx}x^2}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^2\right)}{a^2(a^2-b^2)} - \frac{(2b)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{a\sqrt{-a^2+b^2}} + \frac{(2b)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{a\sqrt{-a^2+b^2}} - \frac{b^2\text{Subst}\left(\int \frac{e^{c+dx}x}{b-\sqrt{-a^2+b^2}+ae^{c+dx}}\,dx, x, x^2\right)}{a(a^2-b^2)\,d} - \frac{b^2\text{Subst}\left(\int \frac{e^{c+dx}x}{b+\sqrt{-a^2+b^2}+ae^{c+dx}}\,dx, x, x^2\right)}{a(a^2-b^2)\,d}$$

$$
\begin{aligned}
&= \frac{b^2x^4}{2a^2\left(a^2-b^2\right)d} + \frac{x^6}{6a^2} - \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^2} - \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&\quad - \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^2} + \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&\quad + \frac{b^2x^4\sinh\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+dx^2\right)\right)} + \frac{b^3\text{Subst}\left(\int\frac{e^{c+dx}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx,x,x^2\right)}{a\left(-a^2+b^2\right)^{3/2}} \\
&\quad - \frac{b^3\text{Subst}\left(\int\frac{e^{c+dx}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx,x,x^2\right)}{a\left(-a^2+b^2\right)^{3/2}} \\
&\quad + \frac{b^2\text{Subst}\left(\int\log\left(1+\frac{ae^{c+dx}}{b-\sqrt{-a^2+b^2}}\right)\,dx,x,x^2\right)}{a^2\left(a^2-b^2\right)d^2} \\
&\quad + \frac{b^2\text{Subst}\left(\int\log\left(1+\frac{ae^{c+dx}}{b+\sqrt{-a^2+b^2}}\right)\,dx,x,x^2\right)}{a^2\left(a^2-b^2\right)d^2} \\
&\quad + \frac{(2b)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^2\right)}{a^2\sqrt{-a^2+b^2}d} \\
&\quad - \frac{(2b)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^2\right)}{a^2\sqrt{-a^2+b^2}d}
\end{aligned}
$$

$$= \frac{b^2x^4}{2a^2(a^2-b^2)d} + \frac{x^6}{6a^2} - \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$- \frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d} + \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$- \frac{2bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{2bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{b^2x^4\sinh(c+dx^2)}{2a(a^2-b^2)d(b+a\cosh(c+dx^2))} + \frac{b^2\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{ax}{b-\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{c+dx^2}\right)}{a^2(a^2-b^2)d^3}$$
$$+ \frac{b^2\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{c+dx^2}\right)}{a^2(a^2-b^2)d^3}$$
$$+ \frac{(2b)\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,x^2\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$- \frac{(2b)\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,x^2\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$- \frac{b^3\mathrm{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,x^2\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{b^3\mathrm{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,x^2\right)}{a^2(-a^2+b^2)^{3/2}d}$$

$$= \frac{b^2x^4}{2a^2(a^2-b^2)d} + \frac{x^6}{6a^2} - \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{b^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$+ \frac{b^3x^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} - \frac{2bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$- \frac{b^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{b^3x^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$+ \frac{2bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b^2x^4\sinh(c+dx^2)}{2a(a^2-b^2)d(b+a\cosh(c+dx^2))}$$
$$+ \frac{(2b)\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{c+dx^2}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$- \frac{(2b)\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{c+dx^2}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$- \frac{b^3\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,x^2\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$+ \frac{b^3\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,x^2\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$

$$= \frac{b^2x^4}{2a^2(a^2-b^2)d} + \frac{x^6}{6a^2} - \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$+ \frac{b^3x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} - \frac{2bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$- \frac{b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{b^3x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$+ \frac{2bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{2b\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$- \frac{2b\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} + \frac{b^2x^4\sinh(c+dx^2)}{2a(a^2-b^2)d(b+a\cosh(c+dx^2))}$$
$$- \frac{b^3\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{c+dx^2}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$+ \frac{b^3\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{c+dx^2}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$

$$= \frac{b^2x^4}{2a^2(a^2-b^2)d} + \frac{x^6}{6a^2} - \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{b^2x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{b^3x^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{bx^4\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$+ \frac{b^3x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} - \frac{2bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$- \frac{b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{b^3x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$+ \frac{2bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{b^3\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$+ \frac{2b\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} + \frac{b^3\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$- \frac{2b\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} + \frac{b^2x^4\sinh(c+dx^2)}{2a(a^2-b^2)d(b+a\cosh(c+dx^2))}$$

Mathematica [A] (verified)

Time = 6.72 (sec) , antiderivative size = 1565, normalized size of antiderivative = 1.57

$$\int \frac{x^5}{(a+b\text{sech}(c+dx^2))^2}\,dx$$

$$= \frac{(b+a\cosh(c+dx^2))\operatorname{sech}^2(c+dx^2)\left(x^6(b+a\cosh(c+dx^2)) - \frac{3be^{2c}(b+a\cosh(c+dx^2))\left(2bd^2e^{2c}\sqrt{(-a^2+b^2)e^{2c}}x^4-2\right.}{}\right.}{}$$

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[In] Integrate[x^5/(a + b*Sech[c + d*x^2])^2,x]
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[Out] ((b + a*Cosh[c + d*x^2])*Sech[c + d*x^2]^2*(x^6*(b + a*Cosh[c + d*x^2]) - (
3*b*E^(2*c)*(b + a*Cosh[c + d*x^2])*(2*b*d^2*E^(2*c)*Sqrt[(-a^2 + b^2)*E^(2
*c)]*x^4 - 2*b*d*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^2*Log[1 + (a*E^(2*c + d*x^2))
/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] - 2*b*d*E^(2*c)*Sqrt[(-a^2 + b^2)*E^
(2*c)]*x^2*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])
] - 2*a^2*d^2*E^c*x^4*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(-a^2 + b^2
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)*E^(2*c)])] + b^2*d^2*E^c*x^4*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(-
a^2 + b^2)*E^(2*c)])] - 2*a^2*d^2*E^(3*c)*x^4*Log[1 + (a*E^(2*c + d*x^2))/(
b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] + b^2*d^2*E^(3*c)*x^4*Log[1 + (a*E^(2*
c + d*x^2))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] - 2*b*d*Sqrt[(-a^2 + b^2)
*E^(2*c)]*x^2*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c
)])] - 2*b*d*E^(2*c)*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^2*Log[1 + (a*E^(2*c + d*x
^2))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] + 2*a^2*d^2*E^c*x^4*Log[1 + (a*E
^(2*c + d*x^2))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] - b^2*d^2*E^c*x^4*Log
[1 + (a*E^(2*c + d*x^2))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] + 2*a^2*d^2*
E^(3*c)*x^4*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]
)] - b^2*d^2*E^(3*c)*x^4*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c + Sqrt[(-a^2 +
b^2)*E^(2*c)])] + 2*(1 + E^(2*c))*(-(b*Sqrt[(-a^2 + b^2)*E^(2*c)]) - 2*a^2*
d*E^c*x^2 + b^2*d*E^c*x^2)*PolyLog[2, -((a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(
-a^2 + b^2)*E^(2*c)]))] - 2*(1 + E^(2*c))*(b*Sqrt[(-a^2 + b^2)*E^(2*c)] - 2
*a^2*d*E^c*x^2 + b^2*d*E^c*x^2)*PolyLog[2, -((a*E^(2*c + d*x^2))/(b*E^c + S
qrt[(-a^2 + b^2)*E^(2*c)]))] + 4*a^2*E^c*PolyLog[3, -((a*E^(2*c + d*x^2))/(
b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 2*b^2*E^c*PolyLog[3, -((a*E^(2*c +
d*x^2))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 4*a^2*E^(3*c)*PolyLog[3, -
((a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 2*b^2*E^(3*c)
*PolyLog[3, -((a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] -
4*a^2*E^c*PolyLog[3, -((a*E^(2*c + d*x^2))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*
c)]))] + 2*b^2*E^c*PolyLog[3, -((a*E^(2*c + d*x^2))/(b*E^c + Sqrt[(-a^2 + b
^2)*E^(2*c)]))] - 4*a^2*E^(3*c)*PolyLog[3, -((a*E^(2*c + d*x^2))/(b*E^c + S
qrt[(-a^2 + b^2)*E^(2*c)]))] + 2*b^2*E^(3*c)*PolyLog[3, -((a*E^(2*c + d*x^2
))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))]))/(d^3*((-a^2 + b^2)*E^(2*c))^(3/
2)*(1 + E^(2*c))) + (3*b^2*x^4*Sech[c]*(-(b*Sinh[c]) + a*Sinh[d*x^2]))/((a
- b)*(a + b)*d)))/(6*a^2*(a + b*Sech[c + d*x^2])^2)
```

Maple [F]

$$\int \frac{x^5}{\left(a+b\,\mathrm{sech}\left(d\,x^2+c\right)\right)^2}dx$$

[In] int(x^5/(a+b*sech(d*x^2+c))^2,x)

[Out] int(x^5/(a+b*sech(d*x^2+c))^2,x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3918 vs. $2(906) = 1812$.

Time = 0.33 (sec) , antiderivative size = 3918, normalized size of antiderivative = 3.94

$$\int \frac{x^5}{(a + b\text{sech}\,(c + dx^2))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5/(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*((a^5 - 2*a^3*b^2 + a*b^4)*d^3*x^6 - 6*(a^3*b^2 - a*b^4)*c^2 + ((a^5 -
2*a^3*b^2 + a*b^4)*d^3*x^6 + 6*(a^3*b^2 - a*b^4)*d^2*x^4 - 6*(a^3*b^2 - a*b
^4)*c^2)*cosh(d*x^2 + c)^2 + ((a^5 - 2*a^3*b^2 + a*b^4)*d^3*x^6 + 6*(a^3*b^
2 - a*b^4)*d^2*x^4 - 6*(a^3*b^2 - a*b^4)*c^2)*sinh(d*x^2 + c)^2 - 6*(2*a^4*
b - a^2*b^3 + (2*a^4*b - a^2*b^3)*cosh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*s
inh(d*x^2 + c)^2 + 2*(2*a^3*b^2 - a*b^4)*cosh(d*x^2 + c) + 2*(2*a^3*b^2 - a
*b^4 + (2*a^4*b - a^2*b^3)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt(-(a^2 - b
^2)/a^2)*polylog(3, -(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2
 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))/a) + 6*(2*a^4*b - a^2*b^
3 + (2*a^4*b - a^2*b^3)*cosh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*sinh(d*x^2
+ c)^2 + 2*(2*a^3*b^2 - a*b^4)*cosh(d*x^2 + c) + 2*(2*a^3*b^2 - a*b^4 + (2*
a^4*b - a^2*b^3)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*p
olylog(3, -(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*
sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))/a) + 2*((a^4*b - 2*a^2*b^3 + b^5)*
d^3*x^6 + 3*(a^2*b^3 - b^5)*d^2*x^4 - 6*(a^2*b^3 - b^5)*c^2)*cosh(d*x^2 + c
) - 6*(a^3*b^2 - a*b^4 + (a^3*b^2 - a*b^4)*cosh(d*x^2 + c)^2 + (a^3*b^2 - a
*b^4)*sinh(d*x^2 + c)^2 + 2*(a^2*b^3 - b^5)*cosh(d*x^2 + c) + 2*(a^2*b^3 -
b^5 + (a^3*b^2 - a*b^4)*cosh(d*x^2 + c))*sinh(d*x^2 + c) - ((2*a^4*b - a^2*
b^3)*d*x^2*cosh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*d*x^2*sinh(d*x^2 + c)^2
+ 2*(2*a^3*b^2 - a*b^4)*d*x^2*cosh(d*x^2 + c) + (2*a^4*b - a^2*b^3)*d*x^2 +
 2*((2*a^4*b - a^2*b^3)*d*x^2*cosh(d*x^2 + c) + (2*a^3*b^2 - a*b^4)*d*x^2)*
sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))*dilog(-(b*cosh(d*x^2 + c) + b*sinh
(d*x^2 + c) + (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2
) + a)/a + 1) - 6*(a^3*b^2 - a*b^4 + (a^3*b^2 - a*b^4)*cosh(d*x^2 + c)^2 +
(a^3*b^2 - a*b^4)*sinh(d*x^2 + c)^2 + 2*(a^2*b^3 - b^5)*cosh(d*x^2 + c) + 2
*(a^2*b^3 - b^5 + (a^3*b^2 - a*b^4)*cosh(d*x^2 + c))*sinh(d*x^2 + c) + ((2*
a^4*b - a^2*b^3)*d*x^2*cosh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*d*x^2*sinh(d
*x^2 + c)^2 + 2*(2*a^3*b^2 - a*b^4)*d*x^2*cosh(d*x^2 + c) + (2*a^4*b - a^2*
b^3)*d*x^2 + 2*((2*a^4*b - a^2*b^3)*d*x^2*cosh(d*x^2 + c) + (2*a^3*b^2 - a*
b^4)*d*x^2)*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))*dilog(-(b*cosh(d*x^2 +
 c) + b*sinh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^
2 - b^2)/a^2) + a)/a + 1) + 3*(2*(a^3*b^2 - a*b^4)*c*cosh(d*x^2 + c)^2 + 2*
(a^3*b^2 - a*b^4)*c*sinh(d*x^2 + c)^2 + 4*(a^2*b^3 - b^5)*c*cosh(d*x^2 + c)
 + 2*(a^3*b^2 - a*b^4)*c + 4*((a^3*b^2 - a*b^4)*c*cosh(d*x^2 + c) + (a^2*b^
3 - b^5)*c)*sinh(d*x^2 + c) - ((2*a^4*b - a^2*b^3)*c^2*cosh(d*x^2 + c)^2 +
```

```
(2*a^4*b - a^2*b^3)*c^2*sinh(d*x^2 + c)^2 + 2*(2*a^3*b^2 - a*b^4)*c^2*cosh(
d*x^2 + c) + (2*a^4*b - a^2*b^3)*c^2 + 2*((2*a^4*b - a^2*b^3)*c^2*cosh(d*x^
2 + c) + (2*a^3*b^2 - a*b^4)*c^2)*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))*
log(2*a*cosh(d*x^2 + c) + 2*a*sinh(d*x^2 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2)
+ 2*b) + 3*(2*(a^3*b^2 - a*b^4)*c*cosh(d*x^2 + c)^2 + 2*(a^3*b^2 - a*b^4)*c
*sinh(d*x^2 + c)^2 + 4*(a^2*b^3 - b^5)*c*cosh(d*x^2 + c) + 2*(a^3*b^2 - a*b
^4)*c + 4*((a^3*b^2 - a*b^4)*c*cosh(d*x^2 + c) + (a^2*b^3 - b^5)*c)*sinh(d*
x^2 + c) + ((2*a^4*b - a^2*b^3)*c^2*cosh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)
*c^2*sinh(d*x^2 + c)^2 + 2*(2*a^3*b^2 - a*b^4)*c^2*cosh(d*x^2 + c) + (2*a^4
*b - a^2*b^3)*c^2 + 2*((2*a^4*b - a^2*b^3)*c^2*cosh(d*x^2 + c) + (2*a^3*b^2
 - a*b^4)*c^2)*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))*log(2*a*cosh(d*x^2
+ c) + 2*a*sinh(d*x^2 + c) - 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) - 3*(2*(a^3*
b^2 - a*b^4)*d*x^2 + 2*((a^3*b^2 - a*b^4)*d*x^2 + (a^3*b^2 - a*b^4)*c)*cosh
(d*x^2 + c)^2 + 2*((a^3*b^2 - a*b^4)*d*x^2 + (a^3*b^2 - a*b^4)*c)*sinh(d*x^
2 + c)^2 + 2*(a^3*b^2 - a*b^4)*c + 4*((a^2*b^3 - b^5)*d*x^2 + (a^2*b^3 - b^
5)*c)*cosh(d*x^2 + c) + 4*((a^2*b^3 - b^5)*d*x^2 + (a^2*b^3 - b^5)*c + ((a^
3*b^2 - a*b^4)*d*x^2 + (a^3*b^2 - a*b^4)*c)*cosh(d*x^2 + c))*sinh(d*x^2 + c
) - ((2*a^4*b - a^2*b^3)*d^2*x^4 - (2*a^4*b - a^2*b^3)*c^2 + ((2*a^4*b - a^
2*b^3)*d^2*x^4 - (2*a^4*b - a^2*b^3)*c^2)*cosh(d*x^2 + c)^2 + ((2*a^4*b - a
^2*b^3)*d^2*x^4 - (2*a^4*b - a^2*b^3)*c^2)*sinh(d*x^2 + c)^2 + 2*((2*a^3*b^
2 - a*b^4)*d^2*x^4 - (2*a^3*b^2 - a*b^4)*c^2)*cosh(d*x^2 + c) + 2*((2*a^3*b
^2 - a*b^4)*d^2*x^4 - (2*a^3*b^2 - a*b^4)*c^2 + ((2*a^4*b - a^2*b^3)*d^2*x^
4 - (2*a^4*b - a^2*b^3)*c^2)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt(-(a^2 -
 b^2)/a^2))*log((b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c)
 + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) - 3*(2*(a^3*b^2 - a*b^
4)*d*x^2 + 2*((a^3*b^2 - a*b^4)*d*x^2 + (a^3*b^2 - a*b^4)*c)*cosh(d*x^2 + c
)^2 + 2*((a^3*b^2 - a*b^4)*d*x^2 + (a^3*b^2 - a*b^4)*c)*sinh(d*x^2 + c)^2 +
 2*(a^3*b^2 - a*b^4)*c + 4*((a^2*b^3 - b^5)*d*x^2 + (a^2*b^3 - b^5)*c)*cosh
(d*x^2 + c) + 4*((a^2*b^3 - b^5)*d*x^2 + (a^2*b^3 - b^5)*c + ((a^3*b^2 - a*
b^4)*d*x^2 + (a^3*b^2 - a*b^4)*c)*cosh(d*x^2 + c))*sinh(d*x^2 + c) + ((2*a^
4*b - a^2*b^3)*d^2*x^4 - (2*a^4*b - a^2*b^3)*c^2 + ((2*a^4*b - a^2*b^3)*d^2
*x^4 - (2*a^4*b - a^2*b^3)*c^2)*cosh(d*x^2 + c)^2 + ((2*a^4*b - a^2*b^3)*d^
2*x^4 - (2*a^4*b - a^2*b^3)*c^2)*sinh(d*x^2 + c)^2 + 2*((2*a^3*b^2 - a*b^4)
*d^2*x^4 - (2*a^3*b^2 - a*b^4)*c^2)*cosh(d*x^2 + c) + 2*((2*a^3*b^2 - a*b^4
)*d^2*x^4 - (2*a^3*b^2 - a*b^4)*c^2 + ((2*a^4*b - a^2*b^3)*d^2*x^4 - (2*a^4
*b - a^2*b^3)*c^2)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)
)*log((b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*sinh(
d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) + 2*((a^4*b - 2*a^2*b^3 + b^5)*d
^3*x^6 + 3*(a^2*b^3 - b^5)*d^2*x^4 - 6*(a^2*b^3 - b^5)*c^2 + ((a^5 - 2*a^3*
b^2 + a*b^4)*d^3*x^6 + 6*(a^3*b^2 - a*b^4)*d^2*x^4 - 6*(a^3*b^2 - a*b^4)*c^
2)*cosh(d*x^2 + c))*sinh(d*x^2 + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d^3*cosh(
d*x^2 + c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^4)*d^3*sinh(d*x^2 + c)^2 + 2*(a^6*b
 - 2*a^4*b^3 + a^2*b^5)*d^3*cosh(d*x^2 + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d
^3 + 2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d^3*cosh(d*x^2 + c) + (a^6*b - 2*a^4*b^
3 + a^2*b^5)*d^3)*sinh(d*x^2 + c))
```

Sympy [F]

$$\int \frac{x^5}{(a+b\mathrm{sech}\,(c+dx^2))^2}\,dx = \int \frac{x^5}{(a+b\,\mathrm{sech}\,(c+dx^2))^2}\,dx$$

```
[In] integrate(x**5/(a+b*sech(d*x**2+c))**2,x)
[Out] Integral(x**5/(a + b*sech(c + d*x**2))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+b\mathrm{sech}\,(c+dx^2))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{x^5}{(a+b\mathrm{sech}\,(c+dx^2))^2}\,dx = \int \frac{x^5}{(b\,\mathrm{sech}\,(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^5/(a+b*sech(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(x^5/(b*sech(d*x^2 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+b\mathrm{sech}\,(c+dx^2))^2}\,dx = \int \frac{x^5}{\left(a+\frac{b}{\cosh(d\,x^2+c)}\right)^2}\,dx$$

```
[In] int(x^5/(a + b/cosh(c + d*x^2))^2,x)
[Out] int(x^5/(a + b/cosh(c + d*x^2))^2, x)
```

3.24 $\int \frac{x^4}{\left(a+b\,\mathbf{sech}\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^4}{(a+b\,\mathrm{sech}\,(c+dx^2))^2}\,dx = \mathrm{Int}\left(\frac{x^4}{(a+b\,\mathrm{sech}\,(c+dx^2))^2},x\right)$$

```
[Out] Unintegrable(x^4/(a+b*sech(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4}{(a+b\,\mathrm{sech}\,(c+dx^2))^2}\,dx = \int \frac{x^4}{(a+b\,\mathrm{sech}\,(c+dx^2))^2}\,dx$$

```
[In] Int[x^4/(a + b*Sech[c + d*x^2])^2,x]
```

```
[Out] Defer[Int][x^4/(a + b*Sech[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^4}{(a+b\,\mathrm{sech}\,(c+dx^2))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 20.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{(a+b\text{sech}(c+dx^2))^2}\,dx=\int \frac{x^4}{(a+b\text{sech}(c+dx^2))^2}\,dx$$

```
[In] Integrate[x^4/(a + b*Sech[c + d*x^2])^2,x]
[Out] Integrate[x^4/(a + b*Sech[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a+b\ \text{sech}\,(d\,x^2+c))^2}dx$$

```
[In] int(x^4/(a+b*sech(d*x^2+c))^2,x)
[Out] int(x^4/(a+b*sech(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^4}{(a+b\text{sech}(c+dx^2))^2}\,dx=\int \frac{x^4}{(b\,\text{sech}\,(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^4/(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(x^4/(b^2*sech(d*x^2 + c)^2 + 2*a*b*sech(d*x^2 + c) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{(a+b\text{sech}(c+dx^2))^2}\,dx=\int \frac{x^4}{(a+b\,\text{sech}\,(c+dx^2))^2}\,dx$$

```
[In] integrate(x**4/(a+b*sech(d*x**2+c))**2,x)
[Out] Integral(x**4/(a + b*sech(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 312, normalized size of antiderivative = 17.33

$$\int \frac{x^4}{(a + b\operatorname{sech}(c + dx^2))^2}\,dx = \int \frac{x^4}{(b\operatorname{sech}(dx^2 + c) + a)^2}\,dx$$

```
[In] integrate(x^4/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5*((a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*x^5*e^(2*d*x^2) - 5*a*b^2*x^3 + (a^3
*d - a*b^2*d)*x^5 - (5*b^3*x^3*e^c - 2*(a^2*b*d*e^c - b^3*d*e^c)*x^5)*e^(d*
x^2))/(a^5*d - a^3*b^2*d + (a^5*d*e^(2*c) - a^3*b^2*d*e^(2*c))*e^(2*d*x^2)
+ 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*e^(d*x^2)) - integrate(-(3*a*b^2*x^2 + (3
*b^3*x^2*e^c - 2*(2*a^2*b*d*e^c - b^3*d*e^c)*x^4)*e^(d*x^2))/(a^5*d - a^3*b
^2*d + (a^5*d*e^(2*c) - a^3*b^2*d*e^(2*c))*e^(2*d*x^2) + 2*(a^4*b*d*e^c - a
^2*b^3*d*e^c)*e^(d*x^2)), x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{(a + b\operatorname{sech}(c + dx^2))^2}\,dx = \int \frac{x^4}{(b\operatorname{sech}(dx^2 + c) + a)^2}\,dx$$

```
[In] integrate(x^4/(a+b*sech(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*sech(d*x^2 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{(a + b\operatorname{sech}(c + dx^2))^2}\,dx = \int \frac{x^4}{\left(a + \frac{b}{\cosh(d\,x^2+c)}\right)^2}\,dx$$

```
[In] int(x^4/(a + b/cosh(c + d*x^2))^2,x)
```

```
[Out] int(x^4/(a + b/cosh(c + d*x^2))^2, x)
```

3.25 $\int \frac{x^3}{\left(a+b\,\text{sech}(c+dx^2)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 555

$$\int \frac{x^3}{(a+b\,\text{sech}(c+dx^2))^2}\,dx = \frac{x^4}{4a^2} + \frac{b^3x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d} - \frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{b^3x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d} + \frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{b^2\log(b+a\cosh(c+dx^2))}{2a^2(a^2-b^2)d^2} + \frac{b^3\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d^2} - \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{b^3\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d^2} + \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b^2x^2\sinh(c+dx^2)}{2a(a^2-b^2)d(b+a\cosh(c+dx^2))}$$

```
[Out] 1/4*x^4/a^2-1/2*b^2*ln(b+a*cosh(d*x^2+c))/a^2/(a^2-b^2)/d^2+1/2*b^3*x^2*ln(
1+a*exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-1/2*b^3*x^2*l
n(1+a*exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+1/2*b^3*pol
ylog(2,-a*exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-1/2*b
^3*polylog(2,-a*exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2
+1/2*b^2*x^2*sinh(d*x^2+c)/a/(a^2-b^2)/d/(b+a*cosh(d*x^2+c))-b*x^2*ln(1+a*e
xp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)+b*x^2*ln(1+a*exp(d
*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)-b*polylog(2,-a*exp(d*x
```

```
^2+c)/(b-(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+b*polylog(2,-a*exp(d*x
^2+c)/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5544, 4276, 3405, 3401, 2296, 2221, 2317, 2438, 2747, 31}

$$\int \frac{x^3}{(a+b\text{sech}(c+dx^2))^2}\,dx = -\frac{b\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2\sqrt{b^2-a^2}}+\frac{b\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2\sqrt{b^2-a^2}} -\frac{b^2\log(a\cosh(c+dx^2)+b)}{2a^2d^2(a^2-b^2)} -\frac{bx^2\log\left(\frac{ae^{c+dx^2}}{b-\sqrt{b^2-a^2}}+1\right)}{a^2d\sqrt{b^2-a^2}}+\frac{bx^2\log\left(\frac{ae^{c+dx^2}}{\sqrt{b^2-a^2}+b}+1\right)}{a^2d\sqrt{b^2-a^2}} +\frac{b^2x^2\sinh(c+dx^2)}{2ad(a^2-b^2)(a\cosh(c+dx^2)+b)} +\frac{b^3\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)}{2a^2d^2(b^2-a^2)^{3/2}}-\frac{b^3\,\text{PolyLog}\left(2,-\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)}{2a^2d^2(b^2-a^2)^{3/2}} +\frac{b^3x^2\log\left(\frac{ae^{c+dx^2}}{b-\sqrt{b^2-a^2}}+1\right)}{2a^2d(b^2-a^2)^{3/2}}-\frac{b^3x^2\log\left(\frac{ae^{c+dx^2}}{\sqrt{b^2-a^2}+b}+1\right)}{2a^2d(b^2-a^2)^{3/2}}+\frac{x^4}{4a^2}$$

```
[In] Int[x^3/(a + b*Sech[c + d*x^2])^2,x]
```

```
[Out] x^4/(4*a^2) + (b^3*x^2*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2])])/(
2*a^2*(-a^2 + b^2)^(3/2)*d) - (b*x^2*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[-a
^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - (b^3*x^2*Log[1 + (a*E^(c + d*x^2))/
(b + Sqrt[-a^2 + b^2])])/(2*a^2*(-a^2 + b^2)^(3/2)*d) + (b*x^2*Log[1 + (a*E
^(c + d*x^2))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - (b^2*Log[
b + a*Cosh[c + d*x^2]])/(2*a^2*(a^2 - b^2)*d^2) + (b^3*PolyLog[2, -((a*E^(c
 + d*x^2))/(b - Sqrt[-a^2 + b^2]))])/(2*a^2*(-a^2 + b^2)^(3/2)*d^2) - (b*Po
lyLog[2, -((a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2
]*d^2) - (b^3*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]))])/(2*a
^2*(-a^2 + b^2)^(3/2)*d^2) + (b*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[-a
^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (b^2*x^2*Sinh[c + d*x^2])/(2*a*(
a^2 - b^2)*d*(b + a*Cosh[c + d*x^2]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
```

```
x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+b\text{sech}(c+dx))^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{x}{a^2}+\frac{b^2x}{a^2(b+a\cosh(c+dx))^2}-\frac{2bx}{a^2(b+a\cosh(c+dx))}\right)dx, x, x^2\right)\\
&= \frac{x^4}{4a^2}-\frac{b\text{Subst}\left(\int \frac{x}{b+a\cosh(c+dx)}\,dx, x, x^2\right)}{a^2}+\frac{b^2\text{Subst}\left(\int \frac{x}{(b+a\cosh(c+dx))^2}\,dx, x, x^2\right)}{2a^2}\\
&= \frac{x^4}{4a^2}+\frac{b^2x^2\sinh\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+dx^2\right)\right)}-\frac{(2b)\text{Subst}\left(\int \frac{e^{c+dx}x}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^2\right)}{a^2}\\
&\quad-\frac{b^3\text{Subst}\left(\int \frac{x}{b+a\cosh(c+dx)}\,dx, x, x^2\right)}{2a^2\left(a^2-b^2\right)}-\frac{b^2\text{Subst}\left(\int \frac{\sinh(c+dx)}{b+a\cosh(c+dx)}\,dx, x, x^2\right)}{2a\left(a^2-b^2\right)d}\\
&= \frac{x^4}{4a^2}+\frac{b^2x^2\sinh\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+dx^2\right)\right)}-\frac{b^3\text{Subst}\left(\int \frac{e^{c+dx}x}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^2\right)}{a^2\left(a^2-b^2\right)}\\
&\quad-\frac{(2b)\text{Subst}\left(\int \frac{e^{c+dx}x}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{a\sqrt{-a^2+b^2}}\\
&\quad+\frac{(2b)\text{Subst}\left(\int \frac{e^{c+dx}x}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{a\sqrt{-a^2+b^2}}\\
&\quad-\frac{b^2\text{Subst}\left(\int \frac{1}{b+x}\,dx, x, a\cosh\left(c+dx^2\right)\right)}{2a^2\left(a^2-b^2\right)d^2}
\end{aligned}$$

$$= \frac{x^4}{4a^2} - \frac{bx^2 \log\left(1 + \frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{bx^2 \log\left(1 + \frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{b^2 \log\left(b + a\cosh\left(c+dx^2\right)\right)}{2a^2\left(a^2-b^2\right)d^2} + \frac{b^2x^2 \sinh\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+dx^2\right)\right)} + \frac{b^3 \text{Subst}\left(\int \frac{e^{c+dx}x}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{a\left(-a^2+b^2\right)^{3/2}} - \frac{b^3 \text{Subst}\left(\int \frac{e^{c+dx}x}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, x^2\right)}{a\left(-a^2+b^2\right)^{3/2}} + \frac{b\text{Subst}\left(\int \log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{b\text{Subst}\left(\int \log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$= \frac{x^4}{4a^2} + \frac{b^3x^2 \log\left(1 + \frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} - \frac{bx^2 \log\left(1 + \frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{b^3x^2 \log\left(1 + \frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} + \frac{bx^2 \log\left(1 + \frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{b^2 \log\left(b + a\cosh\left(c+dx^2\right)\right)}{2a^2\left(a^2-b^2\right)d^2} + \frac{b^2x^2 \sinh\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+dx^2\right)\right)} + \frac{b\text{Subst}\left(\int \frac{\log\left(1+\frac{2ax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{c+dx^2}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{b\text{Subst}\left(\int \frac{\log\left(1+\frac{2ax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{c+dx^2}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{b^3 \text{Subst}\left(\int \log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} + \frac{b^3 \text{Subst}\left(\int \log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d}$$

$$= \frac{x^4}{4a^2} + \frac{b^3x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} - \frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$- \frac{b^3x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} + \frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$- \frac{b^2\log\left(b+a\cosh\left(c+dx^2\right)\right)}{2a^2\left(a^2-b^2\right)d^2} - \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b^2x^2\sinh\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+dx^2\right)\right)}$$
$$- \frac{b^3\text{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{c+dx^2}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d^2}$$
$$+ \frac{b^3\text{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{c+dx^2}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d^2}$$

$$= \frac{x^4}{4a^2} + \frac{b^3x^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} - \frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$- \frac{b^3x^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} + \frac{bx^2\log\left(1+\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$- \frac{b^2\log\left(b+a\cosh\left(c+dx^2\right)\right)}{2a^2\left(a^2-b^2\right)d^2} + \frac{b^3\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d^2}$$
$$- \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{b^3\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d^2}$$
$$+ \frac{b\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b^2x^2\sinh\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+dx^2\right)\right)}$$

Mathematica [A] (verified)

Time = 5.34 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{\left(a+b\text{sech}\left(c+dx^2\right)\right)^2}\,dx$$

$$= \frac{\left(b+a\cosh\left(c+dx^2\right)\right)\text{sech}^2\left(c+dx^2\right)\left(\left(-c+dx^2\right)\left(c+dx^2\right)\left(b+a\cosh\left(c+dx^2\right)\right)-\frac{2b\left(a^2-b^2\right)\left(b+a\cosh\left(c+dx^2\right)\right)}{}\right.}{}$$

```
[In] Integrate[x^3/(a + b*Sech[c + d*x^2])^2,x]
```

```
[Out] ((b + a*Cosh[c + d*x^2])*Sech[c + d*x^2]^2*((-c + d*x^2)*(c + d*x^2)*(b + a
*Cosh[c + d*x^2]) - (2*b*(a^2 - b^2)*(b + a*Cosh[c + d*x^2])*(b*Sqrt[-(a^2
- b^2)^2]*(c + d*x^2) + 4*a^2*Sqrt[-a^2 + b^2]*c*ArcTan[(b + a*E^(c + d*x^2
))/Sqrt[a^2 - b^2]] - 2*b^2*Sqrt[-a^2 + b^2]*c*ArcTan[(b + a*E^(c + d*x^2))
/Sqrt[a^2 - b^2]] - 2*a^2*Sqrt[a^2 - b^2]*(c + d*x^2)*Log[1 + (a*E^(c + d*x
^2))/(b - Sqrt[-a^2 + b^2])] + b^2*Sqrt[a^2 - b^2]*(c + d*x^2)*Log[1 + (a*E
^(c + d*x^2))/(b - Sqrt[-a^2 + b^2])] + 2*a^2*Sqrt[a^2 - b^2]*(c + d*x^2)*L
og[1 + (a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2])] - b^2*Sqrt[a^2 - b^2]*(c +
 d*x^2)*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2])] - b*Sqrt[-(a^2 -
b^2)^2]*Log[a + 2*b*E^(c + d*x^2) + a*E^(2*(c + d*x^2))] + Sqrt[a^2 - b^2]*
(-2*a^2 + b^2)*PolyLog[2, (a*E^(c + d*x^2))/(-b + Sqrt[-a^2 + b^2])] + Sqrt
[a^2 - b^2]*(2*a^2 - b^2)*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b
^2]))]))/(-(a^2 - b^2)^2)^(3/2) + (2*a*b^2*d*x^2*Sinh[c + d*x^2])/((a - b)*
(a + b))))/(4*a^2*d^2*(a + b*Sech[c + d*x^2])^2)
```

Maple [F]

$$\int \frac{x^3}{\left(a+b\,\text{sech}\left(d\,x^2+c\right)\right)^2}dx$$

```
[In] int(x^3/(a+b*sech(d*x^2+c))^2,x)
```

```
[Out] int(x^3/(a+b*sech(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2473 vs. 2(497) = 994.

Time = 0.31 (sec) , antiderivative size = 2473, normalized size of antiderivative = 4.46

$$\int \frac{x^3}{(a+b\text{sech}(c+dx^2))^2} dx = \text{Too large to display}$$

```
[In] integrate(x^3/(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((a^5 - 2*a^3*b^2 + a*b^4)*d^2*x^4 + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*x^4
 + 4*(a^3*b^2 - a*b^4)*d*x^2 + 4*(a^3*b^2 - a*b^4)*c)*cosh(d*x^2 + c)^2 + (
(a^5 - 2*a^3*b^2 + a*b^4)*d^2*x^4 + 4*(a^3*b^2 - a*b^4)*d*x^2 + 4*(a^3*b^2
- a*b^4)*c)*sinh(d*x^2 + c)^2 + 2*(2*a^4*b - a^2*b^3 + (2*a^4*b - a^2*b^3)*
cosh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*sinh(d*x^2 + c)^2 + 2*(2*a^3*b^2 -
a*b^4)*cosh(d*x^2 + c) + 2*(2*a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*cosh(d*
x^2 + c))*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b*cosh(d*x^2 + c)
 + b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 -
 b^2)/a^2) + a)/a + 1) - 2*(2*a^4*b - a^2*b^3 + (2*a^4*b - a^2*b^3)*cosh(d*
x^2 + c)^2 + (2*a^4*b - a^2*b^3)*sinh(d*x^2 + c)^2 + 2*(2*a^3*b^2 - a*b^4)*
cosh(d*x^2 + c) + 2*(2*a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*cosh(d*x^2 + c
))*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b*cosh(d*x^2 + c) + b*si
nh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a
^2) + a)/a + 1) + 2*((2*a^4*b - a^2*b^3)*d*x^2 + ((2*a^4*b - a^2*b^3)*d*x^2
 + (2*a^4*b - a^2*b^3)*c)*cosh(d*x^2 + c)^2 + ((2*a^4*b - a^2*b^3)*d*x^2 +
(2*a^4*b - a^2*b^3)*c)*sinh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*c + 2*((2*a^
3*b^2 - a*b^4)*d*x^2 + (2*a^3*b^2 - a*b^4)*c)*cosh(d*x^2 + c) + 2*((2*a^3*b
^2 - a*b^4)*d*x^2 + (2*a^3*b^2 - a*b^4)*c + ((2*a^4*b - a^2*b^3)*d*x^2 + (2
*a^4*b - a^2*b^3)*c)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^
2)*log((b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c) + a*sinh
(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) - 2*((2*a^4*b - a^2*b^3)*d*x^2
+ ((2*a^4*b - a^2*b^3)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*cosh(d*x^2 + c)^2 + (
(2*a^4*b - a^2*b^3)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*sinh(d*x^2 + c)^2 + (2*a
^4*b - a^2*b^3)*c + 2*((2*a^3*b^2 - a*b^4)*d*x^2 + (2*a^3*b^2 - a*b^4)*c)*c
osh(d*x^2 + c) + 2*((2*a^3*b^2 - a*b^4)*d*x^2 + (2*a^3*b^2 - a*b^4)*c + ((2
*a^4*b - a^2*b^3)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*cosh(d*x^2 + c))*sinh(d*x^
2 + c))*sqrt(-(a^2 - b^2)/a^2)*log((b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) -
 (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) + 4
*(a^3*b^2 - a*b^4)*c + 2*((a^4*b - 2*a^2*b^3 + b^5)*d^2*x^4 + 2*(a^2*b^3 -
b^5)*d*x^2 + 4*(a^2*b^3 - b^5)*c)*cosh(d*x^2 + c) - 2*(a^3*b^2 - a*b^4 + (a
^3*b^2 - a*b^4)*cosh(d*x^2 + c)^2 + (a^3*b^2 - a*b^4)*sinh(d*x^2 + c)^2 + 2
*(a^2*b^3 - b^5)*cosh(d*x^2 + c) + 2*(a^2*b^3 - b^5 + (a^3*b^2 - a*b^4)*cos
h(d*x^2 + c))*sinh(d*x^2 + c) - ((2*a^4*b - a^2*b^3)*c*cosh(d*x^2 + c)^2 +
(2*a^4*b - a^2*b^3)*c*sinh(d*x^2 + c)^2 + 2*(2*a^3*b^2 - a*b^4)*c*cosh(d*x^
2 + c) + (2*a^4*b - a^2*b^3)*c + 2*((2*a^4*b - a^2*b^3)*c*cosh(d*x^2 + c) +
```

```
(2*a^3*b^2 - a*b^4)*c)*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))*log(2*a*co
sh(d*x^2 + c) + 2*a*sinh(d*x^2 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) - 2
*(a^3*b^2 - a*b^4 + (a^3*b^2 - a*b^4)*cosh(d*x^2 + c)^2 + (a^3*b^2 - a*b^4)
*sinh(d*x^2 + c)^2 + 2*(a^2*b^3 - b^5)*cosh(d*x^2 + c) + 2*(a^2*b^3 - b^5 +
 (a^3*b^2 - a*b^4)*cosh(d*x^2 + c))*sinh(d*x^2 + c) + ((2*a^4*b - a^2*b^3)*
c*cosh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*c*sinh(d*x^2 + c)^2 + 2*(2*a^3*b^
2 - a*b^4)*c*cosh(d*x^2 + c) + (2*a^4*b - a^2*b^3)*c + 2*((2*a^4*b - a^2*b^
3)*c*cosh(d*x^2 + c) + (2*a^3*b^2 - a*b^4)*c)*sinh(d*x^2 + c))*sqrt(-(a^2 -
 b^2)/a^2))*log(2*a*cosh(d*x^2 + c) + 2*a*sinh(d*x^2 + c) - 2*a*sqrt(-(a^2
- b^2)/a^2) + 2*b) + 2*((a^4*b - 2*a^2*b^3 + b^5)*d^2*x^4 + 2*(a^2*b^3 - b^
5)*d*x^2 + 4*(a^2*b^3 - b^5)*c + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*x^4 + 4*(a^
3*b^2 - a*b^4)*d*x^2 + 4*(a^3*b^2 - a*b^4)*c)*cosh(d*x^2 + c))*sinh(d*x^2 +
 c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d^2*cosh(d*x^2 + c)^2 + (a^7 - 2*a^5*b^2
+ a^3*b^4)*d^2*sinh(d*x^2 + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d^2*cosh
(d*x^2 + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d^2 + 2*((a^7 - 2*a^5*b^2 + a^3*b
^4)*d^2*cosh(d*x^2 + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^2)*sinh(d*x^2 + c
))
```

Sympy [F]

$$\int \frac{x^3}{(a+b\text{sech}\,(c+dx^2))^2}\,dx = \int \frac{x^3}{(a+b\,\text{sech}\,(c+dx^2))^2}\,dx$$

```
[In] integrate(x**3/(a+b*sech(d*x**2+c))**2,x)
[Out] Integral(x**3/(a + b*sech(c + d*x**2))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+b\text{sech}\,(c+dx^2))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{x^3}{(a + b\text{sech}\,(c + dx^2))^2}\,dx = \int \frac{x^3}{(b\,\text{sech}\,(dx^2 + c) + a)^2}\,dx$$

```
[In] integrate(x^3/(a+b*sech(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(x^3/(b*sech(d*x^2 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a + b\text{sech}\,(c + dx^2))^2}\,dx = \int \frac{x^3}{\left(a + \frac{b}{\cosh(d\,x^2+c)}\right)^2}\,dx$$

```
[In] int(x^3/(a + b/cosh(c + d*x^2))^2,x)
[Out] int(x^3/(a + b/cosh(c + d*x^2))^2, x)
```

3.26 $\int \frac{x^2}{\left(a+b\text{sech}(c+dx^2)\right)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{(a+b\text{sech}(c+dx^2))^2} dx = \text{Int}\left(\frac{x^2}{(a+b\text{sech}(c+dx^2))^2}, x\right)$$

```
[Out] Unintegrable(x^2/(a+b*sech(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{(a+b\text{sech}(c+dx^2))^2} dx = \int \frac{x^2}{(a+b\text{sech}(c+dx^2))^2} dx$$

```
[In] Int[x^2/(a + b*Sech[c + d*x^2])^2,x]
[Out] Defer[Int][x^2/(a + b*Sech[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{(a+b\text{sech}(c+dx^2))^2} dx$$

Mathematica [N/A]

Not integrable

Time = 19.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+b\text{sech}\,(c+dx^2))^2}\,dx = \int \frac{x^2}{(a+b\text{sech}\,(c+dx^2))^2}\,dx$$

```
[In] Integrate[x^2/(a + b*Sech[c + d*x^2])^2,x]
[Out] Integrate[x^2/(a + b*Sech[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+b\ \text{sech}\,(d\,x^2+c))^2}dx$$

```
[In] int(x^2/(a+b*sech(d*x^2+c))^2,x)
[Out] int(x^2/(a+b*sech(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^2}{(a+b\text{sech}\,(c+dx^2))^2}\,dx = \int \frac{x^2}{(b\,\text{sech}\,(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(x^2/(b^2*sech(d*x^2 + c)^2 + 2*a*b*sech(d*x^2 + c) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a+b\text{sech}\,(c+dx^2))^2}\,dx = \int \frac{x^2}{(a+b\,\text{sech}\,(c+dx^2))^2}\,dx$$

```
[In] integrate(x**2/(a+b*sech(d*x**2+c))**2,x)
[Out] Integral(x**2/(a + b*sech(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 300, normalized size of antiderivative = 16.67

$$\int \frac{x^2}{(a + b\text{sech}(c + dx^2))^2} \, dx = \int \frac{x^2}{(b\,\text{sech}\,(dx^2 + c) + a)^2} \, dx$$

```
[In] integrate(x^2/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*x^3*e^(2*d*x^2) - 3*a*b^2*x + (a^3*d
 - a*b^2*d)*x^3 - (3*b^3*x*e^c - 2*(a^2*b*d*e^c - b^3*d*e^c)*x^3)*e^(d*x^2)
)/(a^5*d - a^3*b^2*d + (a^5*d*e^(2*c) - a^3*b^2*d*e^(2*c))*e^(2*d*x^2) + 2*
(a^4*b*d*e^c - a^2*b^3*d*e^c)*e^(d*x^2)) - integrate(-(a*b^2 + (b^3*e^c - 2
*(2*a^2*b*d*e^c - b^3*d*e^c)*x^2)*e^(d*x^2))/(a^5*d - a^3*b^2*d + (a^5*d*e^
(2*c) - a^3*b^2*d*e^(2*c))*e^(2*d*x^2) + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*e^
(d*x^2)), x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a + b\text{sech}(c + dx^2))^2} \, dx = \int \frac{x^2}{(b\,\text{sech}\,(dx^2 + c) + a)^2} \, dx$$

```
[In] integrate(x^2/(a+b*sech(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*sech(d*x^2 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{(a + b\text{sech}(c + dx^2))^2} \, dx = \int \frac{x^2}{\left(a + \frac{b}{\cosh(d\,x^2+c)}\right)^2} \, dx$$

```
[In] int(x^2/(a + b/cosh(c + d*x^2))^2,x)
```

```
[Out] int(x^2/(a + b/cosh(c + d*x^2))^2, x)
```

3.27 $\int \frac{x}{\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 123

$$\int \frac{x}{\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2}\,dx = \frac{x^2}{2a^2} - \frac{b(2a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2\tanh\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)}$$

```
[Out] 1/2*x^2/a^2-b*(2*a^2-b^2)*arctan((a-b)^(1/2)*tanh(1/2*d*x^2+1/2*c)/(a+b)^(1
/2))/a^2/(a-b)^(3/2)/(a+b)^(3/2)/d+1/2*b^2*tanh(d*x^2+c)/a/(a^2-b^2)/d/(a+b
*sech(d*x^2+c))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5544, 3870, 4004, 3916, 2738, 214}

$$\int \frac{x}{\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2}\,dx = -\frac{b(2a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}} + \frac{b^2\tanh\left(c+dx^2\right)}{2ad\left(a^2-b^2\right)\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)} + \frac{x^2}{2a^2}$$

```
[In] Int[x/(a + b*Sech[c + d*x^2])^2,x]
```

```
[Out] x^2/(2*a^2) - (b*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x^2)/2])/Sqr
t[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (b^2*Tanh[c + d*x^2])/(2*a
*(a^2 - b^2)*d*(a + b*Sech[c + d*x^2]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+b\operatorname{sech}(c+dx))^2}\,dx, x, x^2\right)$$

$$= \frac{b^2 \tanh\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)} - \frac{\text{Subst}\left(\int \frac{-a^2+b^2+ab\operatorname{sech}(c+dx)}{a+b\operatorname{sech}(c+dx)}\,dx, x, x^2\right)}{2a\left(a^2-b^2\right)}$$

$$= \frac{x^2}{2a^2} + \frac{b^2 \tanh\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)} - \frac{\left(b(2a^2-b^2)\right)\text{Subst}\left(\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}(c+dx)}\,dx, x, x^2\right)}{2a^2\left(a^2-b^2\right)}$$

$$= \frac{x^2}{2a^2} + \frac{b^2 \tanh\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)} - \frac{\left(2a^2-b^2\right)\text{Subst}\left(\int \frac{1}{1+\frac{a\cosh(c+dx)}{b}}\,dx, x, x^2\right)}{2a^2\left(a^2-b^2\right)}$$

$$= \frac{x^2}{2a^2} + \frac{b^2 \tanh\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)} + \frac{\left(i(2a^2-b^2)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, i\tanh\left(\frac{1}{2}\left(c+dx^2\right)\right)\right)}{a^2\left(a^2-b^2\right)d}$$

$$= \frac{x^2}{2a^2} - \frac{b(2a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}\left(c+dx^2\right)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2 \tanh\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.79

$$\int \frac{x}{\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)^2}\,dx$$

$$= \frac{a\left(\left(a^2-b^2\right)^{3/2}\left(c+dx^2\right)+\left(4a^2b-2b^3\right)\arctan\left(\frac{(-a+b)\tanh\left(\frac{1}{2}\left(c+dx^2\right)\right)}{\sqrt{a^2-b^2}}\right)\right)\cosh\left(c+dx^2\right)+b\left(\left(a^2-b^2\right)^{3/2}(c-\right.}{2a^2(a-b)(a+b)\sqrt{a^2-b^2}d\left(b+a\cos\right.}$$

```
[In] Integrate[x/(a + b*Sech[c + d*x^2])^2,x]
```

```
[Out] (a*((a^2 - b^2)^(3/2)*(c + d*x^2) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh
[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])*Cosh[c + d*x^2] + b*((a^2 - b^2)^(3/2)*(
c + d*x^2) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*x^2)/2])/Sqrt[a
^2 - b^2]] + a*b*Sqrt[a^2 - b^2]*Sinh[c + d*x^2]))/(2*a^2*(a - b)*(a + b)*S
qrt[a^2 - b^2]*d*(b + a*Cosh[c + d*x^2]))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{\frac{\ln\left(1+\tanh\left(\frac{d x^2}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{2b\left(-\frac{ab\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)^2 a-\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)^2 b+a+b\right)}+\frac{\left(2a^2-b^2\right)\arctan\left(\frac{(a-b)\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right.}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right.}{a^2}}{2d}$
default	$\frac{\frac{\ln\left(1+\tanh\left(\frac{d x^2}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{2b\left(-\frac{ab\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)^2 a-\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)^2 b+a+b\right)}+\frac{\left(2a^2-b^2\right)\arctan\left(\frac{(a-b)\tanh\left(\frac{dx^2}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right.}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right.}{a^2}}{2d}$
risch	$\frac{x^2}{2a^2}-\frac{b^2\left(e^{d x^2+c}b+a\right)}{a^2(a^2-b^2)d\left(e^{2d x^2+2c}a+2\,e^{d x^2+c}b+a\right)}-\frac{b\ln\left(e^{d x^2+c}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}+\frac{b^3\ln\left(e^{d x^2+c}+\frac{b\sqrt{-a^2+b^2}+a}{\sqrt{-a^2+b^2}}\right.}{2\sqrt{-a^2+b^2}\,(a+b)(a-b)a}$

```
[In] int(x/(a+b*sech(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(1/a^2*ln(1+tanh(1/2*d*x^2+1/2*c))-2/a^2*b*(-a*b/(a^2-b^2)*tanh(1/2*d
*x^2+1/2*c)/(tanh(1/2*d*x^2+1/2*c)^2*a-tanh(1/2*d*x^2+1/2*c)^2*b+a+b)+(2*a^
2-b^2)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x^2+1/2*c)/(
(a+b)*(a-b))^(1/2)))-1/a^2*ln(tanh(1/2*d*x^2+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. 2(110) = 220.

Time = 0.30 (sec) , antiderivative size = 1314, normalized size of antiderivative = 10.68

$$\int \frac{x}{\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^5 - 2*a^3*b^2 + a*b^4)*d*x^2*cosh(d*x^2 + c)^2 + (a^5 - 2*a^3*b^2
+ a*b^4)*d*x^2*sinh(d*x^2 + c)^2 - 2*a^3*b^2 + 2*a*b^4 + (a^5 - 2*a^3*b^2 +
 a*b^4)*d*x^2 - (2*a^3*b - a*b^3 + (2*a^3*b - a*b^3)*cosh(d*x^2 + c)^2 + (2
*a^3*b - a*b^3)*sinh(d*x^2 + c)^2 + 2*(2*a^2*b^2 - b^4)*cosh(d*x^2 + c) + 2
*(2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqr
t(-a^2 + b^2)*log((a^2*cosh(d*x^2 + c)^2 + a^2*sinh(d*x^2 + c)^2 + 2*a*b*co
sh(d*x^2 + c) - a^2 + 2*b^2 + 2*(a^2*cosh(d*x^2 + c) + a*b)*sinh(d*x^2 + c)
 + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c) + b))/(a*cosh(
d*x^2 + c)^2 + a*sinh(d*x^2 + c)^2 + 2*b*cosh(d*x^2 + c) + 2*(a*cosh(d*x^2
```

```
+ c) + b)*sinh(d*x^2 + c) + a)) - 2*(a^2*b^3 - b^5 - (a^4*b - 2*a^2*b^3 + b
^5)*d*x^2)*cosh(d*x^2 + c) - 2*(a^2*b^3 - b^5 - (a^5 - 2*a^3*b^2 + a*b^4)*d
*x^2*cosh(d*x^2 + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*x^2)*sinh(d*x^2 + c))/((
a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x^2 + c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^4
)*d*sinh(d*x^2 + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cosh(d*x^2 + c) +
 (a^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x^
2 + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x^2 + c)), 1/2*((a^5 - 2*a
^3*b^2 + a*b^4)*d*x^2*cosh(d*x^2 + c)^2 + (a^5 - 2*a^3*b^2 + a*b^4)*d*x^2*s
inh(d*x^2 + c)^2 - 2*a^3*b^2 + 2*a*b^4 + (a^5 - 2*a^3*b^2 + a*b^4)*d*x^2 +
2*(2*a^3*b - a*b^3 + (2*a^3*b - a*b^3)*cosh(d*x^2 + c)^2 + (2*a^3*b - a*b^3
)*sinh(d*x^2 + c)^2 + 2*(2*a^2*b^2 - b^4)*cosh(d*x^2 + c) + 2*(2*a^2*b^2 -
b^4 + (2*a^3*b - a*b^3)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt(a^2 - b^2)*a
rctan(-(a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c) + b)/sqrt(a^2 - b^2)) - 2*(a^
2*b^3 - b^5 - (a^4*b - 2*a^2*b^3 + b^5)*d*x^2)*cosh(d*x^2 + c) - 2*(a^2*b^3
 - b^5 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x^2*cosh(d*x^2 + c) - (a^4*b - 2*a^2*b
^3 + b^5)*d*x^2)*sinh(d*x^2 + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x^2
 + c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^4)*d*sinh(d*x^2 + c)^2 + 2*(a^6*b - 2*a^
4*b^3 + a^2*b^5)*d*cosh(d*x^2 + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^
7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x^2 + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*
d)*sinh(d*x^2 + c))]
```

Sympy [F]

$$\int \frac{x}{(a+b\text{sech}(c+dx^2))^2}\,dx=\int \frac{x}{(a+b\,\text{sech}\,(c+dx^2))^2}\,dx$$

```
[In] integrate(x/(a+b*sech(d*x**2+c))**2,x)
```

```
[Out] Integral(x/(a + b*sech(c + d*x**2))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+b\text{sech}(c+dx^2))^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.20

$$\int \frac{x}{(a+b\operatorname{sech}(c+dx^2))^2}\,dx = -\frac{(2a^2b-b^3)\arctan\left(\frac{ae^{(dx^2+c)}+b}{\sqrt{a^2-b^2}}\right)}{(a^4d-a^2b^2d)\sqrt{a^2-b^2}} - \frac{b^3e^{(dx^2+c)}+ab^2}{(a^4d-a^2b^2d)(ae^{(2dx^2+2c)}+2be^{(dx^2+c)}+a)} + \frac{dx^2+c}{2a^2d}$$

```
[In] integrate(x/(a+b*sech(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a^2*b - b^3)*arctan((a*e^(d*x^2 + c) + b)/sqrt(a^2 - b^2))/((a^4*d - a^
2*b^2*d)*sqrt(a^2 - b^2)) - (b^3*e^(d*x^2 + c) + a*b^2)/((a^4*d - a^2*b^2*d
)*(a*e^(2*d*x^2 + 2*c) + 2*b*e^(d*x^2 + c) + a)) + 1/2*(d*x^2 + c)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.57

$$\int \frac{x}{(a+b\operatorname{sech}(c+dx^2))^2}\,dx = \frac{\frac{b^2}{d\,(a\,b^2-a^3)}+\frac{b^3\,e^{d\,x^2+c}}{a\,d\,(a\,b^2-a^3)}}{a+2\,b\,e^{d\,x^2+c}+a\,e^{2\,d\,x^2+2\,c}} + \frac{x^2}{2\,a^2} + \frac{b\,\ln\left(\frac{2\,b\,x\,e^{d\,x^2+c}\,(2\,a^2-b^2)}{a^3\,(a^2-b^2)}-\frac{2\,b\,x\,\left(a+b\,e^{d\,x^2+c}\right)\,(2\,a^2-b^2)}{a^3\,(a+b)^{3/2}\,(b-a)^{3/2}}\right)\,(2\,a^2-b^2)}{2\,a^2\,d\,(a+b)^{3/2}\,(b-a)^{3/2}} - \frac{b\,\ln\left(\frac{2\,b\,x\,e^{d\,x^2+c}\,(2\,a^2-b^2)}{a^3\,(a^2-b^2)}+\frac{2\,b\,x\,\left(a+b\,e^{d\,x^2+c}\right)\,(2\,a^2-b^2)}{a^3\,(a+b)^{3/2}\,(b-a)^{3/2}}\right)\,(2\,a^2-b^2)}{2\,a^2\,d\,(a+b)^{3/2}\,(b-a)^{3/2}}$$

```
[In] int(x/(a + b/cosh(c + d*x^2))^2,x)
```

```
[Out] (b^2/(d*(a*b^2 - a^3)) + (b^3*exp(c + d*x^2))/(a*d*(a*b^2 - a^3)))/(a + 2*b
*exp(c + d*x^2) + a*exp(2*c + 2*d*x^2)) + x^2/(2*a^2) + (b*log((2*b*x*exp(c
 + d*x^2)*(2*a^2 - b^2))/(a^3*(a^2 - b^2)) - (2*b*x*(a + b*exp(c + d*x^2))*
(2*a^2 - b^2))/(a^3*(a + b)^(3/2)*(b - a)^(3/2)))*(2*a^2 - b^2))/(2*a^2*d*(
a + b)^(3/2)*(b - a)^(3/2)) - (b*log((2*b*x*exp(c + d*x^2)*(2*a^2 - b^2))/(
a^3*(a^2 - b^2)) + (2*b*x*(a + b*exp(c + d*x^2))*(2*a^2 - b^2))/(a^3*(a + b
)^(3/2)*(b - a)^(3/2)))*(2*a^2 - b^2))/(2*a^2*d*(a + b)^(3/2)*(b - a)^(3/2)
)
```

3.28 $\int \frac{1}{x\left(a+b\,\mathbf{sech}\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sech(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Int[1/(x*(a + b*Sech[c + d*x^2])^2),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Sech[c + d*x^2])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 53.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x*(a + b*Sech[c + d*x^2])^2),x]
[Out] Integrate[1/(x*(a + b*Sech[c + d*x^2])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\ \operatorname{sech}\left(d\,x^2+c\right))^2}dx$$

```
[In] int(1/x/(a+b*sech(d*x^2+c))^2,x)
[Out] int(1/x/(a+b*sech(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{sech}\left(dx^2+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x*sech(d*x^2 + c)^2 + 2*a*b*x*sech(d*x^2 + c) + a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] integrate(1/x/(a+b*sech(d*x**2+c))**2,x)
[Out] Integral(1/(x*(a + b*sech(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 249, normalized size of antiderivative = 13.83

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{sech}\left(dx^2+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] -(b^3*e^(d*x^2 + c) + a*b^2)/((a^5*d*e^(2*c) - a^3*b^2*d*e^(2*c))*x^2*e^(2*
d*x^2) + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^2*e^(d*x^2) + (a^5*d - a^3*b^2*d
)*x^2) + log(x)/a^2 - integrate(2*(a*b^2 + (b^3*e^c + (2*a^2*b*d*e^c - b^3*
d*e^c)*x^2)*e^(d*x^2))/((a^5*d*e^(2*c) - a^3*b^2*d*e^(2*c))*x^3*e^(2*d*x^2)
 + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^3*e^(d*x^2) + (a^5*d - a^3*b^2*d)*x^3)
, x)
```

Giac [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{sech}\left(dx^2+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*sech(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sech(d*x^2 + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+\frac{b}{\cosh\left(d\,x^2+c\right)}\right)^2}\,dx$$

```
[In] int(1/(x*(a + b/cosh(c + d*x^2))^2),x)
```

```
[Out] int(1/(x*(a + b/cosh(c + d*x^2))^2), x)
```

3.29 $\int \frac{1}{x^2\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2}\,dx=\mathrm{Int}\left(\frac{1}{x^2\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*sech(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Int[1/(x^2*(a + b*Sech[c + d*x^2])^2),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*Sech[c + d*x^2])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^2\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 26.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b\text{sech}(c + dx^2))^2} dx = \int \frac{1}{x^2 (a + b\text{sech}(c + dx^2))^2} dx$$

```
[In] Integrate[1/(x^2*(a + b*Sech[c + d*x^2])^2),x]
[Out] Integrate[1/(x^2*(a + b*Sech[c + d*x^2])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a + b \operatorname{sech}(d x^2 + c))^2} dx$$

```
[In] int(1/x^2/(a+b*sech(d*x^2+c))^2,x)
[Out] int(1/x^2/(a+b*sech(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^2 (a + b\text{sech}(c + dx^2))^2} dx = \int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)^2 x^2} dx$$

```
[In] integrate(1/x^2/(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^2*sech(d*x^2 + c)^2 + 2*a*b*x^2*sech(d*x^2 + c) + a^2*x^2
), x)
```

Sympy [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 (a+b\text{sech}(c+dx^2))^2} dx = \int \frac{1}{x^2 (a+b\,\text{sech}(c+dx^2))^2} dx$$

```
[In] integrate(1/x**2/(a+b*sech(d*x**2+c))**2,x)
[Out] Integral(1/(x**2*(a + b*sech(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 318, normalized size of antiderivative = 17.67

$$\int \frac{1}{x^2 (a+b\text{sech}(c+dx^2))^2} dx = \int \frac{1}{(b\,\text{sech}(dx^2+c)+a)^2 x^2} dx$$

```
[In] integrate(1/x^2/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
[Out] -((a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*x^2*e^(2*d*x^2) + a*b^2 + (a^3*d - a*b^
2*d)*x^2 + (b^3*e^c + 2*(a^2*b*d*e^c - b^3*d*e^c)*x^2)*e^(d*x^2))/((a^5*d*e
^(2*c) - a^3*b^2*d*e^(2*c))*x^3*e^(2*d*x^2) + 2*(a^4*b*d*e^c - a^2*b^3*d*e^
c)*x^3*e^(d*x^2) + (a^5*d - a^3*b^2*d)*x^3) - integrate((3*a*b^2 + (3*b^3*e
^c + 2*(2*a^2*b*d*e^c - b^3*d*e^c)*x^2)*e^(d*x^2))/((a^5*d*e^(2*c) - a^3*b^
2*d*e^(2*c))*x^4*e^(2*d*x^2) + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^4*e^(d*x^2
) + (a^5*d - a^3*b^2*d)*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a+b\text{sech}(c+dx^2))^2} dx = \int \frac{1}{(b\,\text{sech}(dx^2+c)+a)^2 x^2} dx$$

```
[In] integrate(1/x^2/(a+b*sech(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*sech(d*x^2 + c) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^2 (a + b\text{sech}(c + dx^2))^2} dx = \int \frac{1}{x^2 \left(a + \frac{b}{\cosh(d x^2+c)}\right)^2} dx$$

```
[In] int(1/(x^2*(a + b/cosh(c + d*x^2))^2),x)
[Out] int(1/(x^2*(a + b/cosh(c + d*x^2))^2), x)
```

3.30 $\int \frac{1}{x^3\left(a+b\,\mathrm{sech}(c+dx^2)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2}\,dx=\mathrm{Int}\left(\frac{1}{x^3\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^3/(a+b*sech(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^3\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Int[1/(x^3*(a + b*Sech[c + d*x^2])^2),x]
```

```
[Out] Defer[Int][1/(x^3*(a + b*Sech[c + d*x^2])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^3\left(a+b\,\mathrm{sech}\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 28.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \left(a+b\text{sech}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^3 \left(a+b\text{sech}\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x^3*(a + b*Sech[c + d*x^2])^2),x]
[Out] Integrate[1/(x^3*(a + b*Sech[c + d*x^2])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \left(a+b\ \text{sech}\left(d\,x^2+c\right)\right)^2}dx$$

```
[In] int(1/x^3/(a+b*sech(d*x^2+c))^2,x)
[Out] int(1/x^3/(a+b*sech(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^3 \left(a+b\text{sech}\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{sech}\left(dx^2+c\right)+a\right)^2 x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^3*sech(d*x^2 + c)^2 + 2*a*b*x^3*sech(d*x^2 + c) + a^2*x^3
), x)
```

Sympy [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 (a + b\text{sech}(c + dx^2))^2} dx = \int \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

```
[In] integrate(1/x**3/(a+b*sech(d*x**2+c))**2,x)
[Out] Integral(1/(x**3*(a + b*sech(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 319, normalized size of antiderivative = 17.72

$$\int \frac{1}{x^3 (a + b\text{sech}(c + dx^2))^2} dx = \int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)^2 x^3} dx$$

```
[In] integrate(1/x^3/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
[Out] -1/2*((a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*x^2*e^(2*d*x^2) + 2*a*b^2 + (a^3*d
- a*b^2*d)*x^2 + 2*(b^3*e^c + (a^2*b*d*e^c - b^3*d*e^c)*x^2)*e^(d*x^2))/((a
^5*d*e^(2*c) - a^3*b^2*d*e^(2*c))*x^4*e^(2*d*x^2) + 2*(a^4*b*d*e^c - a^2*b^
3*d*e^c)*x^4*e^(d*x^2) + (a^5*d - a^3*b^2*d)*x^4) - integrate(2*(2*a*b^2 +
(2*b^3*e^c + (2*a^2*b*d*e^c - b^3*d*e^c)*x^2)*e^(d*x^2))/((a^5*d*e^(2*c) -
a^3*b^2*d*e^(2*c))*x^5*e^(2*d*x^2) + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^5*e^
(d*x^2) + (a^5*d - a^3*b^2*d)*x^5), x)
```

Giac [N/A]

Not integrable

Time = 3.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^3 (a + b\text{sech}(c + dx^2))^2} dx = \int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)^2 x^3} dx$$

```
[In] integrate(1/x^3/(a+b*sech(d*x^2+c))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3 (a + b\text{sech}(c + dx^2))^2} dx = \int \frac{1}{x^3 \left(a + \frac{b}{\cosh(d x^2 + c)}\right)^2} dx$$

```
[In] int(1/(x^3*(a + b/cosh(c + d*x^2))^2),x)
[Out] int(1/(x^3*(a + b/cosh(c + d*x^2))^2), x)
```

3.31 $\int \frac{\text{sech}^2\left(\frac{1}{x}\right)}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 6

$$\int \frac{\text{sech}^2\left(\frac{1}{x}\right)}{x^2} dx = -\tanh\left(\frac{1}{x}\right)$$

```
[Out] -tanh(1/x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5544, 3852, 8}

$$\int \frac{\text{sech}^2\left(\frac{1}{x}\right)}{x^2} dx = -\tanh\left(\frac{1}{x}\right)$$

```
[In] Int[Sech[x^(-1)]^2/x^2,x]
```

```
[Out] -Tanh[x^(-1)]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \text{sech}^2(x)\,dx, x, \frac{1}{x}\right)\\ &= -\left(i\text{Subst}\left(\int 1\,dx, x, -i\tanh\left(\frac{1}{x}\right)\right)\right)\\ &= -\tanh\left(\frac{1}{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}^2\left(\frac{1}{x}\right)}{x^2}\,dx = -\tanh\left(\frac{1}{x}\right)$$

```
[In] Integrate[Sech[x^(-1)]^2/x^2,x]
[Out] -Tanh[x^(-1)]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$-\tanh\left(\frac{1}{x}\right)$	7
default	$-\tanh\left(\frac{1}{x}\right)$	7
risch	$\frac{2}{e^{\frac{2}{x}}+1}$	13
parallelrisch	$-\frac{2\tanh\left(\frac{1}{2x}\right)}{1+\tanh\left(\frac{1}{2x}\right)^2}$	21

```
[In] int(sech(1/x)^2/x^2,x,method=_RETURNVERBOSE)
[Out] -tanh(1/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(6) = 12$.

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 4.67

$$\int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2}\,dx = \frac{2}{\cosh\left(\frac{1}{x}\right)^2 + 2\,\cosh\left(\frac{1}{x}\right)\sinh\left(\frac{1}{x}\right) + \sinh\left(\frac{1}{x}\right)^2 + 1}$$

```
[In] integrate(sech(1/x)^2/x^2,x, algorithm="fricas")
[Out] 2/(cosh(1/x)^2 + 2*cosh(1/x)*sinh(1/x) + sinh(1/x)^2 + 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2}\,dx = \int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2}\,dx$$

```
[In] integrate(sech(1/x)**2/x**2,x)
[Out] Integral(sech(1/x)**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2}\,dx = \frac{2}{e^{\frac{2}{x}} + 1}$$

```
[In] integrate(sech(1/x)^2/x^2,x, algorithm="maxima")
[Out] 2/(e^(2/x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2}\,dx = \frac{2}{e^{\frac{2}{x}} + 1}$$

```
[In] integrate(sech(1/x)^2/x^2,x, algorithm="giac")
[Out] 2/(e^(2/x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2}\,dx = \frac{2}{e^{2/x}+1}$$

```
[In] int(1/(x^2*cosh(1/x)^2),x)
[Out] 2/(exp(2/x) + 1)
```

3.32 $\int x^3 \left(a+b\,\mathbf{sech}\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 426

$$\int x^3\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^4}{4}+\frac{4bx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{14ibx^3\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{14ibx^3\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{84ibx^{5/2}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{84ibx^{5/2}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{420ibx^2\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{420ibx^2\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{1680ibx^{3/2}\,\text{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5}-\frac{1680ibx^{3/2}\,\text{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5}-\frac{5040ibx\,\text{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6}+\frac{5040ibx\,\text{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6}+\frac{10080ib\sqrt{x}\,\text{PolyLog}\left(7,-ie^{c+d\sqrt{x}}\right)}{d^7}-\frac{10080ib\sqrt{x}\,\text{PolyLog}\left(7,ie^{c+d\sqrt{x}}\right)}{d^7}-\frac{10080ib\,\text{PolyLog}\left(8,-ie^{c+d\sqrt{x}}\right)}{d^8}+\frac{10080ib\,\text{PolyLog}\left(8,ie^{c+d\sqrt{x}}\right)}{d^8}$$

```
[Out] 1/4*a*x^4+4*b*x^(7/2)*arctan(exp(c+d*x^(1/2)))/d-420*I*b*x^2*polylog(4,-I*e
xp(c+d*x^(1/2)))/d^4+5040*I*b*x*polylog(6,I*exp(c+d*x^(1/2)))/d^6+14*I*b*x^
3*polylog(2,I*exp(c+d*x^(1/2)))/d^2-5040*I*b*x*polylog(6,-I*exp(c+d*x^(1/2)
))/d^6+420*I*b*x^2*polylog(4,I*exp(c+d*x^(1/2)))/d^4-10080*I*b*polylog(7,I*
exp(c+d*x^(1/2)))*x^(1/2)/d^7+84*I*b*x^(5/2)*polylog(3,-I*exp(c+d*x^(1/2)))
/d^3+10080*I*b*polylog(7,-I*exp(c+d*x^(1/2)))*x^(1/2)/d^7-14*I*b*x^3*polylo
g(2,-I*exp(c+d*x^(1/2)))/d^2-1680*I*b*x^(3/2)*polylog(5,I*exp(c+d*x^(1/2)))
/d^5+10080*I*b*polylog(8,I*exp(c+d*x^(1/2)))/d^8-84*I*b*x^(5/2)*polylog(3,I
*exp(c+d*x^(1/2)))/d^3-10080*I*b*polylog(8,-I*exp(c+d*x^(1/2)))/d^8+1680*I*
b*x^(3/2)*polylog(5,-I*exp(c+d*x^(1/2)))/d^5
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {14, 5544, 4265, 2611, 6744, 2320, 6724}

$$\int x^3\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^4}{4}+\frac{4bx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{10080ib\operatorname{PolyLog}\left(8,-ie^{c+d\sqrt{x}}\right)}{d^8}+\frac{10080ib\operatorname{PolyLog}\left(8,ie^{c+d\sqrt{x}}\right)}{d^8}+\frac{10080ib\sqrt{x}\operatorname{PolyLog}\left(7,-ie^{c+d\sqrt{x}}\right)}{d^7}-\frac{10080ib\sqrt{x}\operatorname{PolyLog}\left(7,ie^{c+d\sqrt{x}}\right)}{d^7}-\frac{5040ibx\operatorname{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6}+\frac{5040ibx\operatorname{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6}+\frac{1680ibx^{3/2}\operatorname{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5}-\frac{1680ibx^{3/2}\operatorname{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5}-\frac{420ibx^2\operatorname{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{420ibx^2\operatorname{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{14ibx^3\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{14ibx^3\operatorname{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}$$

```
[In] Int[x^3*(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^4)/4 + (4*b*x^(7/2)*ArcTan[E^(c + d*Sqrt[x])])/d - ((14*I)*b*x^3*PolyL
og[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((14*I)*b*x^3*PolyLog[2, I*E^(c + d*Sq
rt[x])])/d^2 + ((84*I)*b*x^(5/2)*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 -
((84*I)*b*x^(5/2)*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 - ((420*I)*b*x^2*Pol
```

```
yLog[4, (-I)*E^(c + d*Sqrt[x])])/d^4 + ((420*I)*b*x^2*PolyLog[4, I*E^(c + d
*Sqrt[x])])/d^4 + ((1680*I)*b*x^(3/2)*PolyLog[5, (-I)*E^(c + d*Sqrt[x])])/d
^5 - ((1680*I)*b*x^(3/2)*PolyLog[5, I*E^(c + d*Sqrt[x])])/d^5 - ((5040*I)*b
*x*PolyLog[6, (-I)*E^(c + d*Sqrt[x])])/d^6 + ((5040*I)*b*x*PolyLog[6, I*E^(
c + d*Sqrt[x])])/d^6 + ((10080*I)*b*Sqrt[x]*PolyLog[7, (-I)*E^(c + d*Sqrt[x
])])/d^7 - ((10080*I)*b*Sqrt[x]*PolyLog[7, I*E^(c + d*Sqrt[x])])/d^7 - ((10
080*I)*b*PolyLog[8, (-I)*E^(c + d*Sqrt[x])])/d^8 + ((10080*I)*b*PolyLog[8,
I*E^(c + d*Sqrt[x])])/d^8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^3 + bx^3\operatorname{sech}\left(c + d\sqrt{x}\right)\right) dx \\
&= \frac{ax^4}{4} + b\int x^3\operatorname{sech}\left(c + d\sqrt{x}\right) dx \\
&= \frac{ax^4}{4} + (2b)\text{Subst}\left(\int x^7\operatorname{sech}(c + dx)\, dx, x, \sqrt{x}\right) \\
&= \frac{ax^4}{4} + \frac{4bx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{(14ib)\text{Subst}\left(\int x^6\log\left(1 - ie^{c+dx}\right) dx, x, \sqrt{x}\right)}{d} \\
&\quad + \frac{(14ib)\text{Subst}\left(\int x^6\log\left(1 + ie^{c+dx}\right) dx, x, \sqrt{x}\right)}{d} \\
&= \frac{ax^4}{4} + \frac{4bx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14ibx^3\operatorname{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{14ibx^3\operatorname{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{(84ib)\text{Subst}\left(\int x^5\operatorname{PolyLog}\left(2, -ie^{c+dx}\right) dx, x, \sqrt{x}\right)}{d^2} \\
&\quad - \frac{(84ib)\text{Subst}\left(\int x^5\operatorname{PolyLog}\left(2, ie^{c+dx}\right) dx, x, \sqrt{x}\right)}{d^2} \\
&= \frac{ax^4}{4} + \frac{4bx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14ibx^3\operatorname{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{14ibx^3\operatorname{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3, -ie^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3, ie^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{(420ib)\text{Subst}\left(\int x^4\operatorname{PolyLog}\left(3, -ie^{c+dx}\right) dx, x, \sqrt{x}\right)}{d^3} \\
&\quad + \frac{(420ib)\text{Subst}\left(\int x^4\operatorname{PolyLog}\left(3, ie^{c+dx}\right) dx, x, \sqrt{x}\right)}{d^3}
\end{aligned}$$

$$= \frac{ax^4}{4} + \frac{4bx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14ibx^3\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{14ibx^3\operatorname{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{420ibx^2\operatorname{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{420ibx^2\operatorname{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{(1680ib)\operatorname{Subst}\left(\int x^3\operatorname{PolyLog}\left(4,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^4} - \frac{(1680ib)\operatorname{Subst}\left(\int x^3\operatorname{PolyLog}\left(4,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$

$$= \frac{ax^4}{4} + \frac{4bx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14ibx^3\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{14ibx^3\operatorname{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{420ibx^2\operatorname{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{420ibx^2\operatorname{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{1680ibx^{3/2}\operatorname{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{1680ibx^{3/2}\operatorname{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{(5040ib)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(5,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^5} + \frac{(5040ib)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(5,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^5}$$

$$= \frac{ax^4}{4} + \frac{4bx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14ibx^3\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{14ibx^3\operatorname{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{420ibx^2\operatorname{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{420ibx^2\operatorname{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{1680ibx^{3/2}\operatorname{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{1680ibx^{3/2}\operatorname{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{5040ibx\operatorname{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{5040ibx\operatorname{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{(10080ib)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(6,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^6} - \frac{(10080ib)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(6,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^6}$$

$$= \frac{ax^4}{4} + \frac{4bx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14ibx^3\,\text{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{14ibx^3\,\text{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{84ibx^{5/2}\,\text{PolyLog}\left(3, -ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{84ibx^{5/2}\,\text{PolyLog}\left(3, ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{420ibx^2\,\text{PolyLog}\left(4, -ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{420ibx^2\,\text{PolyLog}\left(4, ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{1680ibx^{3/2}\,\text{PolyLog}\left(5, -ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{1680ibx^{3/2}\,\text{PolyLog}\left(5, ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{5040ibx\,\text{PolyLog}\left(6, -ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{5040ibx\,\text{PolyLog}\left(6, ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{10080ib\sqrt{x}\,\text{PolyLog}\left(7, -ie^{c+d\sqrt{x}}\right)}{d^7} - \frac{10080ib\sqrt{x}\,\text{PolyLog}\left(7, ie^{c+d\sqrt{x}}\right)}{d^7} - \frac{(10080ib)\text{Subst}\left(\int \text{PolyLog}\left(7, -ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^7} + \frac{(10080ib)\text{Subst}\left(\int \text{PolyLog}\left(7, ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^7}$$

$$= \frac{ax^4}{4} + \frac{4bx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14ibx^3\,\text{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{14ibx^3\,\text{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{84ibx^{5/2}\,\text{PolyLog}\left(3, -ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{84ibx^{5/2}\,\text{PolyLog}\left(3, ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{420ibx^2\,\text{PolyLog}\left(4, -ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{420ibx^2\,\text{PolyLog}\left(4, ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{1680ibx^{3/2}\,\text{PolyLog}\left(5, -ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{1680ibx^{3/2}\,\text{PolyLog}\left(5, ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{5040ibx\,\text{PolyLog}\left(6, -ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{5040ibx\,\text{PolyLog}\left(6, ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{10080ib\sqrt{x}\,\text{PolyLog}\left(7, -ie^{c+d\sqrt{x}}\right)}{d^7} - \frac{10080ib\sqrt{x}\,\text{PolyLog}\left(7, ie^{c+d\sqrt{x}}\right)}{d^7} - \frac{(10080ib)\text{Subst}\left(\int \frac{\text{PolyLog}(7,-ix)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{d^8} + \frac{(10080ib)\text{Subst}\left(\int \frac{\text{PolyLog}(7,ix)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{d^8}$$

$$= \frac{ax^4}{4} + \frac{4bx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14ibx^3 \operatorname{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{14ibx^3 \operatorname{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{84ibx^{5/2} \operatorname{PolyLog}\left(3, -ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{84ibx^{5/2} \operatorname{PolyLog}\left(3, ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{420ibx^2 \operatorname{PolyLog}\left(4, -ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{420ibx^2 \operatorname{PolyLog}\left(4, ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{1680ibx^{3/2} \operatorname{PolyLog}\left(5, -ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{1680ibx^{3/2} \operatorname{PolyLog}\left(5, ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{5040ibx \operatorname{PolyLog}\left(6, -ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{5040ibx \operatorname{PolyLog}\left(6, ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{10080ib\sqrt{x} \operatorname{PolyLog}\left(7, -ie^{c+d\sqrt{x}}\right)}{d^7} - \frac{10080ib\sqrt{x} \operatorname{PolyLog}\left(7, ie^{c+d\sqrt{x}}\right)}{d^7} - \frac{10080ib \operatorname{PolyLog}\left(8, -ie^{c+d\sqrt{x}}\right)}{d^8} + \frac{10080ib \operatorname{PolyLog}\left(8, ie^{c+d\sqrt{x}}\right)}{d^8}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.97

$$\int x^3\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)\,dx = \frac{ax^4}{4} + \frac{2ib\left(d^7x^{7/2}\log\left(1-ie^{c+d\sqrt{x}}\right) - d^7x^{7/2}\log\left(1+ie^{c+d\sqrt{x}}\right) - 7d^6x^3\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right) + 7d^6x^3\operatorname{PolyLog}$$

```
[In] Integrate[x^3*(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^4)/4 + ((2*I)*b*(d^7*x^(7/2)*Log[1 - I*E^(c + d*Sqrt[x])] - d^7*x^(7/2
)*Log[1 + I*E^(c + d*Sqrt[x])] - 7*d^6*x^3*PolyLog[2, (-I)*E^(c + d*Sqrt[x]
)] + 7*d^6*x^3*PolyLog[2, I*E^(c + d*Sqrt[x])] + 42*d^5*x^(5/2)*PolyLog[3,
(-I)*E^(c + d*Sqrt[x])] - 42*d^5*x^(5/2)*PolyLog[3, I*E^(c + d*Sqrt[x])] -
210*d^4*x^2*PolyLog[4, (-I)*E^(c + d*Sqrt[x])] + 210*d^4*x^2*PolyLog[4, I*E
^(c + d*Sqrt[x])] + 840*d^3*x^(3/2)*PolyLog[5, (-I)*E^(c + d*Sqrt[x])] - 84
0*d^3*x^(3/2)*PolyLog[5, I*E^(c + d*Sqrt[x])] - 2520*d^2*x*PolyLog[6, (-I)*
E^(c + d*Sqrt[x])] + 2520*d^2*x*PolyLog[6, I*E^(c + d*Sqrt[x])] + 5040*d*Sq
rt[x]*PolyLog[7, (-I)*E^(c + d*Sqrt[x])] - 5040*d*Sqrt[x]*PolyLog[7, I*E^(c
 + d*Sqrt[x])] - 5040*PolyLog[8, (-I)*E^(c + d*Sqrt[x])] + 5040*PolyLog[8,
I*E^(c + d*Sqrt[x])]))/d^8
```

Maple [F]

$$\int x^3 \left(a + b\ \mathrm{sech}\left(c + d\sqrt{x}\right)\right) dx$$

```
[In] int(x^3*(a+b*sech(c+d*x^(1/2))),x)
[Out] int(x^3*(a+b*sech(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x^3 \left(a + b\mathrm{sech}\left(c + d\sqrt{x}\right)\right)\ dx = \int \left(b\ \mathrm{sech}\left(d\sqrt{x} + c\right) + a\right)x^3\ dx$$

```
[In] integrate(x^3*(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(b*x^3*sech(d*sqrt(x) + c) + a*x^3, x)
```

Sympy [F]

$$\int x^3 \left(a + b\mathrm{sech}\left(c + d\sqrt{x}\right)\right)\ dx = \int x^3 \left(a + b\ \mathrm{sech}\left(c + d\sqrt{x}\right)\right)\ dx$$

```
[In] integrate(x**3*(a+b*sech(c+d*x**(1/2))),x)
[Out] Integral(x**3*(a + b*sech(c + d*sqrt(x))), x)
```

Maxima [F]

$$\int x^3 \left(a + b\mathrm{sech}\left(c + d\sqrt{x}\right)\right)\ dx = \int \left(b\ \mathrm{sech}\left(d\sqrt{x} + c\right) + a\right)x^3\ dx$$

```
[In] integrate(x^3*(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")
[Out] 1/4*a*x^4 + 2*b*integrate(x^3*e^(d*sqrt(x) + c)/(e^(2*d*sqrt(x) + 2*c) + 1)
, x)
```

Giac [F]

$$\int x^3\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)x^3\,dx$$

```
[In] integrate(x^3*(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int x^3\left(a+\frac{b}{\cosh\left(c+d\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x^3*(a + b/cosh(c + d*x^(1/2))),x)
[Out] int(x^3*(a + b/cosh(c + d*x^(1/2))), x)
```

3.33 $\int x^2 \left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 310

$$\int x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^3}{3}+\frac{4bx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{10ibx^2\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{10ibx^2\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{40ibx^{3/2}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{40ibx^{3/2}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{120ibx\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{120ibx\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5}-\frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5}-\frac{240ib\,\text{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6}+\frac{240ib\,\text{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6}$$

```
[Out] 1/3*a*x^3+4*b*x^(5/2)*arctan(exp(c+d*x^(1/2)))/d-10*I*b*x^2*polylog(2,-I*ex
p(c+d*x^(1/2)))/d^2+10*I*b*x^2*polylog(2,I*exp(c+d*x^(1/2)))/d^2+40*I*b*x^(
3/2)*polylog(3,-I*exp(c+d*x^(1/2)))/d^3-40*I*b*x^(3/2)*polylog(3,I*exp(c+d*
x^(1/2)))/d^3-120*I*b*x*polylog(4,-I*exp(c+d*x^(1/2)))/d^4+120*I*b*x*polylo
g(4,I*exp(c+d*x^(1/2)))/d^4-240*I*b*polylog(6,-I*exp(c+d*x^(1/2)))/d^6+240*
```

```
I*b*polylog(6,I*exp(c+d*x^(1/2)))/d^6+240*I*b*polylog(5,-I*exp(c+d*x^(1/2))
)*x^(1/2)/d^5-240*I*b*polylog(5,I*exp(c+d*x^(1/2)))*x^(1/2)/d^5
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {14, 5544, 4265, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=&\frac{ax^3}{3}+\frac{4bx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{240ib\,\text{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6}\\&+\frac{240ib\,\text{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6}\\&+\frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5}\\&-\frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5}\\&-\frac{120ibx\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4}\\&+\frac{120ibx\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4}\\&+\frac{40ibx^{3/2}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}\\&-\frac{40ibx^{3/2}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}\\&-\frac{10ibx^2\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}\\&+\frac{10ibx^2\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}\end{aligned}$$

```
[In] Int[x^2*(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^3)/3 + (4*b*x^(5/2)*ArcTan[E^(c + d*Sqrt[x])])/d - ((10*I)*b*x^2*PolyL
og[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((10*I)*b*x^2*PolyLog[2, I*E^(c + d*Sq
rt[x])])/d^2 + ((40*I)*b*x^(3/2)*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 -
((40*I)*b*x^(3/2)*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 - ((120*I)*b*x*PolyL
og[4, (-I)*E^(c + d*Sqrt[x])])/d^4 + ((120*I)*b*x*PolyLog[4, I*E^(c + d*Sqr
t[x])])/d^4 + ((240*I)*b*Sqrt[x]*PolyLog[5, (-I)*E^(c + d*Sqrt[x])])/d^5 -
((240*I)*b*Sqrt[x]*PolyLog[5, I*E^(c + d*Sqrt[x])])/d^5 - ((240*I)*b*PolyLo
g[6, (-I)*E^(c + d*Sqrt[x])])/d^6 + ((240*I)*b*PolyLog[6, I*E^(c + d*Sqrt[x
])])/d^6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \left(ax^2 + bx^2\text{sech}\left(c + d\sqrt{x}\right)\right) dx$$

$$= \frac{ax^3}{3} + b\int x^2\text{sech}\left(c + d\sqrt{x}\right) dx$$

$$= \frac{ax^3}{3} + (2b)\text{Subst}\left(\int x^5\text{sech}(c + dx)\,dx, x, \sqrt{x}\right)$$

$$= \frac{ax^3}{3} + \frac{4bx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{(10ib)\text{Subst}\left(\int x^4\log\left(1 - ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d} + \frac{(10ib)\text{Subst}\left(\int x^4\log\left(1 + ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d}$$

$$= \frac{ax^3}{3} + \frac{4bx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10ibx^2\,\text{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{10ibx^2\,\text{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{(40ib)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2, -ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2} - \frac{(40ib)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2, ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$

$$= \frac{ax^3}{3} + \frac{4bx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10ibx^2\,\text{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{10ibx^2\,\text{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{40ibx^{3/2}\,\text{PolyLog}\left(3, -ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{40ibx^{3/2}\,\text{PolyLog}\left(3, ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{(120ib)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3, -ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^3} + \frac{(120ib)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3, ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^3}$$

$$
\begin{aligned}
&= \frac{ax^3}{3} + \frac{4bx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10ibx^2\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{10ibx^2\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{40ibx^{3/2}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{40ibx^{3/2}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{120ibx\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{120ibx\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} \\
&\quad + \frac{(240ib)\text{Subst}\left(\int x\,\text{PolyLog}\left(4,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^4} \\
&\quad - \frac{(240ib)\text{Subst}\left(\int x\,\text{PolyLog}\left(4,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^4} \\
&= \frac{ax^3}{3} + \frac{4bx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10ibx^2\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{10ibx^2\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{40ibx^{3/2}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{40ibx^{3/2}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{120ibx\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{120ibx\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} \\
&\quad + \frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} \\
&\quad - \frac{(240ib)\text{Subst}\left(\int \text{PolyLog}\left(5,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^5} \\
&\quad + \frac{(240ib)\text{Subst}\left(\int \text{PolyLog}\left(5,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^5} \\
&= \frac{ax^3}{3} + \frac{4bx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10ibx^2\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{10ibx^2\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{40ibx^{3/2}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{40ibx^{3/2}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{120ibx\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} \\
&\quad + \frac{120ibx\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} \\
&\quad - \frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{(240ib)\text{Subst}\left(\int \frac{\text{PolyLog}(5,-ix)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^6} \\
&\quad + \frac{(240ib)\text{Subst}\left(\int \frac{\text{PolyLog}(5,ix)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^6}
\end{aligned}
$$

$$= \frac{ax^3}{3} + \frac{4bx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10ibx^2\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{10ibx^2\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{40ibx^{3/2}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{40ibx^{3/2}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{120ibx\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{120ibx\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{240ib\,\text{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{240ib\,\text{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx = \frac{ax^3}{3} + \frac{2ib\left(d^5x^{5/2}\log\left(1-ie^{c+d\sqrt{x}}\right) - d^5x^{5/2}\log\left(1+ie^{c+d\sqrt{x}}\right) - 5d^4x^2\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right) + 5d^4x^2\,\text{PolyLog}\right.}{}$$

```
[In] Integrate[x^2*(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^3)/3 + ((2*I)*b*(d^5*x^(5/2)*Log[1 - I*E^(c + d*Sqrt[x])] - d^5*x^(5/2
)*Log[1 + I*E^(c + d*Sqrt[x])] - 5*d^4*x^2*PolyLog[2, (-I)*E^(c + d*Sqrt[x]
)] + 5*d^4*x^2*PolyLog[2, I*E^(c + d*Sqrt[x])] + 20*d^3*x^(3/2)*PolyLog[3,
(-I)*E^(c + d*Sqrt[x])] - 20*d^3*x^(3/2)*PolyLog[3, I*E^(c + d*Sqrt[x])] -
60*d^2*x*PolyLog[4, (-I)*E^(c + d*Sqrt[x])] + 60*d^2*x*PolyLog[4, I*E^(c +
d*Sqrt[x])] + 120*d*Sqrt[x]*PolyLog[5, (-I)*E^(c + d*Sqrt[x])] - 120*d*Sqrt
[x]*PolyLog[5, I*E^(c + d*Sqrt[x])] - 120*PolyLog[6, (-I)*E^(c + d*Sqrt[x])
] + 120*PolyLog[6, I*E^(c + d*Sqrt[x])]))/d^6
```

Maple [F]

$$\int x^2\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)dx$$

```
[In] int(x^2*(a+b*sech(c+d*x^(1/2))),x)
```

```
[Out] int(x^2*(a+b*sech(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)x^2\,dx$$

```
[In] integrate(x^2*(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(b*x^2*sech(d*sqrt(x) + c) + a*x^2, x)
```

Sympy [F]

$$\int x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int x^2\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx$$

```
[In] integrate(x**2*(a+b*sech(c+d*x**(1/2))),x)
[Out] Integral(x**2*(a + b*sech(c + d*sqrt(x))), x)
```

Maxima [F]

$$\int x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)x^2\,dx$$

```
[In] integrate(x^2*(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")
[Out] 1/3*a*x^3 + 2*b*integrate(x^2*e^(d*sqrt(x) + c)/(e^(2*d*sqrt(x) + 2*c) + 1)
, x)
```

Giac [F]

$$\int x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)x^2\,dx$$

```
[In] integrate(x^2*(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int x^2\left(a+\frac{b}{\cosh\left(c+d\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x^2*(a + b/cosh(c + d*x^(1/2))),x)
[Out] int(x^2*(a + b/cosh(c + d*x^(1/2))), x)
```

3.34 $\int x\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 194

$$\begin{aligned}\int x\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)\,dx=&\ \frac{ax^2}{2}+\frac{4bx^{3/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d}\\&-\frac{6ibx\,\mathrm{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{6ibx\,\mathrm{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}\\&+\frac{12ib\sqrt{x}\,\mathrm{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}\\&-\frac{12ib\sqrt{x}\,\mathrm{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}\\&-\frac{12ib\,\mathrm{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{12ib\,\mathrm{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4}\end{aligned}$$

```
[Out] 1/2*a*x^2+4*b*x^(3/2)*arctan(exp(c+d*x^(1/2)))/d-6*I*b*x*polylog(2,-I*exp(c
+d*x^(1/2)))/d^2+6*I*b*x*polylog(2,I*exp(c+d*x^(1/2)))/d^2-12*I*b*polylog(4
,-I*exp(c+d*x^(1/2)))/d^4+12*I*b*polylog(4,I*exp(c+d*x^(1/2)))/d^4+12*I*b*p
olylog(3,-I*exp(c+d*x^(1/2)))*x^(1/2)/d^3-12*I*b*polylog(3,I*exp(c+d*x^(1/2
)))*x^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used

= {14, 5544, 4265, 2611, 6744, 2320, 6724}

$$\int x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^2}{2}+\frac{4bx^{3/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{12ib\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{12ib\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{6ibx\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{6ibx\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}$$

```
[In] Int[x*(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^2)/2 + (4*b*x^(3/2)*ArcTan[E^(c + d*Sqrt[x])])/d - ((6*I)*b*x*PolyLog[
2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((6*I)*b*x*PolyLog[2, I*E^(c + d*Sqrt[x])
])/d^2 + ((12*I)*b*Sqrt[x]*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((12*I
)*b*Sqrt[x]*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 - ((12*I)*b*PolyLog[4, (-I
)*E^(c + d*Sqrt[x])])/d^4 + ((12*I)*b*PolyLog[4, I*E^(c + d*Sqrt[x])])/d^4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ax + bx\operatorname{sech}\left(c + d\sqrt{x}\right)\right) dx \\
&= \frac{ax^2}{2} + b\int x\operatorname{sech}\left(c + d\sqrt{x}\right) dx \\
&= \frac{ax^2}{2} + (2b)\operatorname{Subst}\left(\int x^3\operatorname{sech}(c+dx)\,dx, x, \sqrt{x}\right) \\
&= \frac{ax^2}{2} + \frac{4bx^{3/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{(6ib)\operatorname{Subst}\left(\int x^2\log\left(1 - ie^{c+dx}\right) dx, x, \sqrt{x}\right)}{d} \\
&\quad + \frac{(6ib)\operatorname{Subst}\left(\int x^2\log\left(1 + ie^{c+dx}\right) dx, x, \sqrt{x}\right)}{d} \\
&= \frac{ax^2}{2} + \frac{4bx^{3/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{6ibx\operatorname{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{6ibx\operatorname{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{(12ib)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(2, -ie^{c+dx}\right) dx, x, \sqrt{x}\right)}{d^2} \\
&\quad - \frac{(12ib)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(2, ie^{c+dx}\right) dx, x, \sqrt{x}\right)}{d^2}
\end{aligned}
$$

$$= \frac{ax^2}{2} + \frac{4bx^{3/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{6ibx\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{6ibx\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{(12ib)\text{Subst}\left(\int \text{PolyLog}\left(3,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3} + \frac{(12ib)\text{Subst}\left(\int \text{PolyLog}\left(3,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= \frac{ax^2}{2} + \frac{4bx^{3/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{6ibx\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{6ibx\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{(12ib)\text{Subst}\left(\int \frac{\text{PolyLog}(3,-ix)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^4} + \frac{(12ib)\text{Subst}\left(\int \frac{\text{PolyLog}(3,ix)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^4}$$

$$= \frac{ax^2}{2} + \frac{4bx^{3/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{6ibx\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{6ibx\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{12ib\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{12ib\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.07

$$\int x\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx = \frac{ax^2}{2} + \frac{2ib\left(d^3x^{3/2}\log\left(1-ie^{c+d\sqrt{x}}\right) - d^3x^{3/2}\log\left(1+ie^{c+d\sqrt{x}}\right) - 3d^2x\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right) + 3d^2x\,\text{PolyLog}\left(\right.\right.}{}$$

```
[In] Integrate[x*(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^2)/2 + ((2*I)*b*(d^3*x^(3/2)*Log[1 - I*E^(c + d*Sqrt[x])] - d^3*x^(3/2
)*Log[1 + I*E^(c + d*Sqrt[x])] - 3*d^2*x*PolyLog[2, (-I)*E^(c + d*Sqrt[x])]
 + 3*d^2*x*PolyLog[2, I*E^(c + d*Sqrt[x])] + 6*d*Sqrt[x]*PolyLog[3, (-I)*E^
(c + d*Sqrt[x])] - 6*d*Sqrt[x]*PolyLog[3, I*E^(c + d*Sqrt[x])] - 6*PolyLog[
4, (-I)*E^(c + d*Sqrt[x])] + 6*PolyLog[4, I*E^(c + d*Sqrt[x])]))/d^4
```

Maple [F]

$$\int x\left(a+b\ \mathrm{sech}\left(c+d\sqrt{x}\right)\right)dx$$

```
[In] int(x*(a+b*sech(c+d*x^(1/2))),x)
[Out] int(x*(a+b*sech(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\,\mathrm{sech}\left(d\sqrt{x}+c\right)+a\right)x\,dx$$

```
[In] integrate(x*(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(b*x*sech(d*sqrt(x) + c) + a*x, x)
```

Sympy [F]

$$\int x\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int x\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)\,dx$$

```
[In] integrate(x*(a+b*sech(c+d*x**(1/2))),x)
[Out] Integral(x*(a + b*sech(c + d*sqrt(x))), x)
```

Maxima [F]

$$\int x\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\,\mathrm{sech}\left(d\sqrt{x}+c\right)+a\right)x\,dx$$

```
[In] integrate(x*(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")
[Out] 1/2*a*x^2 + 2*b*integrate(x*e^(d*sqrt(x) + c)/(e^(2*d*sqrt(x) + 2*c) + 1),
x)
```

Giac [F]

$$\int x\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)x\,dx$$

```
[In] integrate(x*(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int x\left(a+\frac{b}{\cosh\left(c+d\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x*(a + b/cosh(c + d*x^(1/2))),x)
[Out] int(x*(a + b/cosh(c + d*x^(1/2))), x)
```

3.35 $\int \frac{a+b\text{sech}(c+d\sqrt{x})}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\text{sech}(c+d\sqrt{x})}{x} dx = a\log(x) + b\text{Int}\left(\frac{\text{sech}(c+d\sqrt{x})}{x}, x\right)$$

```
[Out] a*ln(x)+b*Unintegrable(sech(c+d*x^(1/2))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{sech}(c+d\sqrt{x})}{x} dx = \int \frac{a+b\text{sech}(c+d\sqrt{x})}{x} dx$$

```
[In] Int[(a + b*Sech[c + d*Sqrt[x]])/x,x]
```

```
[Out] a*Log[x] + b*Defer[Int][Sech[c + d*Sqrt[x]]/x, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{a}{x} + \frac{b\text{sech}(c+d\sqrt{x})}{x}\right) dx \\ &= a\log(x) + b\int \frac{\text{sech}(c+d\sqrt{x})}{x} dx \end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 11.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x}\,dx$$

```
[In] Integrate[(a + b*Sech[c + d*Sqrt[x]])/x,x]
[Out] Integrate[(a + b*Sech[c + d*Sqrt[x]])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a+b\,\operatorname{sech}\left(c+d\sqrt{x}\right)}{x}dx$$

```
[In] int((a+b*sech(c+d*x^(1/2)))/x,x)
[Out] int((a+b*sech(c+d*x^(1/2)))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{b\,\operatorname{sech}\left(d\sqrt{x}+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x,x, algorithm="fricas")
[Out] integral((b*sech(d*sqrt(x) + c) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{a+b\,\operatorname{sech}\left(c+d\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate((a+b*sech(c+d*x**(1/2)))/x,x)
[Out] Integral((a + b*sech(c + d*sqrt(x)))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{b\,\text{sech}\left(d\sqrt{x}+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x,x, algorithm="maxima")
[Out] 2*b*integrate(e^(d*sqrt(x) + c)/(x*e^(2*d*sqrt(x) + 2*c) + x), x) + a*log(x
)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{b\,\text{sech}\left(d\sqrt{x}+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x,x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}}{x}\,dx$$

```
[In] int((a + b/cosh(c + d*x^(1/2)))/x,x)
[Out] int((a + b/cosh(c + d*x^(1/2)))/x, x)
```

3.36 $\int \frac{a+b\text{sech}(c+d\sqrt{x})}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\text{sech}(c+d\sqrt{x})}{x^2}\,dx=-\frac{a}{x}+b\text{Int}\left(\frac{\text{sech}(c+d\sqrt{x})}{x^2},x\right)$$

```
[Out] -a/x+b*Unintegrable(sech(c+d*x^(1/2))/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{sech}(c+d\sqrt{x})}{x^2}\,dx=\int \frac{a+b\text{sech}(c+d\sqrt{x})}{x^2}\,dx$$

```
[In] Int[(a + b*Sech[c + d*Sqrt[x]])/x^2,x]
[Out] -(a/x) + b*Defer[Int][Sech[c + d*Sqrt[x]]/x^2, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{a}{x^2}+\frac{b\text{sech}(c+d\sqrt{x})}{x^2}\right)\,dx\\&=-\frac{a}{x}+b\int \frac{\text{sech}(c+d\sqrt{x})}{x^2}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 10.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] Integrate[(a + b*Sech[c + d*Sqrt[x]])/x^2,x]
[Out] Integrate[(a + b*Sech[c + d*Sqrt[x]])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a+b\ \text{sech}\left(c+d\sqrt{x}\right)}{x^2}dx$$

```
[In] int((a+b*sech(c+d*x^(1/2)))/x^2,x)
[Out] int((a+b*sech(c+d*x^(1/2)))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\,\text{sech}\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x^2,x, algorithm="fricas")
[Out] integral((b*sech(d*sqrt(x) + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\,\text{sech}\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] integrate((a+b*sech(c+d*x**(1/2)))/x**2,x)
[Out] Integral((a + b*sech(c + d*sqrt(x)))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\,\text{sech}\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x^2,x, algorithm="maxima")
[Out] 2*b*integrate(e^(d*sqrt(x) + c)/(x^2*e^(2*d*sqrt(x) + 2*c) + x^2), x) - a/x
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\,\text{sech}\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x^2,x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}}{x^2}\,dx$$

```
[In] int((a + b/cosh(c + d*x^(1/2)))/x^2,x)
[Out] int((a + b/cosh(c + d*x^(1/2)))/x^2, x)
```

3.37 $\int x^3 \left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2 dx$

Optimal result

Integrand size = 20, antiderivative size = 677

$$
\begin{aligned}
\int x^3\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx = {} & \frac{2b^2x^{7/2}}{d}+\frac{a^2x^4}{4}+\frac{8abx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} \\
& -\frac{14b^2x^3\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
& -\frac{28iabx^3\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} \\
& +\frac{28iabx^3\operatorname{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} \\
& -\frac{42b^2x^{5/2}\operatorname{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} \\
& +\frac{168iabx^{5/2}\operatorname{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} \\
& -\frac{168iabx^{5/2}\operatorname{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} \\
& +\frac{105b^2x^2\operatorname{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} \\
& -\frac{840iabx^2\operatorname{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} \\
& +\frac{840iabx^2\operatorname{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} \\
& -\frac{210b^2x^{3/2}\operatorname{PolyLog}\left(4,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5} \\
& +\frac{3360iabx^{3/2}\operatorname{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} \\
& -\frac{3360iabx^{3/2}\operatorname{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} \\
& +\frac{315b^2x\operatorname{PolyLog}\left(5,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6} \\
& -\frac{10080iabx\operatorname{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6} \\
& +\frac{10080iabx\operatorname{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6} \\
& -\frac{315b^2\sqrt{x}\operatorname{PolyLog}\left(6,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^7} \\
& +\frac{20160iab\sqrt{x}\operatorname{PolyLog}\left(7,-ie^{c+d\sqrt{x}}\right)}{d^7} \\
& -\frac{20160iab\sqrt{x}\operatorname{PolyLog}\left(7,ie^{c+d\sqrt{x}}\right)}{d^7} \\
& +\frac{315b^2\operatorname{PolyLog}\left(7,-e^{2\left(c+d\sqrt{x}\right)}\right)}{2d^8} \\
& 20160iab\operatorname{PolyLog}\left(8,-ie^{c+d\sqrt{x}}\right)
\end{aligned}
$$

```
[Out] 168*I*a*b*x^(5/2)*polylog(3,-I*exp(c+d*x^(1/2)))/d^3+28*I*a*b*x^3*polylog(2
,I*exp(c+d*x^(1/2)))/d^2+840*I*a*b*x^2*polylog(4,I*exp(c+d*x^(1/2)))/d^4+33
60*I*a*b*x^(3/2)*polylog(5,-I*exp(c+d*x^(1/2)))/d^5+10080*I*a*b*x*polylog(6
,I*exp(c+d*x^(1/2)))/d^6+20160*I*a*b*polylog(7,-I*exp(c+d*x^(1/2)))*x^(1/2)
/d^7+8*a*b*x^(7/2)*arctan(exp(c+d*x^(1/2)))/d-20160*I*a*b*polylog(8,-I*exp(
c+d*x^(1/2)))/d^8+20160*I*a*b*polylog(8,I*exp(c+d*x^(1/2)))/d^8-28*I*a*b*x^
3*polylog(2,-I*exp(c+d*x^(1/2)))/d^2-168*I*a*b*x^(5/2)*polylog(3,I*exp(c+d*
x^(1/2)))/d^3-840*I*a*b*x^2*polylog(4,-I*exp(c+d*x^(1/2)))/d^4-3360*I*a*b*x
^(3/2)*polylog(5,I*exp(c+d*x^(1/2)))/d^5-10080*I*a*b*x*polylog(6,-I*exp(c+d
*x^(1/2)))/d^6-20160*I*a*b*polylog(7,I*exp(c+d*x^(1/2)))*x^(1/2)/d^7-14*b^2
*x^3*ln(1+exp(2*c+2*d*x^(1/2)))/d^2-42*b^2*x^(5/2)*polylog(2,-exp(2*c+2*d*x
^(1/2)))/d^3+105*b^2*x^2*polylog(3,-exp(2*c+2*d*x^(1/2)))/d^4-210*b^2*x^(3/
2)*polylog(4,-exp(2*c+2*d*x^(1/2)))/d^5+315*b^2*x*polylog(5,-exp(2*c+2*d*x^
(1/2)))/d^6-315*b^2*polylog(6,-exp(2*c+2*d*x^(1/2)))*x^(1/2)/d^7+2*b^2*x^(7
/2)*tanh(c+d*x^(1/2))/d+315/2*b^2*polylog(7,-exp(2*c+2*d*x^(1/2)))/d^8+2*b^
2*x^(7/2)/d+1/4*a^2*x^4
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 677, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules

used = {5544, 4275, 4265, 2611, 6744, 2320, 6724, 4269, 3799, 2221}

$$\int x^3\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{a^2x^4}{4}+\frac{8abx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d}$$
$$-\frac{20160iab\operatorname{PolyLog}\left(8,-ie^{c+d\sqrt{x}}\right)}{d^8}$$
$$+\frac{20160iab\operatorname{PolyLog}\left(8,ie^{c+d\sqrt{x}}\right)}{d^8}$$
$$+\frac{20160iab\sqrt{x}\operatorname{PolyLog}\left(7,-ie^{c+d\sqrt{x}}\right)}{d^7}$$
$$-\frac{20160iab\sqrt{x}\operatorname{PolyLog}\left(7,ie^{c+d\sqrt{x}}\right)}{d^7}$$
$$-\frac{10080iabx\operatorname{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6}$$
$$+\frac{10080iabx\operatorname{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6}$$
$$+\frac{3360iabx^{3/2}\operatorname{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5}$$
$$-\frac{3360iabx^{3/2}\operatorname{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5}$$
$$-\frac{840iabx^2\operatorname{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4}$$
$$+\frac{840iabx^2\operatorname{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4}$$
$$+\frac{168iabx^{5/2}\operatorname{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}$$
$$-\frac{168iabx^{5/2}\operatorname{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}$$
$$-\frac{28iabx^3\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}$$
$$+\frac{28iabx^3\operatorname{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}$$
$$+\frac{315b^2\operatorname{PolyLog}\left(7,-e^{2\left(c+d\sqrt{x}\right)}\right)}{2d^8}$$
$$-\frac{315b^2\sqrt{x}\operatorname{PolyLog}\left(6,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^7}$$
$$+\frac{315b^2x\operatorname{PolyLog}\left(5,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6}$$
$$-\frac{210b^2x^{3/2}\operatorname{PolyLog}\left(4,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5}$$
$$+\frac{105b^2x^2\operatorname{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4}$$
$$-\frac{42b^2x^{5/2}\operatorname{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}$$

```
[In] Int[x^3*(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*b^2*x^(7/2))/d + (a^2*x^4)/4 + (8*a*b*x^(7/2)*ArcTan[E^(c + d*Sqrt[x])])
/d - (14*b^2*x^3*Log[1 + E^(2*(c + d*Sqrt[x]))])/d^2 - ((28*I)*a*b*x^3*Poly
Log[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((28*I)*a*b*x^3*PolyLog[2, I*E^(c + d
*Sqrt[x])])/d^2 - (42*b^2*x^(5/2)*PolyLog[2, -E^(2*(c + d*Sqrt[x]))])/d^3 +
 ((168*I)*a*b*x^(5/2)*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((168*I)*a*
b*x^(5/2)*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 + (105*b^2*x^2*PolyLog[3, -E
^(2*(c + d*Sqrt[x]))])/d^4 - ((840*I)*a*b*x^2*PolyLog[4, (-I)*E^(c + d*Sqrt
[x])])/d^4 + ((840*I)*a*b*x^2*PolyLog[4, I*E^(c + d*Sqrt[x])])/d^4 - (210*b
^2*x^(3/2)*PolyLog[4, -E^(2*(c + d*Sqrt[x]))])/d^5 + ((3360*I)*a*b*x^(3/2)*
PolyLog[5, (-I)*E^(c + d*Sqrt[x])])/d^5 - ((3360*I)*a*b*x^(3/2)*PolyLog[5,
I*E^(c + d*Sqrt[x])])/d^5 + (315*b^2*x*PolyLog[5, -E^(2*(c + d*Sqrt[x]))])/
d^6 - ((10080*I)*a*b*x*PolyLog[6, (-I)*E^(c + d*Sqrt[x])])/d^6 + ((10080*I)
*a*b*x*PolyLog[6, I*E^(c + d*Sqrt[x])])/d^6 - (315*b^2*Sqrt[x]*PolyLog[6, -
E^(2*(c + d*Sqrt[x]))])/d^7 + ((20160*I)*a*b*Sqrt[x]*PolyLog[7, (-I)*E^(c +
 d*Sqrt[x])])/d^7 - ((20160*I)*a*b*Sqrt[x]*PolyLog[7, I*E^(c + d*Sqrt[x])])
/d^7 + (315*b^2*PolyLog[7, -E^(2*(c + d*Sqrt[x]))])/(2*d^8) - ((20160*I)*a*
b*PolyLog[8, (-I)*E^(c + d*Sqrt[x])])/d^8 + ((20160*I)*a*b*PolyLog[8, I*E^(
c + d*Sqrt[x])])/d^8 + (2*b^2*x^(7/2)*Tanh[c + d*Sqrt[x]])/d
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^7(a+b\text{sech}(c+dx))^2\,dx,x,\sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(a^2x^7+2abx^7\text{sech}(c+dx)+b^2x^7\text{sech}^2(c+dx)\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{a^2x^4}{4}+(4ab)\text{Subst}\left(\int x^7\text{sech}(c+dx)\,dx,x,\sqrt{x}\right)\\
&\quad+\left(2b^2\right)\text{Subst}\left(\int x^7\text{sech}^2(c+dx)\,dx,x,\sqrt{x}\right)\\
&= \frac{a^2x^4}{4}+\frac{8abx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d}+\frac{2b^2x^{7/2}\tanh\left(c+d\sqrt{x}\right)}{d}\\
&\quad-\frac{(28iab)\text{Subst}\left(\int x^6\log\left(1-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d}\\
&\quad+\frac{(28iab)\text{Subst}\left(\int x^6\log\left(1+ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d}\\
&\quad-\frac{\left(14b^2\right)\text{Subst}\left(\int x^6\tanh(c+dx)\,dx,x,\sqrt{x}\right)}{d}\\
&= \frac{2b^2x^{7/2}}{d}+\frac{a^2x^4}{4}+\frac{8abx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{28iabx^3\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}\\
&\quad+\frac{28iabx^3\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{2b^2x^{7/2}\tanh\left(c+d\sqrt{x}\right)}{d}\\
&\quad+\frac{(168iab)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^2}\\
&\quad-\frac{(168iab)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^2}\\
&\quad-\frac{\left(28b^2\right)\text{Subst}\left(\int \frac{e^{2(c+dx)}x^6}{1+e^{2(c+dx)}}\,dx,x,\sqrt{x}\right)}{d}\\
&= \frac{2b^2x^{7/2}}{d}+\frac{a^2x^4}{4}+\frac{8abx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{14b^2x^3\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2}\\
&\quad-\frac{28iabx^3\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{28iabx^3\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}\\
&\quad+\frac{168iabx^{5/2}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{168iabx^{5/2}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}\\
&\quad+\frac{2b^2x^{7/2}\tanh\left(c+d\sqrt{x}\right)}{d}-\frac{(840iab)\text{Subst}\left(\int x^4\,\text{PolyLog}\left(3,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3}\\
&\quad+\frac{(840iab)\text{Subst}\left(\int x^4\,\text{PolyLog}\left(3,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3}\\
&\quad+\frac{\left(84b^2\right)\text{Subst}\left(\int x^5\log\left(1+e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2}
\end{aligned}$$

$$= \frac{2b^2x^{7/2}}{d} + \frac{a^2x^4}{4} + \frac{8abx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14b^2x^3\log\left(1+e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{28iabx^3\,\mathrm{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{28iabx^3\,\mathrm{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} - \frac{42b^2x^{5/2}\,\mathrm{PolyLog}\left(2,-e^{2(c+d\sqrt{x})}\right)}{d^3} + \frac{168iabx^{5/2}\,\mathrm{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{168iabx^{5/2}\,\mathrm{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{840iabx^2\,\mathrm{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{840iabx^2\,\mathrm{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{2b^2x^{7/2}\tanh\left(c+d\sqrt{x}\right)}{d} + \frac{(3360iab)\mathrm{Subst}\left(\int x^3\,\mathrm{PolyLog}\left(4,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^4} - \frac{(3360iab)\mathrm{Subst}\left(\int x^3\,\mathrm{PolyLog}\left(4,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^4} + \frac{(210b^2)\,\mathrm{Subst}\left(\int x^4\,\mathrm{PolyLog}\left(2,-e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= \frac{2b^2x^{7/2}}{d} + \frac{a^2x^4}{4} + \frac{8abx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14b^2x^3\log\left(1+e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{28iabx^3\,\mathrm{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{28iabx^3\,\mathrm{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} - \frac{42b^2x^{5/2}\,\mathrm{PolyLog}\left(2,-e^{2(c+d\sqrt{x})}\right)}{d^3} + \frac{168iabx^{5/2}\,\mathrm{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{168iabx^{5/2}\,\mathrm{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{105b^2x^2\,\mathrm{PolyLog}\left(3,-e^{2(c+d\sqrt{x})}\right)}{d^4} - \frac{840iabx^2\,\mathrm{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{840iabx^2\,\mathrm{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{3360iabx^{3/2}\,\mathrm{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{3360iabx^{3/2}\,\mathrm{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} + \frac{2b^2x^{7/2}\tanh\left(c+d\sqrt{x}\right)}{d} - \frac{(10080iab)\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(5,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^5} + \frac{(10080iab)\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(5,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^5} - \frac{(420b^2)\,\mathrm{Subst}\left(\int x^3\,\mathrm{PolyLog}\left(3,-e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$

$$
\begin{aligned}
&= \frac{2b^2x^{7/2}}{d} + \frac{a^2x^4}{4} + \frac{8abx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14b^2x^3\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
&\quad - \frac{28iabx^3\,\mathrm{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{28iabx^3\,\mathrm{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad - \frac{42b^2x^{5/2}\,\mathrm{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{168iabx^{5/2}\,\mathrm{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{168iabx^{5/2}\,\mathrm{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{105b^2x^2\,\mathrm{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} \\
&\quad - \frac{840iabx^2\,\mathrm{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{840iabx^2\,\mathrm{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} \\
&\quad - \frac{210b^2x^{3/2}\,\mathrm{PolyLog}\left(4,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5} + \frac{3360iabx^{3/2}\,\mathrm{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} \\
&\quad - \frac{3360iabx^{3/2}\,\mathrm{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{10080iabx\,\mathrm{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6} \\
&\quad + \frac{10080iabx\,\mathrm{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{2b^2x^{7/2}\tanh\left(c+d\sqrt{x}\right)}{d} \\
&\quad + \frac{(20160iab)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(6,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^6} \\
&\quad - \frac{(20160iab)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(6,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^6} \\
&\quad + \frac{(630b^2)\,\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(4,-e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b^2x^{7/2}}{d} + \frac{a^2x^4}{4} + \frac{8abx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14b^2x^3\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
&- \frac{28iabx^3\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{28iabx^3\operatorname{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} \\
&- \frac{42b^2x^{5/2}\operatorname{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{168iabx^{5/2}\operatorname{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} \\
&- \frac{168iabx^{5/2}\operatorname{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{105b^2x^2\operatorname{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} \\
&- \frac{840iabx^2\operatorname{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{840iabx^2\operatorname{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} \\
&- \frac{210b^2x^{3/2}\operatorname{PolyLog}\left(4,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5} + \frac{3360iabx^{3/2}\operatorname{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} \\
&- \frac{3360iabx^{3/2}\operatorname{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} + \frac{315b^2x\operatorname{PolyLog}\left(5,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6} \\
&- \frac{10080iabx\operatorname{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{10080iabx\operatorname{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6} \\
&+ \frac{20160iab\sqrt{x}\operatorname{PolyLog}\left(7,-ie^{c+d\sqrt{x}}\right)}{d^7} - \frac{20160iab\sqrt{x}\operatorname{PolyLog}\left(7,ie^{c+d\sqrt{x}}\right)}{d^7} \\
&+ \frac{2b^2x^{7/2}\tanh\left(c+d\sqrt{x}\right)}{d} - \frac{(20160iab)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(7,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^7} \\
&+ \frac{(20160iab)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(7,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^7} \\
&- \frac{(630b^2)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(5,-e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^6}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b^2x^{7/2}}{d} + \frac{a^2x^4}{4} + \frac{8abx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14b^2x^3\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
&\quad - \frac{28iabx^3\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{28iabx^3\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad - \frac{42b^2x^{5/2}\,\text{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{168iabx^{5/2}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{168iabx^{5/2}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{105b^2x^2\,\text{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} \\
&\quad - \frac{840iabx^2\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{840iabx^2\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} \\
&\quad - \frac{210b^2x^{3/2}\,\text{PolyLog}\left(4,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5} + \frac{3360iabx^{3/2}\,\text{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} \\
&\quad - \frac{3360iabx^{3/2}\,\text{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} + \frac{315b^2x\,\text{PolyLog}\left(5,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6} \\
&\quad - \frac{10080iabx\,\text{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{10080iabx\,\text{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6} \\
&\quad - \frac{315b^2\sqrt{x}\,\text{PolyLog}\left(6,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^7} + \frac{20160iab\sqrt{x}\,\text{PolyLog}\left(7,-ie^{c+d\sqrt{x}}\right)}{d^7} \\
&\quad - \frac{20160iab\sqrt{x}\,\text{PolyLog}\left(7,ie^{c+d\sqrt{x}}\right)}{d^7} + \frac{2b^2x^{7/2}\tanh\left(c+d\sqrt{x}\right)}{d} \\
&\quad - \frac{(20160iab)\text{Subst}\left(\int\frac{\text{PolyLog}(7,-ix)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^8} \\
&\quad + \frac{(20160iab)\text{Subst}\left(\int\frac{\text{PolyLog}(7,ix)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^8} \\
&\quad + \frac{(315b^2)\,\text{Subst}\left(\int\text{PolyLog}\left(6,-e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^7}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b^2x^{7/2}}{d} + \frac{a^2x^4}{4} + \frac{8abx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{14b^2x^3\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
&- \frac{28iabx^3\,\mathrm{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{28iabx^3\,\mathrm{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} \\
&- \frac{42b^2x^{5/2}\,\mathrm{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{168iabx^{5/2}\,\mathrm{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} \\
&- \frac{168iabx^{5/2}\,\mathrm{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{105b^2x^2\,\mathrm{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} \\
&- \frac{840iabx^2\,\mathrm{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{840iabx^2\,\mathrm{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} \\
&- \frac{210b^2x^{3/2}\,\mathrm{PolyLog}\left(4,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5} + \frac{3360iabx^{3/2}\,\mathrm{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} \\
&- \frac{3360iabx^{3/2}\,\mathrm{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} + \frac{315b^2x\,\mathrm{PolyLog}\left(5,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6} \\
&- \frac{10080iabx\,\mathrm{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{10080iabx\,\mathrm{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6} \\
&- \frac{315b^2\sqrt{x}\,\mathrm{PolyLog}\left(6,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^7} + \frac{20160iab\sqrt{x}\,\mathrm{PolyLog}\left(7,-ie^{c+d\sqrt{x}}\right)}{d^7} \\
&- \frac{20160iab\sqrt{x}\,\mathrm{PolyLog}\left(7,ie^{c+d\sqrt{x}}\right)}{d^7} - \frac{20160iab\,\mathrm{PolyLog}\left(8,-ie^{c+d\sqrt{x}}\right)}{d^8} \\
&+ \frac{20160iab\,\mathrm{PolyLog}\left(8,ie^{c+d\sqrt{x}}\right)}{d^8} + \frac{2b^2x^{7/2}\tanh\left(c+d\sqrt{x}\right)}{d} \\
&+ \frac{(315b^2)\,\mathrm{Subst}\left(\int \frac{\mathrm{PolyLog}(6,-x)}{x}\,dx, x, e^{2\left(c+d\sqrt{x}\right)}\right)}{2d^8}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b^2x^{7/2}}{d} + \frac{a^2x^4}{4} + \frac{8abx^{7/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} \\
&- \frac{14b^2x^3\log\left(1+e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{28iabx^3\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} \\
&+ \frac{28iabx^3\operatorname{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} - \frac{42b^2x^{5/2}\operatorname{PolyLog}\left(2,-e^{2(c+d\sqrt{x})}\right)}{d^3} \\
&+ \frac{168iabx^{5/2}\operatorname{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{168iabx^{5/2}\operatorname{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} \\
&+ \frac{105b^2x^2\operatorname{PolyLog}\left(3,-e^{2(c+d\sqrt{x})}\right)}{d^4} - \frac{840iabx^2\operatorname{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} \\
&+ \frac{840iabx^2\operatorname{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} - \frac{210b^2x^{3/2}\operatorname{PolyLog}\left(4,-e^{2(c+d\sqrt{x})}\right)}{d^5} \\
&+ \frac{3360iabx^{3/2}\operatorname{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{3360iabx^{3/2}\operatorname{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} \\
&+ \frac{315b^2x\operatorname{PolyLog}\left(5,-e^{2(c+d\sqrt{x})}\right)}{d^6} - \frac{10080iabx\operatorname{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6} \\
&+ \frac{10080iabx\operatorname{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6} - \frac{315b^2\sqrt{x}\operatorname{PolyLog}\left(6,-e^{2(c+d\sqrt{x})}\right)}{d^7} \\
&+ \frac{20160iab\sqrt{x}\operatorname{PolyLog}\left(7,-ie^{c+d\sqrt{x}}\right)}{d^7} - \frac{20160iab\sqrt{x}\operatorname{PolyLog}\left(7,ie^{c+d\sqrt{x}}\right)}{d^7} \\
&+ \frac{315b^2\operatorname{PolyLog}\left(7,-e^{2(c+d\sqrt{x})}\right)}{2d^8} - \frac{20160iab\operatorname{PolyLog}\left(8,-ie^{c+d\sqrt{x}}\right)}{d^8} \\
&+ \frac{20160iab\operatorname{PolyLog}\left(8,ie^{c+d\sqrt{x}}\right)}{d^8} + \frac{2b^2x^{7/2}\tanh\left(c+d\sqrt{x}\right)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 8.60 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.10

$$
\int x^3\left(a+b\operatorname{sech}(c+d\sqrt{x})\right)^2\,dx
$$

$$
= \frac{\cosh\left(c+d\sqrt{x}\right)\left(a+b\operatorname{sech}(c+d\sqrt{x})\right)^2\left(\frac{16b^2e^{2c}x^{7/2}\cosh(c+d\sqrt{x})}{d(1+e^{2c})} + a^2x^4\cosh\left(c+d\sqrt{x}\right) + \frac{2ib\cosh(c+d\sqrt{x})\left(8ad^7x^7\right.}{}\right.}{}
$$

```
[In] Integrate[x^3*(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (Cosh[c + d*Sqrt[x]]*(a + b*Sech[c + d*Sqrt[x]])^2*((16*b^2*E^(2*c)*x^(7/2)
*Cosh[c + d*Sqrt[x]])/(d*(1 + E^(2*c))) + a^2*x^4*Cosh[c + d*Sqrt[x]] + ((2
*I)*b*Cosh[c + d*Sqrt[x]]*(8*a*d^7*x^(7/2)*Log[1 - I*E^(c + d*Sqrt[x])] - 8
```

```
*a*d^7*x^(7/2)*Log[1 + I*E^(c + d*Sqrt[x])] + (28*I)*b*d^6*x^3*Log[1 + E^(2
*(c + d*Sqrt[x]))] - 56*a*d^6*x^3*PolyLog[2, (-I)*E^(c + d*Sqrt[x])] + 56*a
*d^6*x^3*PolyLog[2, I*E^(c + d*Sqrt[x])] + (84*I)*b*d^5*x^(5/2)*PolyLog[2,
-E^(2*(c + d*Sqrt[x]))] + 336*a*d^5*x^(5/2)*PolyLog[3, (-I)*E^(c + d*Sqrt[x
])] - 336*a*d^5*x^(5/2)*PolyLog[3, I*E^(c + d*Sqrt[x])] - (210*I)*b*d^4*x^2
*PolyLog[3, -E^(2*(c + d*Sqrt[x]))] - 1680*a*d^4*x^2*PolyLog[4, (-I)*E^(c +
 d*Sqrt[x])] + 1680*a*d^4*x^2*PolyLog[4, I*E^(c + d*Sqrt[x])] + (420*I)*b*d
^3*x^(3/2)*PolyLog[4, -E^(2*(c + d*Sqrt[x]))] + 6720*a*d^3*x^(3/2)*PolyLog[
5, (-I)*E^(c + d*Sqrt[x])] - 6720*a*d^3*x^(3/2)*PolyLog[5, I*E^(c + d*Sqrt[
x])] - (630*I)*b*d^2*x*PolyLog[5, -E^(2*(c + d*Sqrt[x]))] - 20160*a*d^2*x*P
olyLog[6, (-I)*E^(c + d*Sqrt[x])] + 20160*a*d^2*x*PolyLog[6, I*E^(c + d*Sqr
t[x])] + (630*I)*b*d*Sqrt[x]*PolyLog[6, -E^(2*(c + d*Sqrt[x]))] + 40320*a*d
*Sqrt[x]*PolyLog[7, (-I)*E^(c + d*Sqrt[x])] - 40320*a*d*Sqrt[x]*PolyLog[7,
I*E^(c + d*Sqrt[x])] - (315*I)*b*PolyLog[7, -E^(2*(c + d*Sqrt[x]))] - 40320
*a*PolyLog[8, (-I)*E^(c + d*Sqrt[x])] + 40320*a*PolyLog[8, I*E^(c + d*Sqrt[
x])]))/d^8 + (8*b^2*x^(7/2)*Sech[c]*Sinh[d*Sqrt[x]])/d))/(4*(b + a*Cosh[c +
 d*Sqrt[x]])^2)
```

Maple [F]

$$\int x^3 \left(a + b\ \mathrm{sech}\left(c + d\sqrt{x}\right)\right)^2 dx$$

[In] `int(x^3*(a+b*sech(c+d*x^(1/2)))^2,x)`

[Out] `int(x^3*(a+b*sech(c+d*x^(1/2)))^2,x)`

Fricas [F]

$$\int x^3 \left(a + b\mathrm{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int \left(b\ \mathrm{sech}\left(d\sqrt{x} + c\right) + a\right)^2 x^3 \, dx$$

[In] `integrate(x^3*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")`

[Out] `integral(b^2*x^3*sech(d*sqrt(x) + c)^2 + 2*a*b*x^3*sech(d*sqrt(x) + c) + a^2*x^3, x)`

Sympy [F]

$$\int x^3 \left(a + b\text{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int x^3 \left(a + b \operatorname{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx$$

```
[In] integrate(x**3*(a+b*sech(c+d*x**(1/2)))**2,x)
[Out] Integral(x**3*(a + b*sech(c + d*sqrt(x)))**2, x)
```

Maxima [F]

$$\int x^3 \left(a + b\text{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int \left(b \operatorname{sech}\left(d\sqrt{x} + c\right) + a\right)^2 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] 1/4*(a^2*d*x^4*e^(2*d*sqrt(x) + 2*c) + a^2*d*x^4 - 16*b^2*x^(7/2))/(d*e^(2*
d*sqrt(x) + 2*c) + d) + integrate(2*(2*a*b*d*x^3*e^(d*sqrt(x) + c) + 7*b^2*
x^(5/2))/(d*e^(2*d*sqrt(x) + 2*c) + d), x)
```

Giac [F]

$$\int x^3 \left(a + b\text{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int \left(b \operatorname{sech}\left(d\sqrt{x} + c\right) + a\right)^2 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b\text{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int x^3 \left(a + \frac{b}{\cosh\left(c + d\sqrt{x}\right)}\right)^2 \, dx$$

```
[In] int(x^3*(a + b/cosh(c + d*x^(1/2)))^2,x)
[Out] int(x^3*(a + b/cosh(c + d*x^(1/2)))^2, x)
```

3.38 $\int x^2\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 497

$$\int x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{2b^2x^{5/2}}{d}+\frac{a^2x^3}{3}+\frac{8abx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{10b^2x^2\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{20iabx^2\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{20iabx^2\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}-\frac{20b^2x^{3/2}\,\text{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{80iabx^{3/2}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{80iabx^{3/2}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}+\frac{30b^2x\,\text{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{240iabx\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{240iabx\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4}-\frac{30b^2\sqrt{x}\,\text{PolyLog}\left(4,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5}+\frac{480iab\sqrt{x}\,\text{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5}-\frac{480iab\sqrt{x}\,\text{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5}+\frac{15b^2\,\text{PolyLog}\left(5,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6}-\frac{480iab\,\text{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6}+\frac{480iab\,\text{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6}+\frac{2b^2x^{5/2}\tanh\left(c+d\sqrt{x}\right)}{d}$$

```
[Out] 2*b^2*x^(5/2)/d+1/3*a^2*x^3+8*a*b*x^(5/2)*arctan(exp(c+d*x^(1/2)))/d-10*b^2
*x^2*ln(1+exp(2*c+2*d*x^(1/2)))/d^2+20*I*a*b*x^2*polylog(2,I*exp(c+d*x^(1/2
)))/d^2-240*I*a*b*x*polylog(4,-I*exp(c+d*x^(1/2)))/d^4-20*b^2*x^(3/2)*polyl
og(2,-exp(2*c+2*d*x^(1/2)))/d^3+480*I*a*b*polylog(6,I*exp(c+d*x^(1/2)))/d^6
```

```
+240*I*a*b*x*polylog(4,I*exp(c+d*x^(1/2)))/d^4+30*b^2*x*polylog(3,-exp(2*c+
2*d*x^(1/2)))/d^4-20*I*a*b*x^2*polylog(2,-I*exp(c+d*x^(1/2)))/d^2-80*I*a*b*
x^(3/2)*polylog(3,I*exp(c+d*x^(1/2)))/d^3+15*b^2*polylog(5,-exp(2*c+2*d*x^(
1/2)))/d^6-480*I*a*b*polylog(6,-I*exp(c+d*x^(1/2)))/d^6+80*I*a*b*x^(3/2)*po
lylog(3,-I*exp(c+d*x^(1/2)))/d^3-30*b^2*polylog(4,-exp(2*c+2*d*x^(1/2)))*x^
(1/2)/d^5-480*I*a*b*polylog(5,I*exp(c+d*x^(1/2)))*x^(1/2)/d^5+480*I*a*b*pol
ylog(5,-I*exp(c+d*x^(1/2)))*x^(1/2)/d^5+2*b^2*x^(5/2)*tanh(c+d*x^(1/2))/d
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules

used = {5544, 4275, 4265, 2611, 6744, 2320, 6724, 4269, 3799, 2221}

$$\int x^2\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{a^2x^3}{3}+\frac{8abx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{480iab\operatorname{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6}+\frac{480iab\operatorname{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6}+\frac{480iab\sqrt{x}\operatorname{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5}-\frac{480iab\sqrt{x}\operatorname{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5}-\frac{240iabx\operatorname{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{240iabx\operatorname{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{80iabx^{3/2}\operatorname{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{80iabx^{3/2}\operatorname{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{20iabx^2\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{20iabx^2\operatorname{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{15b^2\operatorname{PolyLog}\left(5,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6}-\frac{30b^2\sqrt{x}\operatorname{PolyLog}\left(4,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5}+\frac{30b^2x\operatorname{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{20b^2x^{3/2}\operatorname{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}-\frac{10b^2x^2\log\left(e^{2\left(c+d\sqrt{x}\right)}+1\right)}{d^2}+\frac{2b^2x^{5/2}\tanh\left(c+d\sqrt{x}\right)}{d}+\frac{2b^2x^{5/2}}{d}$$

```
[In] Int[x^2*(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*b^2*x^(5/2))/d + (a^2*x^3)/3 + (8*a*b*x^(5/2)*ArcTan[E^(c + d*Sqrt[x])])
/d - (10*b^2*x^2*Log[1 + E^(2*(c + d*Sqrt[x]))])/d^2 - ((20*I)*a*b*x^2*Poly
```

```
Log[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((20*I)*a*b*x^2*PolyLog[2, I*E^(c + d
*Sqrt[x])])/d^2 - (20*b^2*x^(3/2)*PolyLog[2, -E^(2*(c + d*Sqrt[x]))])/d^3 +
 ((80*I)*a*b*x^(3/2)*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((80*I)*a*b*
x^(3/2)*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 + (30*b^2*x*PolyLog[3, -E^(2*(
c + d*Sqrt[x]))])/d^4 - ((240*I)*a*b*x*PolyLog[4, (-I)*E^(c + d*Sqrt[x])])/
d^4 + ((240*I)*a*b*x*PolyLog[4, I*E^(c + d*Sqrt[x])])/d^4 - (30*b^2*Sqrt[x]
*PolyLog[4, -E^(2*(c + d*Sqrt[x]))])/d^5 + ((480*I)*a*b*Sqrt[x]*PolyLog[5,
(-I)*E^(c + d*Sqrt[x])])/d^5 - ((480*I)*a*b*Sqrt[x]*PolyLog[5, I*E^(c + d*S
qrt[x])])/d^5 + (15*b^2*PolyLog[5, -E^(2*(c + d*Sqrt[x]))])/d^6 - ((480*I)*
a*b*PolyLog[6, (-I)*E^(c + d*Sqrt[x])])/d^6 + ((480*I)*a*b*PolyLog[6, I*E^(
c + d*Sqrt[x])])/d^6 + (2*b^2*x^(5/2)*Tanh[c + d*Sqrt[x]])/d
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int x^5(a+b\,\text{sech}(c+dx))^2\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(a^2x^5+2abx^5\text{sech}(c+dx)+b^2x^5\text{sech}^2(c+dx)\right)\,dx, x, \sqrt{x}\right)$$

$$= \frac{a^2x^3}{3} + (4ab)\text{Subst}\left(\int x^5\text{sech}(c+dx)\,dx, x, \sqrt{x}\right) + (2b^2)\,\text{Subst}\left(\int x^5\text{sech}^2(c+dx)\,dx, x, \sqrt{x}\right)$$

$$= \frac{a^2x^3}{3} + \frac{8abx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} + \frac{2b^2x^{5/2}\tanh\left(c+d\sqrt{x}\right)}{d} - \frac{(20iab)\text{Subst}\left(\int x^4\log\left(1-ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d} + \frac{(20iab)\text{Subst}\left(\int x^4\log\left(1+ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d} - \frac{(10b^2)\,\text{Subst}\left(\int x^4\tanh(c+dx)\,dx, x, \sqrt{x}\right)}{d}$$

$$= \frac{2b^2x^{5/2}}{d} + \frac{a^2x^3}{3} + \frac{8abx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{20iabx^2\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{20iabx^2\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{2b^2x^{5/2}\tanh\left(c+d\sqrt{x}\right)}{d} + \frac{(80iab)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2,-ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2} - \frac{(80iab)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2,ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2} - \frac{(20b^2)\,\text{Subst}\left(\int \frac{e^{2(c+dx)}x^4}{1+e^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{d}$$

$$= \frac{2b^2x^{5/2}}{d} + \frac{a^2x^3}{3} + \frac{8abx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10b^2x^2\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{20iabx^2\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{20iabx^2\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{80iabx^{3/2}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{80iabx^{3/2}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{2b^2x^{5/2}\tanh\left(c+d\sqrt{x}\right)}{d} - \frac{(240iab)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3,-ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^3} + \frac{(240iab)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3,ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^3} + \frac{(40b^2)\,\text{Subst}\left(\int x^3\log\left(1+e^{2(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$

$$= \frac{2b^2x^{5/2}}{d} + \frac{a^2x^3}{3} + \frac{8abx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10b^2x^2\log\left(1+e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{20iabx^2\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{20iabx^2\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} - \frac{20b^2x^{3/2}\,\text{PolyLog}\left(2,-e^{2(c+d\sqrt{x})}\right)}{d^3} + \frac{80iabx^{3/2}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{80iabx^{3/2}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{240iabx\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{240iabx\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{2b^2x^{5/2}\tanh\left(c+d\sqrt{x}\right)}{d} + \frac{(480iab)\text{Subst}\left(\int x\,\text{PolyLog}\left(4,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^4} - \frac{(480iab)\text{Subst}\left(\int x\,\text{PolyLog}\left(4,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^4} + \frac{(60b^2)\,\text{Subst}\left(\int x^2\,\text{PolyLog}\left(2,-e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= \frac{2b^2x^{5/2}}{d} + \frac{a^2x^3}{3} + \frac{8abx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10b^2x^2\log\left(1+e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{20iabx^2\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{20iabx^2\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} - \frac{20b^2x^{3/2}\,\text{PolyLog}\left(2,-e^{2(c+d\sqrt{x})}\right)}{d^3} + \frac{80iabx^{3/2}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{80iabx^{3/2}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{30b^2x\,\text{PolyLog}\left(3,-e^{2(c+d\sqrt{x})}\right)}{d^4} - \frac{240iabx\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{240iabx\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{480iab\sqrt{x}\,\text{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{480iab\sqrt{x}\,\text{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} + \frac{2b^2x^{5/2}\tanh\left(c+d\sqrt{x}\right)}{d} - \frac{(480iab)\text{Subst}\left(\int \text{PolyLog}\left(5,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^5} + \frac{(480iab)\text{Subst}\left(\int \text{PolyLog}\left(5,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^5} - \frac{(60b^2)\,\text{Subst}\left(\int x\,\text{PolyLog}\left(3,-e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$

$$
\begin{aligned}
&= \frac{2b^2x^{5/2}}{d} + \frac{a^2x^3}{3} + \frac{8abx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10b^2x^2\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
&\quad - \frac{20iabx^2\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{20iabx^2\operatorname{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad - \frac{20b^2x^{3/2}\operatorname{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{80iabx^{3/2}\operatorname{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{80iabx^{3/2}\operatorname{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{30b^2x\operatorname{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} \\
&\quad - \frac{240iabx\operatorname{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{240iabx\operatorname{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} \\
&\quad - \frac{30b^2\sqrt{x}\operatorname{PolyLog}\left(4,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5} + \frac{480iab\sqrt{x}\operatorname{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} \\
&\quad - \frac{480iab\sqrt{x}\operatorname{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} + \frac{2b^2x^{5/2}\tanh\left(c+d\sqrt{x}\right)}{d} \\
&\quad - \frac{(480iab)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(5,-ix)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{d^6} \\
&\quad + \frac{(480iab)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(5,ix)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{d^6} \\
&\quad + \frac{\left(30b^2\right)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(4,-e^{2(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^5}
\end{aligned}
$$

$$= \frac{2b^2x^{5/2}}{d} + \frac{a^2x^3}{3} + \frac{8abx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10b^2x^2\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{20iabx^2\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{20iabx^2\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} - \frac{20b^2x^{3/2}\,\text{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{80iabx^{3/2}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{80iabx^{3/2}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{30b^2x\,\text{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} - \frac{240iabx\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{240iabx\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} - \frac{30b^2\sqrt{x}\,\text{PolyLog}\left(4,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5} + \frac{480iab\sqrt{x}\,\text{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{480iab\sqrt{x}\,\text{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{480iab\,\text{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{480iab\,\text{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{2b^2x^{5/2}\tanh\left(c+d\sqrt{x}\right)}{d} + \frac{(15b^2)\,\text{Subst}\left(\int\frac{\text{PolyLog}(4,-x)}{x}\,dx,x,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6}$$

$$= \frac{2b^2x^{5/2}}{d} + \frac{a^2x^3}{3} + \frac{8abx^{5/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10b^2x^2\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{20iabx^2\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{20iabx^2\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} - \frac{20b^2x^{3/2}\,\text{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{80iabx^{3/2}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{80iabx^{3/2}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{30b^2x\,\text{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} - \frac{240iabx\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{240iabx\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} - \frac{30b^2\sqrt{x}\,\text{PolyLog}\left(4,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5} + \frac{480iab\sqrt{x}\,\text{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{480iab\sqrt{x}\,\text{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} + \frac{15b^2\,\text{PolyLog}\left(5,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^6} - \frac{480iab\,\text{PolyLog}\left(6,-ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{480iab\,\text{PolyLog}\left(6,ie^{c+d\sqrt{x}}\right)}{d^6} + \frac{2b^2x^{5/2}\tanh\left(c+d\sqrt{x}\right)}{d}$$

Mathematica [A] (verified)

Time = 7.13 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.17

$$\int x^2 \left(a + b\text{sech}\left(c + d\sqrt{x}\right)\right)^2 dx$$
$$= \frac{\cosh\left(c + d\sqrt{x}\right)\left(a + b\text{sech}\left(c + d\sqrt{x}\right)\right)^2 \left(\frac{12b^2e^{2c}x^{5/2}\cosh(c+d\sqrt{x})}{d(1+e^{2c})} + a^2x^3\cosh\left(c + d\sqrt{x}\right) + \frac{3ib\cosh(c+d\sqrt{x})\left(4ad^5\right.}{}\right.}{}$$

```
[In] Integrate[x^2*(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (Cosh[c + d*Sqrt[x]]*(a + b*Sech[c + d*Sqrt[x]])^2*((12*b^2*E^(2*c)*x^(5/2)
*Cosh[c + d*Sqrt[x]])/(d*(1 + E^(2*c))) + a^2*x^3*Cosh[c + d*Sqrt[x]] + ((3
*I)*b*Cosh[c + d*Sqrt[x]]*(4*a*d^5*x^(5/2)*Log[1 - I*E^(c + d*Sqrt[x])] - 4
*a*d^5*x^(5/2)*Log[1 + I*E^(c + d*Sqrt[x])] + (10*I)*b*d^4*x^2*Log[1 + E^(2
*(c + d*Sqrt[x]))] - 20*a*d^4*x^2*PolyLog[2, (-I)*E^(c + d*Sqrt[x])] + 20*a
*d^4*x^2*PolyLog[2, I*E^(c + d*Sqrt[x])] + (20*I)*b*d^3*x^(3/2)*PolyLog[2,
-E^(2*(c + d*Sqrt[x]))] + 80*a*d^3*x^(3/2)*PolyLog[3, (-I)*E^(c + d*Sqrt[x]
)] - 80*a*d^3*x^(3/2)*PolyLog[3, I*E^(c + d*Sqrt[x])] - (30*I)*b*d^2*x*Poly
Log[3, -E^(2*(c + d*Sqrt[x]))] - 240*a*d^2*x*PolyLog[4, (-I)*E^(c + d*Sqrt[
x])] + 240*a*d^2*x*PolyLog[4, I*E^(c + d*Sqrt[x])] + (30*I)*b*d*Sqrt[x]*Pol
yLog[4, -E^(2*(c + d*Sqrt[x]))] + 480*a*d*Sqrt[x]*PolyLog[5, (-I)*E^(c + d*
Sqrt[x])] - 480*a*d*Sqrt[x]*PolyLog[5, I*E^(c + d*Sqrt[x])] - (15*I)*b*Poly
Log[5, -E^(2*(c + d*Sqrt[x]))] - 480*a*PolyLog[6, (-I)*E^(c + d*Sqrt[x])] +
 480*a*PolyLog[6, I*E^(c + d*Sqrt[x])]))/d^6 + (6*b^2*x^(5/2)*Sech[c]*Sinh[
d*Sqrt[x]])/d))/(3*(b + a*Cosh[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int x^2 \left(a + b\ \text{sech}\left(c + d\sqrt{x}\right)\right)^2 dx$$

```
[In] int(x^2*(a+b*sech(c+d*x^(1/2)))^2,x)
```

```
[Out] int(x^2*(a+b*sech(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x^2 \left(a + b\text{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int \left(b \operatorname{sech}\left(d\sqrt{x} + c\right) + a\right)^2 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*sech(d*sqrt(x) + c)^2 + 2*a*b*x^2*sech(d*sqrt(x) + c) + a^
2*x^2, x)
```

Sympy [F]

$$\int x^2 \left(a + b\text{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int x^2 \left(a + b \operatorname{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx$$

```
[In] integrate(x**2*(a+b*sech(c+d*x**(1/2)))**2,x)
[Out] Integral(x**2*(a + b*sech(c + d*sqrt(x)))**2, x)
```

Maxima [F]

$$\int x^2 \left(a + b\text{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int \left(b \operatorname{sech}\left(d\sqrt{x} + c\right) + a\right)^2 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] 1/3*(a^2*d*x^3*e^(2*d*sqrt(x) + 2*c) + a^2*d*x^3 - 12*b^2*x^(5/2))/(d*e^(2*
d*sqrt(x) + 2*c) + d) + integrate(2*(2*a*b*d*x^2*e^(d*sqrt(x) + c) + 5*b^2*
x^(3/2))/(d*e^(2*d*sqrt(x) + 2*c) + d), x)
```

Giac [F]

$$\int x^2 \left(a + b\text{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int \left(b \operatorname{sech}\left(d\sqrt{x} + c\right) + a\right)^2 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2 \, dx = \int x^2 \left(a+\frac{b}{\cosh\left(c+d\sqrt{x}\right)}\right)^2 \, dx$$

```
[In] int(x^2*(a + b/cosh(c + d*x^(1/2)))^2,x)
[Out] int(x^2*(a + b/cosh(c + d*x^(1/2)))^2, x)
```

3.39 $\int x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 319

$$\begin{aligned}\int x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx=&\frac{2b^2x^{3/2}}{d}+\frac{a^2x^2}{2}+\frac{8abx^{3/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d}\\&-\frac{6b^2x\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2}\\&-\frac{12iabx\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}\\&+\frac{12iabx\operatorname{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}\\&-\frac{6b^2\sqrt{x}\operatorname{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}\\&+\frac{24iab\sqrt{x}\operatorname{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}\\&-\frac{24iab\sqrt{x}\operatorname{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}\\&+\frac{3b^2\operatorname{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4}\\&-\frac{24iab\operatorname{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4}\\&+\frac{24iab\operatorname{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{2b^2x^{3/2}\tanh\left(c+d\sqrt{x}\right)}{d}\end{aligned}$$

```
[Out] 2*b^2*x^(3/2)/d+1/2*a^2*x^2+8*a*b*x^(3/2)*arctan(exp(c+d*x^(1/2)))/d-6*b^2*
x*ln(1+exp(2*c+2*d*x^(1/2)))/d^2-12*I*a*b*x*polylog(2,-I*exp(c+d*x^(1/2)))/
d^2+12*I*a*b*x*polylog(2,I*exp(c+d*x^(1/2)))/d^2+3*b^2*polylog(3,-exp(2*c+2
```

```
*d*x^(1/2)))/d^4-24*I*a*b*polylog(4,-I*exp(c+d*x^(1/2)))/d^4+24*I*a*b*polyl
og(4,I*exp(c+d*x^(1/2)))/d^4-6*b^2*polylog(2,-exp(2*c+2*d*x^(1/2)))*x^(1/2)
/d^3+24*I*a*b*polylog(3,-I*exp(c+d*x^(1/2)))*x^(1/2)/d^3-24*I*a*b*polylog(3
,I*exp(c+d*x^(1/2)))*x^(1/2)/d^3+2*b^2*x^(3/2)*tanh(c+d*x^(1/2))/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5544, 4275, 4265, 2611, 6744, 2320, 6724, 4269, 3799, 2221}

$$\begin{aligned}\int x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx = {} & \frac{a^2x^2}{2}+\frac{8abx^{3/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} \\ & -\frac{24iab\operatorname{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} \\ & +\frac{24iab\operatorname{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} \\ & +\frac{24iab\sqrt{x}\operatorname{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} \\ & -\frac{24iab\sqrt{x}\operatorname{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} \\ & -\frac{12iabx\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} \\ & +\frac{12iabx\operatorname{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} \\ & +\frac{3b^2\operatorname{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} \\ & -\frac{6b^2\sqrt{x}\operatorname{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} \\ & -\frac{6b^2x\log\left(e^{2\left(c+d\sqrt{x}\right)}+1\right)}{d^2} \\ & +\frac{2b^2x^{3/2}\tanh\left(c+d\sqrt{x}\right)}{d}+\frac{2b^2x^{3/2}}{d}\end{aligned}$$

```
[In] Int[x*(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*b^2*x^(3/2))/d + (a^2*x^2)/2 + (8*a*b*x^(3/2)*ArcTan[E^(c + d*Sqrt[x])])
/d - (6*b^2*x*Log[1 + E^(2*(c + d*Sqrt[x]))])/d^2 - ((12*I)*a*b*x*PolyLog[2
, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((12*I)*a*b*x*PolyLog[2, I*E^(c + d*Sqrt[x
])])/d^2 - (6*b^2*Sqrt[x]*PolyLog[2, -E^(2*(c + d*Sqrt[x]))])/d^3 + ((24*I)
*a*b*Sqrt[x]*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((24*I)*a*b*Sqrt[x]*
PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 + (3*b^2*PolyLog[3, -E^(2*(c + d*Sqrt[
```

```
x]))])/d^4 - ((24*I)*a*b*PolyLog[4, (-I)*E^(c + d*Sqrt[x])])/d^4 + ((24*I)*
a*b*PolyLog[4, I*E^(c + d*Sqrt[x])])/d^4 + (2*b^2*x^(3/2)*Tanh[c + d*Sqrt[x
]])/d
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
```

```
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^3(a+b\text{sech}(c+dx))^2\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(a^2x^3+2abx^3\text{sech}(c+dx)+b^2x^3\text{sech}^2(c+dx)\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{a^2x^2}{2}+(4ab)\text{Subst}\left(\int x^3\text{sech}(c+dx)\,dx, x, \sqrt{x}\right)\\
&\quad+\left(2b^2\right)\text{Subst}\left(\int x^3\text{sech}^2(c+dx)\,dx, x, \sqrt{x}\right)
\end{aligned}$$

$$=\frac{a^2x^2}{2}+\frac{8abx^{3/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d}+\frac{2b^2x^{3/2}\tanh\left(c+d\sqrt{x}\right)}{d}-\frac{(12iab)\text{Subst}\left(\int x^2\log\left(1-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d}+\frac{(12iab)\text{Subst}\left(\int x^2\log\left(1+ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d}-\frac{(6b^2)\,\text{Subst}\left(\int x^2\tanh(c+dx)\,dx,x,\sqrt{x}\right)}{d}$$

$$=\frac{2b^2x^{3/2}}{d}+\frac{a^2x^2}{2}+\frac{8abx^{3/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{12iabx\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{12iabx\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{2b^2x^{3/2}\tanh\left(c+d\sqrt{x}\right)}{d}+\frac{(24iab)\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^2}-\frac{(24iab)\text{Subst}\left(\int x\,\text{PolyLog}\left(2,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^2}-\frac{(12b^2)\,\text{Subst}\left(\int\frac{e^{2(c+dx)}x^2}{1+e^{2(c+dx)}}\,dx,x,\sqrt{x}\right)}{d}$$

$$=\frac{2b^2x^{3/2}}{d}+\frac{a^2x^2}{2}+\frac{8abx^{3/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{6b^2x\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{12iabx\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{12iabx\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{24iab\sqrt{x}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{24iab\sqrt{x}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}+\frac{2b^2x^{3/2}\tanh\left(c+d\sqrt{x}\right)}{d}-\frac{(24iab)\text{Subst}\left(\int\text{PolyLog}\left(3,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3}+\frac{(24iab)\text{Subst}\left(\int\text{PolyLog}\left(3,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^3}+\frac{(12b^2)\,\text{Subst}\left(\int x\log\left(1+e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2}$$

$$
\begin{aligned}
&= \frac{2b^2x^{3/2}}{d} + \frac{a^2x^2}{2} + \frac{8abx^{3/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{6b^2x\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
&\quad - \frac{12iabx\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{12iabx\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad - \frac{6b^2\sqrt{x}\,\text{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{24iab\sqrt{x}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{24iab\sqrt{x}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{2b^2x^{3/2}\tanh\left(c+d\sqrt{x}\right)}{d} \\
&\quad - \frac{(24iab)\text{Subst}\left(\int \frac{\text{PolyLog}(3,-ix)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^4} \\
&\quad + \frac{(24iab)\text{Subst}\left(\int \frac{\text{PolyLog}(3,ix)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^4} \\
&\quad + \frac{(6b^2)\,\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b^2x^{3/2}}{d} + \frac{a^2x^2}{2} + \frac{8abx^{3/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{6b^2x\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
&\quad - \frac{12iabx\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{12iabx\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} \\
&\quad - \frac{6b^2\sqrt{x}\,\text{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{24iab\sqrt{x}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} \\
&\quad - \frac{24iab\sqrt{x}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{24iab\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} \\
&\quad + \frac{24iab\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{2b^2x^{3/2}\tanh\left(c+d\sqrt{x}\right)}{d} \\
&\quad + \frac{(3b^2)\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4}
\end{aligned}
$$

$$= \frac{2b^2x^{3/2}}{d} + \frac{a^2x^2}{2} + \frac{8abx^{3/2}\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{6b^2x\log\left(1+e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{12iabx\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{12iabx\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} - \frac{6b^2\sqrt{x}\,\text{PolyLog}\left(2,-e^{2(c+d\sqrt{x})}\right)}{d^3} + \frac{24iab\sqrt{x}\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{24iab\sqrt{x}\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{3b^2\,\text{PolyLog}\left(3,-e^{2(c+d\sqrt{x})}\right)}{d^4} - \frac{24iab\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{24iab\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{2b^2x^{3/2}\tanh\left(c+d\sqrt{x}\right)}{d}$$

Mathematica [A] (warning: unable to verify)

Time = 5.82 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.46

$$\int x\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx$$

$$= \frac{\cosh\left(c+d\sqrt{x}\right)\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2\left(a^2x^2\cosh\left(c+d\sqrt{x}\right)+\frac{2b\cosh\left(c+d\sqrt{x}\right)\left(4be^{2c}x^{3/2}+\frac{i\left(1+e^{2c}\right)\left(12ibd^2x\log\left(1-ie^{\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[x*(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (Cosh[c + d*Sqrt[x]]*(a + b*Sech[c + d*Sqrt[x]])^2*(a^2*x^2*Cosh[c + d*Sqrt
[x]] + (2*b*Cosh[c + d*Sqrt[x]]*(4*b*E^(2*c)*x^(3/2) + (I*(1 + E^(2*c))*((1
2*I)*b*d^2*x*Log[1 - I*E^(c + d*Sqrt[x])] + 4*a*d^3*x^(3/2)*Log[1 - I*E^(c
+ d*Sqrt[x])] + (12*I)*b*d^2*x*Log[1 + I*E^(c + d*Sqrt[x])] - 4*a*d^3*x^(3/
2)*Log[1 + I*E^(c + d*Sqrt[x])] - (6*I)*b*d^2*x*Log[1 + E^(2*(c + d*Sqrt[x]
))] - 12*((-I)*b*d*Sqrt[x] + a*d^2*x)*PolyLog[2, (-I)*E^(c + d*Sqrt[x])] +
12*(I*b*d*Sqrt[x] + a*d^2*x)*PolyLog[2, I*E^(c + d*Sqrt[x])] + 24*a*d*Sqrt[
x]*PolyLog[3, (-I)*E^(c + d*Sqrt[x])] - 24*a*d*Sqrt[x]*PolyLog[3, I*E^(c +
d*Sqrt[x])] - (3*I)*b*PolyLog[3, -E^(2*(c + d*Sqrt[x]))] - 24*a*PolyLog[4,
(-I)*E^(c + d*Sqrt[x])] + 24*a*PolyLog[4, I*E^(c + d*Sqrt[x])]))/d^3))/(d*(
1 + E^(2*c))) + (4*b^2*x^(3/2)*Sech[c]*Sinh[d*Sqrt[x]])/d))/(2*(b + a*Cosh[
c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int x\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2dx$$

```
[In] int(x*(a+b*sech(c+d*x^(1/2)))^2,x)
[Out] int(x*(a+b*sech(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\,\mathrm{sech}\left(d\sqrt{x}+c\right)+a\right)^2x\,dx$$

```
[In] integrate(x*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(b^2*x*sech(d*sqrt(x) + c)^2 + 2*a*b*x*sech(d*sqrt(x) + c) + a^2*x,
 x)
```

Sympy [F]

$$\int x\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx$$

```
[In] integrate(x*(a+b*sech(c+d*x**(1/2)))**2,x)
[Out] Integral(x*(a + b*sech(c + d*sqrt(x)))**2, x)
```

Maxima [F]

$$\int x\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\,\mathrm{sech}\left(d\sqrt{x}+c\right)+a\right)^2x\,dx$$

```
[In] integrate(x*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] 1/2*(a^2*d*x^2*e^(2*d*sqrt(x) + 2*c) + a^2*d*x^2 - 8*b^2*x^(3/2))/(d*e^(2*d
*sqrt(x) + 2*c) + d) + integrate(2*(2*a*b*d*x*e^(d*sqrt(x) + c) + 3*b^2*sqr
t(x))/(d*e^(2*d*sqrt(x) + 2*c) + d), x)
```

Giac [F]

$$\int x(a + b\text{sech}(c + d\sqrt{x}))^2 \, dx = \int (b\,\text{sech}\left(d\sqrt{x} + c\right) + a)^2 x \, dx$$

```
[In] integrate(x*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a + b\text{sech}(c + d\sqrt{x}))^2 \, dx = \int x \left(a + \frac{b}{\cosh\left(c + d\sqrt{x}\right)}\right)^2 dx$$

```
[In] int(x*(a + b/cosh(c + d*x^(1/2)))^2,x)
[Out] int(x*(a + b/cosh(c + d*x^(1/2)))^2, x)
```

3.40 $\int \frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx = \text{Int}\left(\frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x},x\right)$$

```
[Out] Unintegrable((a+b*sech(c+d*x^(1/2)))^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx = \int \frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

```
[In] Int[(a + b*Sech[c + d*Sqrt[x]])^2/x,x]
```

```
[Out] Defer[Int][(a + b*Sech[c + d*Sqrt[x]])^2/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 98.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

```
[In] Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/x,x]
[Out] Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\,\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^{2}}{x}dx$$

```
[In] int((a+b*sech(c+d*x^(1/2)))^2/x,x)
[Out] int((a+b*sech(c+d*x^(1/2)))^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)^{2}}{x}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2/x,x, algorithm="fricas")
[Out] integral((b^2*sech(d*sqrt(x) + c)^2 + 2*a*b*sech(d*sqrt(x) + c) + a^2)/x, x
)
```

Sympy [N/A]

Not integrable

Time = 13.81 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\mathrm{sech}\,\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

```
[In] integrate((a+b*sech(c+d*x**(1/2)))**2/x,x)
[Out] Integral((a + b*sech(c + d*sqrt(x)))**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 86, normalized size of antiderivative = 4.30

$$\int \frac{\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\,\mathrm{sech}\,\left(d\sqrt{x}+c\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2/x,x, algorithm="maxima")
[Out] a^2*log(x) - 4*b^2*sqrt(x)/(d*x*e^(2*d*sqrt(x) + 2*c) + d*x) + integrate(2*
(2*a*b*d*x*e^(d*sqrt(x) + c) - b^2*sqrt(x))/(d*x^2*e^(2*d*sqrt(x) + 2*c) +
d*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\,\mathrm{sech}\,\left(d\sqrt{x}+c\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2/x,x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}\right)^2}{x}\,dx$$

```
[In] int((a + b/cosh(c + d*x^(1/2)))^2/x,x)
[Out] int((a + b/cosh(c + d*x^(1/2)))^2/x, x)
```

3.41 $\int \frac{\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\mathrm{Int}\left(\frac{\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*sech(c+d*x^(1/2)))^2/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$

```
[In] Int[(a + b*Sech[c + d*Sqrt[x]])^2/x^2,x]
```

```
[Out] Defer[Int][(a + b*Sech[c + d*Sqrt[x]])^2/x^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 37.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$

```
[In] Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/x^2,x]
[Out] Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\,\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^2}dx$$

```
[In] int((a+b*sech(c+d*x^(1/2)))^2/x^2,x)
[Out] int((a+b*sech(c+d*x^(1/2)))^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\,\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*sech(d*sqrt(x) + c)^2 + 2*a*b*sech(d*sqrt(x) + c) + a^2)/x^2,
 x)
```

Sympy [N/A]

Not integrable

Time = 2.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sech(c+d*x**(1/2)))**2/x**2,x)
[Out] Integral((a + b*sech(c + d*sqrt(x)))**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 110, normalized size of antiderivative = 5.50

$$\int \frac{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2/x^2,x, algorithm="maxima")
[Out] -(a^2*d*x*e^(2*d*sqrt(x) + 2*c) + a^2*d*x + 4*b^2*sqrt(x))/(d*x^2*e^(2*d*sq
rt(x) + 2*c) + d*x^2) + integrate(2*(2*a*b*d*x*e^(d*sqrt(x) + c) - 3*b^2*sq
rt(x))/(d*x^3*e^(2*d*sqrt(x) + 2*c) + d*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2/x^2,x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}\right)^2}{x^2}\,dx$$

```
[In] int((a + b/cosh(c + d*x^(1/2)))^2/x^2,x)
[Out] int((a + b/cosh(c + d*x^(1/2)))^2/x^2, x)
```

3.42 $\int \frac{x^3}{a+b\,\mathrm{sech}(c+d\sqrt{x})}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 961

$$\int \frac{x^3}{a+b\operatorname{sech}\left(c+d\sqrt{x}\right)}\,dx=\frac{x^4}{4a}-\frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$-\frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+\frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+\frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$-\frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$-\frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$+\frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$+\frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$-\frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$-\frac{5040bx\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}$$
$$+\frac{5040bx\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}$$
$$+\frac{10080b\sqrt{x}\operatorname{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7}$$
$$-\frac{10080b\sqrt{x}\operatorname{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7}$$
$$-\frac{10080b\operatorname{PolyLog}\left(8,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^8}$$
$$+\frac{10080b\operatorname{PolyLog}\left(8,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^8}$$

```
[Out] 1/4*x^4/a-2*b*x^(7/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a/d/(-a
^2+b^2)^(1/2)+2*b*x^(7/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d
/(-a^2+b^2)^(1/2)-14*b*x^3*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2
)))/a/d^2/(-a^2+b^2)^(1/2)+14*b*x^3*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+
b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)+84*b*x^(5/2)*polylog(3,-a*exp(c+d*x^(1/
2))/(b-(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)-84*b*x^(5/2)*polylog(3,-a*
exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)-420*b*x^2*pol
ylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a/d^4/(-a^2+b^2)^(1/2)+420
*b*x^2*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d^4/(-a^2+b^2)
^(1/2)+1680*b*x^(3/2)*polylog(5,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a
/d^5/(-a^2+b^2)^(1/2)-1680*b*x^(3/2)*polylog(5,-a*exp(c+d*x^(1/2))/(b+(-a^2
+b^2)^(1/2)))/a/d^5/(-a^2+b^2)^(1/2)-5040*b*x*polylog(6,-a*exp(c+d*x^(1/2))
/(b-(-a^2+b^2)^(1/2)))/a/d^6/(-a^2+b^2)^(1/2)+5040*b*x*polylog(6,-a*exp(c+d
*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d^6/(-a^2+b^2)^(1/2)-10080*b*polylog(8,-a
*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a/d^8/(-a^2+b^2)^(1/2)+10080*b*poly
log(8,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d^8/(-a^2+b^2)^(1/2)+1008
0*b*polylog(7,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^7/(-a^2
+b^2)^(1/2)-10080*b*polylog(7,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(
1/2)/a/d^7/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 961, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used

$= \{5544, 4276, 3401, 2296, 2221, 2611, 6744, 2320, 6724\}$

$$\int \frac{x^3}{a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)}\,dx = \frac{x^4}{4a} - \frac{2b\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}}+1\right)x^{7/2}}{a\sqrt{b^2-a^2}d} + \frac{2b\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}}+1\right)x^{7/2}}{a\sqrt{b^2-a^2}d}$$
$$- \frac{14b\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)x^3}{a\sqrt{b^2-a^2}d^2}$$
$$+ \frac{14b\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)x^3}{a\sqrt{b^2-a^2}d^2}$$
$$+ \frac{84b\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)x^{5/2}}{a\sqrt{b^2-a^2}d^3}$$
$$- \frac{84b\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)x^{5/2}}{a\sqrt{b^2-a^2}d^3}$$
$$- \frac{420b\,\mathrm{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)x^2}{a\sqrt{b^2-a^2}d^4}$$
$$+ \frac{420b\,\mathrm{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)x^2}{a\sqrt{b^2-a^2}d^4}$$
$$+ \frac{1680b\,\mathrm{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)x^{3/2}}{a\sqrt{b^2-a^2}d^5}$$
$$- \frac{1680b\,\mathrm{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)x^{3/2}}{a\sqrt{b^2-a^2}d^5}$$
$$- \frac{5040b\,\mathrm{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)x}{a\sqrt{b^2-a^2}d^6}$$
$$+ \frac{5040b\,\mathrm{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)x}{a\sqrt{b^2-a^2}d^6}$$
$$+ \frac{10080b\,\mathrm{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)\sqrt{x}}{a\sqrt{b^2-a^2}d^7}$$
$$- \frac{10080b\,\mathrm{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)\sqrt{x}}{a\sqrt{b^2-a^2}d^7}$$
$$- \frac{10080b\,\mathrm{PolyLog}\left(8,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{a\sqrt{b^2-a^2}d^8}$$
$$+ \frac{10080b\,\mathrm{PolyLog}\left(8,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{a\sqrt{b^2-a^2}d^8}$$

```
[In] Int[x^3/(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] x^4/(4*a) - (2*b*x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2
])])/(a*Sqrt[-a^2 + b^2]*d) + (2*b*x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b
 + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (14*b*x^3*PolyLog[2, -((a*E
^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (14*
b*x^3*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[
-a^2 + b^2]*d^2) + (84*b*x^(5/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sq
rt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (84*b*x^(5/2)*PolyLog[3, -((a
*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (4
20*b*x^2*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sq
rt[-a^2 + b^2]*d^4) + (420*b*x^2*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sq
rt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (1680*b*x^(3/2)*PolyLog[5, -(
(a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5) -
(1680*b*x^(3/2)*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))]
)/(a*Sqrt[-a^2 + b^2]*d^5) - (5040*b*x*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(
b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^6) + (5040*b*x*PolyLog[6, -(
(a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^6) +
(10080*b*Sqrt[x]*PolyLog[7, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))
])/(a*Sqrt[-a^2 + b^2]*d^7) - (10080*b*Sqrt[x]*PolyLog[7, -((a*E^(c + d*Sqr
t[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^7) - (10080*b*PolyLo
g[8, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*
d^8) + (10080*b*PolyLog[8, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))]
)/(a*Sqrt[-a^2 + b^2]*d^8)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^7}{a+b\text{sech}(c+dx)}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{x^7}{a} - \frac{bx^7}{a(b+a\cosh(c+dx))}\right)\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^4}{4a} - \frac{(2b)\text{Subst}\left(\int \frac{x^7}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{x^4}{4a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^7}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{x^4}{4a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^7}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}} + \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^7}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}$$

$$= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{(14b)\text{Subst}\left(\int x^6\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d} - \frac{(14b)\text{Subst}\left(\int x^6\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}$$

$$= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{(84b)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{(84b)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$- \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{(420b)\operatorname{Subst}\left(\int x^4\operatorname{PolyLog}\left(3,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$+ \frac{(420b)\operatorname{Subst}\left(\int x^4\operatorname{PolyLog}\left(3,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$- \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$+ \frac{(1680b)\operatorname{Subst}\left(\int x^3\operatorname{PolyLog}\left(4,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$- \frac{(1680b)\operatorname{Subst}\left(\int x^3\operatorname{PolyLog}\left(4,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4}$$

$$= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} - \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} - \frac{(5040b)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(5,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{(5040b)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(5,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^5}$$

$$= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{14bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} - \frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} - \frac{5040bx\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} + \frac{5040bx\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} + \frac{(10080b)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(6,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^6} - \frac{(10080b)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(6,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^6}$$

$$
\begin{aligned}
&= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
&- \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
&+ \frac{84bx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84bx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
&- \frac{420bx^2\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\
&+ \frac{1680bx^{3/2}\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} - \frac{1680bx^{3/2}\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} \\
&- \frac{5040bx\,\text{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} + \frac{5040bx\,\text{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} \\
&+ \frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} - \frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} \\
&- \frac{(10080b)\text{Subst}\left(\int \text{PolyLog}\left(7,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^7} \\
&+ \frac{(10080b)\text{Subst}\left(\int \text{PolyLog}\left(7,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^7}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
&- \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{14bx^3\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
&+ \frac{84bx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84bx^{5/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
&- \frac{420bx^2\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\
&+ \frac{1680bx^{3/2}\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} - \frac{1680bx^{3/2}\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} \\
&- \frac{5040bx\,\text{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} + \frac{5040bx\,\text{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} \\
&+ \frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} - \frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} \\
&- \frac{(10080b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(7,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{a\sqrt{-a^2+b^2}d^8} \\
&+ \frac{(10080b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(7,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{a\sqrt{-a^2+b^2}d^8}
\end{aligned}
$$

$$= \frac{x^4}{4a} - \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$- \frac{14bx^3\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{14bx^3\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{84bx^{5/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84bx^{5/2}\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{420bx^2\,\mathrm{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\,\mathrm{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$+ \frac{1680bx^{3/2}\,\mathrm{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} - \frac{1680bx^{3/2}\,\mathrm{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$- \frac{5040bx\,\mathrm{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} + \frac{5040bx\,\mathrm{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}$$
$$+ \frac{10080b\sqrt{x}\,\mathrm{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} - \frac{10080b\sqrt{x}\,\mathrm{PolyLog}\left(7,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7}$$
$$- \frac{10080b\,\mathrm{PolyLog}\left(8,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^8} + \frac{10080b\,\mathrm{PolyLog}\left(8,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^8}$$

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 721, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)}\,dx$$
$$= \frac{\sqrt{-a^2+b^2}d^8x^4 - 8bd^7x^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right) + 8bd^7x^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right) - 56bd^6x^3\,\mathrm{PolyLog}\left(2,\frac{a}{-b+}\right.}{}$$

```
[In] Integrate[x^3/(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (Sqrt[-a^2 + b^2]*d^8*x^4 - 8*b*d^7*x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(
b - Sqrt[-a^2 + b^2])] + 8*b*d^7*x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b +
 Sqrt[-a^2 + b^2])] - 56*b*d^6*x^3*PolyLog[2, (a*E^(c + d*Sqrt[x]))/(-b + S
qrt[-a^2 + b^2])] + 56*b*d^6*x^3*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sq
rt[-a^2 + b^2]))] + 336*b*d^5*x^(5/2)*PolyLog[3, (a*E^(c + d*Sqrt[x]))/(-b
+ Sqrt[-a^2 + b^2])] - 336*b*d^5*x^(5/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))
/(b + Sqrt[-a^2 + b^2]))] - 1680*b*d^4*x^2*PolyLog[4, (a*E^(c + d*Sqrt[x]))
/(-b + Sqrt[-a^2 + b^2])] + 1680*b*d^4*x^2*PolyLog[4, -((a*E^(c + d*Sqrt[x]
))/(b + Sqrt[-a^2 + b^2]))] + 6720*b*d^3*x^(3/2)*PolyLog[5, (a*E^(c + d*Sqr
t[x]))/(-b + Sqrt[-a^2 + b^2])] - 6720*b*d^3*x^(3/2)*PolyLog[5, -((a*E^(c +
```

```
 d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))] - 20160*b*d^2*x*PolyLog[6, (a*E^(c +
d*Sqrt[x]))/(-b + Sqrt[-a^2 + b^2])] + 20160*b*d^2*x*PolyLog[6, -((a*E^(c +
 d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))] + 40320*b*d*Sqrt[x]*PolyLog[7, (a*E^(
c + d*Sqrt[x]))/(-b + Sqrt[-a^2 + b^2])] - 40320*b*d*Sqrt[x]*PolyLog[7, -((
a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))] - 40320*b*PolyLog[8, (a*E^(c
+ d*Sqrt[x]))/(-b + Sqrt[-a^2 + b^2])] + 40320*b*PolyLog[8, -((a*E^(c + d*S
qrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(4*a*Sqrt[-a^2 + b^2]*d^8)
```

Maple [F]

$$\int \frac{x^3}{a + b\operatorname{sech}\left(c + d\sqrt{x}\right)}dx$$

[In] int(x^3/(a+b*sech(c+d*x^(1/2))),x)

[Out] int(x^3/(a+b*sech(c+d*x^(1/2))),x)

Fricas [F]

$$\int \frac{x^3}{a + b\text{sech}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^3}{b\operatorname{sech}\left(d\sqrt{x} + c\right) + a}\,dx$$

[In] integrate(x^3/(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")

[Out] integral(x^3/(b*sech(d*sqrt(x) + c) + a), x)

Sympy [F]

$$\int \frac{x^3}{a + b\text{sech}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^3}{a + b\operatorname{sech}\left(c + d\sqrt{x}\right)}\,dx$$

[In] integrate(x**3/(a+b*sech(c+d*x**(1/2))),x)

[Out] Integral(x**3/(a + b*sech(c + d*sqrt(x))), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{a + b\text{sech}\left(c + d\sqrt{x}\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{x^3}{a + b\text{sech}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^3}{b\,\text{sech}\left(d\sqrt{x} + c\right) + a}\,dx$$

```
[In] integrate(x^3/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*sech(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a + b\text{sech}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^3}{a + \frac{b}{\cosh\left(c+d\sqrt{x}\right)}}\,dx$$

```
[In] int(x^3/(a + b/cosh(c + d*x^(1/2))),x)
```

```
[Out] int(x^3/(a + b/cosh(c + d*x^(1/2))), x)
```

3.43 $\int \frac{x^2}{a+b\,\mathbf{sech}\left(c+d\sqrt{x}\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 721

$$\int \frac{x^2}{a+b\operatorname{sech}\left(c+d\sqrt{x}\right)}\,dx=\frac{x^3}{3a}-\frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$-\frac{10bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+\frac{10bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+\frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$-\frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$-\frac{120bx\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$+\frac{120bx\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$+\frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$-\frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$-\frac{240b\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}$$
$$+\frac{240b\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}$$

```
[Out] 1/3*x^3/a-2*b*x^(5/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a/d/(-a
^2+b^2)^(1/2)+2*b*x^(5/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d
/(-a^2+b^2)^(1/2)-10*b*x^2*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2
)))/a/d^2/(-a^2+b^2)^(1/2)+10*b*x^2*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+
b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)+40*b*x^(3/2)*polylog(3,-a*exp(c+d*x^(1/
2))/(b-(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)-40*b*x^(3/2)*polylog(3,-a*
exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)-120*b*x*polyl
og(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a/d^4/(-a^2+b^2)^(1/2)+120*b
*x*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d^4/(-a^2+b^2)^(1/
```

```
2)-240*b*polylog(6,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a/d^6/(-a^2+b^
2)^(1/2)+240*b*polylog(6,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d^6/(-
a^2+b^2)^(1/2)+240*b*polylog(5,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^
(1/2)/a/d^5/(-a^2+b^2)^(1/2)-240*b*polylog(5,-a*exp(c+d*x^(1/2))/(b+(-a^2+b
^2)^(1/2)))*x^(1/2)/a/d^5/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {5544, 4276, 3401, 2296, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int \frac{x^2}{a+b\text{sech}\left(c+d\sqrt{x}\right)}dx = &-\frac{240b\,\text{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^6\sqrt{b^2-a^2}} \\ &+\frac{240b\,\text{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^6\sqrt{b^2-a^2}} \\ &+\frac{240b\sqrt{x}\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} \\ &-\frac{240b\sqrt{x}\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} \\ &-\frac{120bx\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} \\ &+\frac{120bx\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} \\ &+\frac{40bx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} \\ &-\frac{40bx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} \\ &-\frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} \\ &+\frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}-\frac{2bx^{5/2}\log\left(\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}+1\right)}{ad\sqrt{b^2-a^2}} \\ &+\frac{2bx^{5/2}\log\left(\frac{ae^{c+d\sqrt{x}}}{\sqrt{b^2-a^2}+b}+1\right)}{ad\sqrt{b^2-a^2}}+\frac{x^3}{3a}\end{aligned}$$

```
[In] Int[x^2/(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] x^3/(3*a) - (2*b*x^(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2
])])/(a*Sqrt[-a^2 + b^2]*d) + (2*b*x^(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b
 + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (10*b*x^2*PolyLog[2, -((a*E
^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (10*
b*x^2*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[
-a^2 + b^2]*d^2) + (40*b*x^(3/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sq
rt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (40*b*x^(3/2)*PolyLog[3, -((a
*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (1
20*b*x*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt
[-a^2 + b^2]*d^4) + (120*b*x*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-
a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (240*b*Sqrt[x]*PolyLog[5, -((a*E^
(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5) - (240*
b*Sqrt[x]*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*S
qrt[-a^2 + b^2]*d^5) - (240*b*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[
-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^6) + (240*b*PolyLog[6, -((a*E^(c + d*
Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^6)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= 2\text{Subst}\left(\int \frac{x^5}{a+b\,\text{sech}(c+dx)}\,dx, x, \sqrt{x}\right) \\ &= 2\text{Subst}\left(\int \left(\frac{x^5}{a} - \frac{bx^5}{a(b+a\cosh(c+dx))}\right)\,dx, x, \sqrt{x}\right) \end{aligned}$$

$$= \frac{x^3}{3a} - \frac{(2b)\text{Subst}\left(\int \frac{x^5}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{x^3}{3a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^5}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{x^3}{3a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^5}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}} + \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^5}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}$$

$$= \frac{x^3}{3a} - \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{(10b)\text{Subst}\left(\int x^4\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d} - \frac{(10b)\text{Subst}\left(\int x^4\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}$$

$$= \frac{x^3}{3a} - \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{(40b)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{(40b)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$
\begin{aligned}
&= \frac{x^3}{3a} - \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
&\quad - \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
&\quad + \frac{40bx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{40bx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
&\quad - \frac{(120b)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3} \\
&\quad + \frac{(120b)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x^3}{3a} - \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
&\quad - \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
&\quad + \frac{40bx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{40bx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
&\quad - \frac{120bx\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{120bx\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\
&\quad + \frac{(240b)\text{Subst}\left(\int x\,\text{PolyLog}\left(4,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4} \\
&\quad - \frac{(240b)\text{Subst}\left(\int x\,\text{PolyLog}\left(4,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4}
\end{aligned}
$$

$$= \frac{x^3}{3a} - \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{10bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{10bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{120bx\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{120bx\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} - \frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} - \frac{(240b)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(5,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{(240b)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(5,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^5}$$

$$= \frac{x^3}{3a} - \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{10bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{10bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{120bx\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{120bx\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} - \frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} - \frac{(240b)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(5,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{a\sqrt{-a^2+b^2}d^6} + \frac{(240b)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(5,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{a\sqrt{-a^2+b^2}d^6}$$

$$= \frac{x^3}{3a} - \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$- \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{10bx^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{40bx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{40bx^{3/2}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{120bx\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{120bx\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$+ \frac{240b\sqrt{x}\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} - \frac{240b\sqrt{x}\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$- \frac{240b\,\text{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} + \frac{240b\,\text{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 547, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{a+b\,\text{sech}\left(c+d\sqrt{x}\right)}\,dx$$
$$= \frac{\sqrt{-a^2+b^2}d^6x^3 - 6bd^5x^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right) + 6bd^5x^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right) - 30bd^4x^2\,\text{PolyLog}\left(2,\right.}{}$$

```
[In] Integrate[x^2/(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (Sqrt[-a^2 + b^2]*d^6*x^3 - 6*b*d^5*x^(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(
b - Sqrt[-a^2 + b^2])] + 6*b*d^5*x^(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b +
 Sqrt[-a^2 + b^2])] - 30*b*d^4*x^2*PolyLog[2, (a*E^(c + d*Sqrt[x]))/(-b + S
qrt[-a^2 + b^2])] + 30*b*d^4*x^2*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sq
rt[-a^2 + b^2]))] + 120*b*d^3*x^(3/2)*PolyLog[3, (a*E^(c + d*Sqrt[x]))/(-b
+ Sqrt[-a^2 + b^2])] - 120*b*d^3*x^(3/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))
/(b + Sqrt[-a^2 + b^2]))] - 360*b*d^2*x*PolyLog[4, (a*E^(c + d*Sqrt[x]))/(-
b + Sqrt[-a^2 + b^2])] + 360*b*d^2*x*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b
+ Sqrt[-a^2 + b^2]))] + 720*b*d*Sqrt[x]*PolyLog[5, (a*E^(c + d*Sqrt[x]))/(-
b + Sqrt[-a^2 + b^2])] - 720*b*d*Sqrt[x]*PolyLog[5, -((a*E^(c + d*Sqrt[x]))
/(b + Sqrt[-a^2 + b^2]))] - 720*b*PolyLog[6, (a*E^(c + d*Sqrt[x]))/(-b + Sq
rt[-a^2 + b^2])] + 720*b*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2
+ b^2]))])/(3*a*Sqrt[-a^2 + b^2]*d^6)
```

Maple [F]

$$\int \frac{x^2}{a + b\ \mathrm{sech}\left(c + d\sqrt{x}\right)} dx$$

```
[In] int(x^2/(a+b*sech(c+d*x^(1/2))),x)
[Out] int(x^2/(a+b*sech(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x^2}{a + b\mathrm{sech}\left(c + d\sqrt{x}\right)}\, dx = \int \frac{x^2}{b\,\mathrm{sech}\left(d\sqrt{x} + c\right) + a}\, dx$$

```
[In] integrate(x^2/(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(x^2/(b*sech(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x^2}{a + b\mathrm{sech}\left(c + d\sqrt{x}\right)}\, dx = \int \frac{x^2}{a + b\,\mathrm{sech}\left(c + d\sqrt{x}\right)}\, dx$$

```
[In] integrate(x**2/(a+b*sech(c+d*x**(1/2))),x)
[Out] Integral(x**2/(a + b*sech(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{a + b\mathrm{sech}\left(c + d\sqrt{x}\right)}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{x^2}{a + b\text{sech}\left(c + d\sqrt{x}\right)} \, dx = \int \frac{x^2}{b \,\text{sech}\left(d\sqrt{x} + c\right) + a} \, dx$$

```
[In] integrate(x^2/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(x^2/(b*sech(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a + b\text{sech}\left(c + d\sqrt{x}\right)} \, dx = \int \frac{x^2}{a + \frac{b}{\cosh\left(c + d\sqrt{x}\right)}} \, dx$$

```
[In] int(x^2/(a + b/cosh(c + d*x^(1/2))),x)
[Out] int(x^2/(a + b/cosh(c + d*x^(1/2))), x)
```

3.44 $\int \frac{x}{a+b\operatorname{sech}(c+d\sqrt{x})}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 481

$$\begin{aligned}\int \frac{x}{a+b\operatorname{sech}\left(c+d\sqrt{x}\right)}\,dx &= \frac{x^2}{2a}-\frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d}\\
&\quad+\frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d}-\frac{6bx\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^2}\\
&\quad+\frac{6bx\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^2}\\
&\quad+\frac{12b\sqrt{x}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^3}\\
&\quad-\frac{12b\sqrt{x}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^3}\\
&\quad-\frac{12b\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^4}\\
&\quad+\frac{12b\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^4}\end{aligned}$$

```
[Out] 1/2*x^2/a-2*b*x^(3/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a/d/(-a
^2+b^2)^(1/2)+2*b*x^(3/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d
/(-a^2+b^2)^(1/2)-6*b*x*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))
/a/d^2/(-a^2+b^2)^(1/2)+6*b*x*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(
1/2)))/a/d^2/(-a^2+b^2)^(1/2)-12*b*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b
^2)^(1/2)))/a/d^4/(-a^2+b^2)^(1/2)+12*b*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-
```

```
a^2+b^2)^(1/2)))/a/d^4/(-a^2+b^2)^(1/2)+12*b*polylog(3,-a*exp(c+d*x^(1/2))/
(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^3/(-a^2+b^2)^(1/2)-12*b*polylog(3,-a*exp(
c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^3/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5544, 4276, 3401, 2296, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{x}{a+b\text{sech}\left(c+d\sqrt{x}\right)}\,dx = -\frac{12b\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}}+\frac{12b\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}}+\frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}-\frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}-\frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}+\frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}-\frac{2bx^{3/2}\log\left(\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}+1\right)}{ad\sqrt{b^2-a^2}}+\frac{2bx^{3/2}\log\left(\frac{ae^{c+d\sqrt{x}}}{\sqrt{b^2-a^2}+b}+1\right)}{ad\sqrt{b^2-a^2}}+\frac{x^2}{2a}$$

```
[In] Int[x/(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] x^2/(2*a) - (2*b*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2
])])/(a*Sqrt[-a^2 + b^2]*d) + (2*b*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b
 + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (6*b*x*PolyLog[2, -((a*E^(c
 + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (6*b*x*
PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 +
 b^2]*d^2) + (12*b*Sqrt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^
2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (12*b*Sqrt[x]*PolyLog[3, -((a*E^(c
+ d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (12*b*Po
lyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b
^2]*d^4) + (12*b*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))
])/(a*Sqrt[-a^2 + b^2]*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^3}{a+b\text{sech}(c+dx)}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^3}{a}-\frac{bx^3}{a(b+a\cosh(c+dx))}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{x^2}{2a}-\frac{(2b)\text{Subst}\left(\int \frac{x^3}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{x^2}{2a}-\frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^3}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{x^2}{2a}-\frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^3}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}\\
&\quad+\frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^3}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}\\
&= \frac{x^2}{2a}-\frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}\\
&\quad+\frac{(6b)\text{Subst}\left(\int x^2\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}\\
&\quad-\frac{(6b)\text{Subst}\left(\int x^2\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}
\end{aligned}
$$

$$= \frac{x^2}{2a} - \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{(12b)\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{(12b)\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$= \frac{x^2}{2a} - \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{(12b)\text{Subst}\left(\int \text{PolyLog}\left(3,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3} + \frac{(12b)\text{Subst}\left(\int \text{PolyLog}\left(3,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$= \frac{x^2}{2a} - \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{(12b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(3,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{c+d\sqrt{x}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{(12b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(3,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{c+d\sqrt{x}}\right)}{a\sqrt{-a^2+b^2}d^4}$$

$$= \frac{x^2}{2a} - \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{6bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{12b\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{12b\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.78

$$\int \frac{x}{a+b\text{sech}\left(c+d\sqrt{x}\right)}\,dx$$
$$= \frac{\sqrt{-a^2+b^2}d^4x^2 - 4bd^3x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right) + 4bd^3x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right) - 12bd^2x\,\text{PolyLog}\left(2,\frac{\cdots}{-b\cdots}\right.}{}$$

```
[In] Integrate[x/(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (Sqrt[-a^2 + b^2]*d^4*x^2 - 4*b*d^3*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(
b - Sqrt[-a^2 + b^2])] + 4*b*d^3*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b +
 Sqrt[-a^2 + b^2])] - 12*b*d^2*x*PolyLog[2, (a*E^(c + d*Sqrt[x]))/(-b + Sqr
t[-a^2 + b^2])] + 12*b*d^2*x*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-
a^2 + b^2]))] + 24*b*d*Sqrt[x]*PolyLog[3, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[
-a^2 + b^2])] - 24*b*d*Sqrt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt
[-a^2 + b^2]))] - 24*b*PolyLog[4, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[-a^2 + b
^2])] + 24*b*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(
2*a*Sqrt[-a^2 + b^2]*d^4)
```

Maple [F]

$$\int \frac{x}{a+b\,\text{sech}\left(c+d\sqrt{x}\right)}dx$$

```
[In] int(x/(a+b*sech(c+d*x^(1/2))),x)
```

```
[Out] int(x/(a+b*sech(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x}{a + b\text{sech}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x}{b\,\text{sech}\left(d\sqrt{x} + c\right) + a}\,dx$$

```
[In] integrate(x/(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(x/(b*sech(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x}{a + b\text{sech}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x}{a + b\,\text{sech}\left(c + d\sqrt{x}\right)}\,dx$$

```
[In] integrate(x/(a+b*sech(c+d*x**(1/2))),x)
[Out] Integral(x/(a + b*sech(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a + b\text{sech}\left(c + d\sqrt{x}\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{x}{a + b\text{sech}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x}{b\,\text{sech}\left(d\sqrt{x} + c\right) + a}\,dx$$

```
[In] integrate(x/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(x/(b*sech(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+b\operatorname{sech}\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x}{a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}}\,dx$$

```
[In] int(x/(a + b/cosh(c + d*x^(1/2))),x)
[Out] int(x/(a + b/cosh(c + d*x^(1/2))), x)
```

3.45 $\int \frac{1}{x\left(a+b\,\mathbf{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\operatorname{Int}\left(\frac{1}{x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sech(c+d*x^(1/2))),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Int[1/(x*(a + b*Sech[c + d*Sqrt[x]])),x]
[Out] Defer[Int][1/(x*(a + b*Sech[c + d*Sqrt[x]])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Integrate[1/(x*(a + b*Sech[c + d*Sqrt[x]])),x]
[Out] Integrate[1/(x*(a + b*Sech[c + d*Sqrt[x]])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(a+b\ \text{sech}\left(c+d\sqrt{x}\right)\right)}dx$$

```
[In] int(1/x/(a+b*sech(c+d*x^(1/2))),x)
[Out] int(1/x/(a+b*sech(c+d*x^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(1/(b*x*sech(d*sqrt(x) + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*sech(c+d*x**(1/2))),x)
[Out] Integral(1/(x*(a + b*sech(c + d*sqrt(x)))), x)
```

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")
[Out] -2*b*integrate(e^(d*sqrt(x) + c)/(a^2*x*e^(2*d*sqrt(x) + 2*c) + 2*a*b*x*e^(
d*sqrt(x) + c) + a^2*x), x) + log(x)/a
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(1/((b*sech(d*sqrt(x) + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\,\left(a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}\right)}\,dx$$

```
[In] int(1/(x*(a + b/cosh(c + d*x^(1/2)))),x)
[Out] int(1/(x*(a + b/cosh(c + d*x^(1/2)))), x)
```

3.46 $\int \frac{a+b\text{sech}(c+d\sqrt{x})}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\text{sech}(c+d\sqrt{x})}{x^2} dx = -\frac{a}{x} + b\text{Int}\left(\frac{\text{sech}(c+d\sqrt{x})}{x^2}, x\right)$$

```
[Out] -a/x+b*Unintegrable(sech(c+d*x^(1/2))/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{sech}(c+d\sqrt{x})}{x^2} dx = \int \frac{a+b\text{sech}(c+d\sqrt{x})}{x^2} dx$$

```
[In] Int[(a + b*Sech[c + d*Sqrt[x]])/x^2,x]
[Out] -(a/x) + b*Defer[Int][Sech[c + d*Sqrt[x]]/x^2, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{a}{x^2} + \frac{b\text{sech}(c+d\sqrt{x})}{x^2}\right) dx \\ &= -\frac{a}{x} + b\int \frac{\text{sech}(c+d\sqrt{x})}{x^2} dx \end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] Integrate[(a + b*Sech[c + d*Sqrt[x]])/x^2,x]
[Out] Integrate[(a + b*Sech[c + d*Sqrt[x]])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a+b\ \text{sech}\left(c+d\sqrt{x}\right)}{x^2}dx$$

```
[In] int((a+b*sech(c+d*x^(1/2)))/x^2,x)
[Out] int((a+b*sech(c+d*x^(1/2)))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\,\text{sech}\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x^2,x, algorithm="fricas")
[Out] integral((b*sech(d*sqrt(x) + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\,\text{sech}\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] integrate((a+b*sech(c+d*x**(1/2)))/x**2,x)
[Out] Integral((a + b*sech(c + d*sqrt(x)))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{a + b\text{sech}\left(c + d\sqrt{x}\right)}{x^2}\,dx = \int \frac{b\,\text{sech}\left(d\sqrt{x} + c\right) + a}{x^2}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x^2,x, algorithm="maxima")
[Out] 2*b*integrate(e^(d*sqrt(x) + c)/(x^2*e^(2*d*sqrt(x) + 2*c) + x^2), x) - a/x
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{sech}\left(c + d\sqrt{x}\right)}{x^2}\,dx = \int \frac{b\,\text{sech}\left(d\sqrt{x} + c\right) + a}{x^2}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x^2,x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b\text{sech}\left(c + d\sqrt{x}\right)}{x^2}\,dx = \int \frac{a + \frac{b}{\cosh\left(c+d\sqrt{x}\right)}}{x^2}\,dx$$

```
[In] int((a + b/cosh(c + d*x^(1/2)))/x^2,x)
[Out] int((a + b/cosh(c + d*x^(1/2)))/x^2, x)
```

3.47 $\int \frac{x^3}{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 2851

$$
\begin{aligned}
\int \frac{x^3}{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx &= \frac{2b^2x^{7/2}}{a^2\left(a^2-b^2\right)d}+\frac{x^4}{4a^2}-\frac{14b^2x^3\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}\\
&+\frac{2b^3x^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}\\
&-\frac{4bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}-\frac{14b^2x^3\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}\\
&-\frac{2b^3x^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}\\
&+\frac{4bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}\\
&-\frac{84b^2x^{5/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}\\
&+\frac{14b^3x^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}\\
&-\frac{28bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}\\
&-\frac{84b^2x^{5/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}\\
&-\frac{14b^3x^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}\\
&+\frac{28bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}\\
&+\frac{420b^2x^2\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}\\
&-\frac{84b^3x^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}\\
&+\frac{168bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}\\
&+\frac{420b^2x^2\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}\\
&+\frac{84b^3x^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}\\
&\quad 168bx^{5/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{\ldots}\right)
\end{aligned}
$$

```
[Out] 2*b^2*x^(7/2)*sinh(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*cosh(c+d*x^(1/2)))-4*b*x
^(7/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)
+4*b*x^(7/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)
^(1/2)-28*b*x^3*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^2
/(-a^2+b^2)^(1/2)+28*b*x^3*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2
)))/a^2/d^2/(-a^2+b^2)^(1/2)+168*b*x^(5/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b
-(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-168*b*x^(5/2)*polylog(3,-a*exp
(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-840*b*x^2*poly
log(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^4/(-a^2+b^2)^(1/2)+84
0*b*x^2*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^4/(-a^2+b
^2)^(1/2)+3360*b*x^(3/2)*polylog(5,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2))
)/a^2/d^5/(-a^2+b^2)^(1/2)-3360*b*x^(3/2)*polylog(5,-a*exp(c+d*x^(1/2))/(b+
(-a^2+b^2)^(1/2)))/a^2/d^5/(-a^2+b^2)^(1/2)-10080*b*x*polylog(6,-a*exp(c+d*
x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^6/(-a^2+b^2)^(1/2)+10080*b*x*polylog(6
,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^6/(-a^2+b^2)^(1/2)-10080*b
^2*polylog(6,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2
)/d^7-10080*b^2*polylog(6,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)
/a^2/(a^2-b^2)/d^7-10080*b^3*polylog(7,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1
/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^7+10080*b^3*polylog(7,-a*exp(c+d*x^(1/
2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^7+20160*b*polylog(
7,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^7/(-a^2+b^2)^(1/2
)-20160*b*polylog(7,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d
^7/(-a^2+b^2)^(1/2)-14*b^2*x^3*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2))
)/a^2/(a^2-b^2)/d^2+2*b^3*x^(7/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/
2)))/a^2/(-a^2+b^2)^(3/2)/d-14*b^2*x^3*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2
)^(1/2)))/a^2/(a^2-b^2)/d^2-2*b^3*x^(7/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+
b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-84*b^2*x^(5/2)*polylog(2,-a*exp(c+d*x^(
1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^3+14*b^3*x^3*polylog(2,-a*exp(c
+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-84*b^2*x^(5/2)*p
olylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-14*b^3
*x^3*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/
2)/d^2+420*b^2*x^2*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/
(a^2-b^2)/d^4-84*b^3*x^(5/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1
/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+420*b^2*x^2*polylog(3,-a*exp(c+d*x^(1/2))/(b
+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^4+84*b^3*x^(5/2)*polylog(3,-a*exp(c+d*x
^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3-1680*b^2*x^(3/2)*pol
ylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^5+420*b^3*
x^2*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2
)/d^4-1680*b^2*x^(3/2)*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/
a^2/(a^2-b^2)/d^5-420*b^3*x^2*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(
1/2)))/a^2/(-a^2+b^2)^(3/2)/d^4+5040*b^2*x*polylog(5,-a*exp(c+d*x^(1/2))/(b
-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^6-1680*b^3*x^(3/2)*polylog(5,-a*exp(c+d
*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^5+5040*b^2*x*polylog
(5,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^6+1680*b^3*x^(
3/2)*polylog(5,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/
```

```
2)/d^5+5040*b^3*x*polylog(6,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(
-a^2+b^2)^(3/2)/d^6-5040*b^3*x*polylog(6,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^
(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^6+10080*b^2*polylog(7,-a*exp(c+d*x^(1/2))/(b
-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^8+10080*b^2*polylog(7,-a*exp(c+d*x^(1/2
))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^8+10080*b^3*polylog(8,-a*exp(c+d*x
^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^8-10080*b^3*polylog(8,
-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^8-20160*b*
polylog(8,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^8/(-a^2+b^2)^(1/2
)+20160*b*polylog(8,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^8/(-a^2
+b^2)^(1/2)+2*b^2*x^(7/2)/a^2/(a^2-b^2)/d+1/4*x^4/a^2
```

Rubi [A] (verified)

Time = 2.78 (sec) , antiderivative size = 2851, normalized size of antiderivative = 1.00, number of steps used = 61, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules

used = {5544, 4276, 3405, 3401, 2296, 2221, 2611, 6744, 2320, 6724, 5681}

$$\int \frac{x^3}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{x^4}{4a^2} - \frac{4b\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}}+1\right)x^{7/2}}{a^2\sqrt{b^2-a^2}d} + \frac{2b^3\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}}+1\right)x^{7/2}}{a^2\left(b^2-a^2\right)^{3/2}d} + \frac{4b\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}}+1\right)x^{7/2}}{a^2\sqrt{b^2-a^2}d} - \frac{2b^3\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}}+1\right)x^{7/2}}{a^2\left(b^2-a^2\right)^{3/2}d} + \frac{2b^2\sinh\left(c+d\sqrt{x}\right)x^{7/2}}{a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)} + \frac{2b^2x^{7/2}}{a^2\left(a^2-b^2\right)d} - \frac{14b^2\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}}+1\right)x^3}{a^2\left(a^2-b^2\right)d^2} - \frac{14b^2\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}}+1\right)x^3}{a^2\left(a^2-b^2\right)d^2} - \frac{28b\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)x^3}{a^2\sqrt{b^2-a^2}d^2} + \frac{14b^3\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)x^3}{a^2\left(b^2-a^2\right)^{3/2}d^2} + \frac{28b\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)x^3}{a^2\sqrt{b^2-a^2}d^2} - \frac{14b^3\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)x^3}{a^2\left(b^2-a^2\right)^{3/2}d^2} - \frac{84b^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)x^{5/2}}{a^2\left(a^2-b^2\right)d^3} - \frac{84b^2\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)x^{5/2}}{a^2\left(a^2-b^2\right)d^3} + \frac{168b\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)x^{5/2}}{a^2\sqrt{b^2-a^2}d^3} - \frac{84b^3\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)x^{5/2}}{a^2\left(b^2-a^2\right)^{3/2}d^3} - \frac{168b\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)x^{5/2}}{a^2\sqrt{b^2-a^2}d^3} + \frac{84b^3\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)x^{5/2}}{a^2\left(b^2-a^2\right)^{3/2}d^3} + \frac{420b^2\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)x^2}{a^2\left(a^2-b^2\right)d^4} + \frac{420b^2\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)x^2}{a^2\left(a^2-b^2\right)d^4}$$

```
[In] Int[x^3/(a + b*Sech[c + d*Sqrt[x]])^2,x]

[Out] (2*b^2*x^(7/2))/(a^2*(a^2 - b^2)*d) + x^4/(4*a^2) - (14*b^2*x^3*Log[1 + (a*
E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) + (2*b^3*
x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 +
 b^2)^(3/2)*d) - (4*b*x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2
+ b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - (14*b^2*x^3*Log[1 + (a*E^(c + d*Sqrt[x
]))/(b + Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) - (2*b^3*x^(7/2)*Log[1 +
 (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d)
+ (4*b*x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*
Sqrt[-a^2 + b^2]*d) - (84*b^2*x^(5/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b
 - Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^3) + (14*b^3*x^3*PolyLog[2, -((a
*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2)
- (28*b*x^3*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a
^2*Sqrt[-a^2 + b^2]*d^2) - (84*b^2*x^(5/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]
))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^3) - (14*b^3*x^3*PolyLog[2,
 -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*
d^2) + (28*b*x^3*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))
])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (420*b^2*x^2*PolyLog[3, -((a*E^(c + d*Sqrt[
x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^4) - (84*b^3*x^(5/2)*Poly
Log[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^
(3/2)*d^3) + (168*b*x^(5/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a
^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^3) + (420*b^2*x^2*PolyLog[3, -((a*E^(
c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^4) + (84*b^3*x
^(5/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-
a^2 + b^2)^(3/2)*d^3) - (168*b*x^(5/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(
b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^3) - (1680*b^2*x^(3/2)*Pol
yLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*
d^5) + (420*b^3*x^2*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2
]))])/(a^2*(-a^2 + b^2)^(3/2)*d^4) - (840*b*x^2*PolyLog[4, -((a*E^(c + d*Sq
rt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^4) - (1680*b^2*x^
(3/2)*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(a^
2 - b^2)*d^5) - (420*b^3*x^2*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-
a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^4) + (840*b*x^2*PolyLog[4, -((a*E^
(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^4) + (50
40*b^2*x*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*
(a^2 - b^2)*d^6) - (1680*b^3*x^(3/2)*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b
- Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^5) + (3360*b*x^(3/2)*PolyL
og[5, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^
2]*d^5) + (5040*b^2*x*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b
^2]))])/(a^2*(a^2 - b^2)*d^6) + (1680*b^3*x^(3/2)*PolyLog[5, -((a*E^(c + d*
Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^5) - (3360*b*
x^(3/2)*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*S
qrt[-a^2 + b^2]*d^5) - (10080*b^2*Sqrt[x]*PolyLog[6, -((a*E^(c + d*Sqrt[x])
)/(b - Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^7) + (5040*b^3*x*PolyLog[6,
```

```
-((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d
^6) - (10080*b*x*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))
])/(a^2*Sqrt[-a^2 + b^2]*d^6) - (10080*b^2*Sqrt[x]*PolyLog[6, -((a*E^(c + d
*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^7) - (5040*b^3*x*Po
lyLog[6, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2
)^(3/2)*d^6) + (10080*b*x*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2
 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^6) + (10080*b^2*PolyLog[7, -((a*E^(c +
d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^8) - (10080*b^3*Sq
rt[x]*PolyLog[7, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a
^2 + b^2)^(3/2)*d^7) + (20160*b*Sqrt[x]*PolyLog[7, -((a*E^(c + d*Sqrt[x]))/
(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^7) + (10080*b^2*PolyLog[7
, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^8) +
 (10080*b^3*Sqrt[x]*PolyLog[7, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2
]))])/(a^2*(-a^2 + b^2)^(3/2)*d^7) - (20160*b*Sqrt[x]*PolyLog[7, -((a*E^(c
+ d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^7) + (10080
*b^3*PolyLog[8, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^
2 + b^2)^(3/2)*d^8) - (20160*b*PolyLog[8, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt
[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^8) - (10080*b^3*PolyLog[8, -((a*E^
(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^8) + (
20160*b*PolyLog[8, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*S
qrt[-a^2 + b^2]*d^8) + (2*b^2*x^(7/2)*Sinh[c + d*Sqrt[x]])/(a*(a^2 - b^2)*d
*(b + a*Cosh[c + d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^7}{(a+b\text{sech}(c+dx))^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^7}{a^2}+\frac{b^2x^7}{a^2(b+a\cosh(c+dx))^2}-\frac{2bx^7}{a^2(b+a\cosh(c+dx))}\right)dx, x, \sqrt{x}\right)\\
&= \frac{x^4}{4a^2}-\frac{(4b)\text{Subst}\left(\int \frac{x^7}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^7}{(b+a\cosh(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= \frac{x^4}{4a^2}+\frac{2b^2x^{7/2}\sinh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}-\frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^7}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&\quad-\frac{(2b^3)\,\text{Subst}\left(\int \frac{x^7}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}-\frac{(14b^2)\,\text{Subst}\left(\int \frac{x^6\sinh(c+dx)}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}\\
&= \frac{2b^2x^{7/2}}{a^2\left(a^2-b^2\right)d}+\frac{x^4}{4a^2}+\frac{2b^2x^{7/2}\sinh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}\\
&\quad-\frac{(4b^3)\,\text{Subst}\left(\int \frac{e^{c+dx}x^7}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}\\
&\quad-\frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^7}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}\\
&\quad+\frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^7}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}\\
&\quad-\frac{(14b^2)\,\text{Subst}\left(\int \frac{e^{c+dx}x^6}{b-\sqrt{-a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}\\
&\quad-\frac{(14b^2)\,\text{Subst}\left(\int \frac{e^{c+dx}x^6}{b+\sqrt{-a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2b^2x^{7/2}}{a^2\left(a^2-b^2\right)d} + \frac{x^4}{4a^2} - \frac{14b^2x^3\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^2} \\
&\quad - \frac{4bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{14b^2x^3\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^2} \\
&\quad + \frac{4bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2b^2x^{7/2}\sinh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)} \\
&\quad + \frac{\left(4b^3\right)\text{Subst}\left(\int\frac{e^{c+dx}x^7}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx,x,\sqrt{x}\right)}{a\left(-a^2+b^2\right)^{3/2}} \\
&\quad - \frac{\left(4b^3\right)\text{Subst}\left(\int\frac{e^{c+dx}x^7}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx,x,\sqrt{x}\right)}{a\left(-a^2+b^2\right)^{3/2}} \\
&\quad + \frac{\left(84b^2\right)\text{Subst}\left(\int x^5\log\left(1+\frac{ae^{c+dx}}{b-\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\left(a^2-b^2\right)d^2} \\
&\quad + \frac{\left(84b^2\right)\text{Subst}\left(\int x^5\log\left(1+\frac{ae^{c+dx}}{b+\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\left(a^2-b^2\right)d^2} \\
&\quad + \frac{(28b)\text{Subst}\left(\int x^6\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&\quad - \frac{(28b)\text{Subst}\left(\int x^6\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}
\end{aligned}
$$

$$= \frac{2b^2x^{7/2}}{a^2(a^2-b^2)d} + \frac{x^4}{4a^2} - \frac{14b^2x^3\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{2b^3x^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} - \frac{4bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$- \frac{14b^2x^3\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2b^3x^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{4bx^{7/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{84b^2x^{5/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{28bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{84b^2x^{5/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$+ \frac{28bx^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{2b^2x^{7/2}\sinh\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{(420b^2)\operatorname{Subst}\left(\int x^4\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx}}{b-\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^3}$$
$$+ \frac{(420b^2)\operatorname{Subst}\left(\int x^4\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx}}{b+\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^3}$$
$$+ \frac{(168b)\operatorname{Subst}\left(\int x^5\operatorname{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$- \frac{(168b)\operatorname{Subst}\left(\int x^5\operatorname{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$- \frac{(14b^3)\operatorname{Subst}\left(\int x^6\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{(14b^3)\operatorname{Subst}\left(\int x^6\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

= Too large to display

Mathematica [A] (verified)

Time = 10.37 (sec) , antiderivative size = 3035, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[x^3/(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (x^4*(b + a*Cosh[c + d*Sqrt[x]])^2*Sech[c + d*Sqrt[x]]^2)/(4*a^2*(a + b*Sec
h[c + d*Sqrt[x]])^2) + (2*b*E^c*(b + a*Cosh[c + d*Sqrt[x]])^2*(2*b*E^c*x^(7
/2) - ((1 + E^(2*c))*(7*b*d^6*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^3*Log[1 + (a*E^(
2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] + 2*a^2*d^7*E^c*x^(
7/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]
- b^2*d^7*E^c*x^(7/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 +
 b^2)*E^(2*c)])] + 7*b*d^6*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^3*Log[1 + (a*E^(2*c
 + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] - 2*a^2*d^7*E^c*x^(7/2
)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] + b
^2*d^7*E^c*x^(7/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^
2)*E^(2*c)])] + 7*d^5*(6*b*Sqrt[(-a^2 + b^2)*E^(2*c)] + 2*a^2*d*E^c*Sqrt[x]
 - b^2*d*E^c*Sqrt[x])*x^(5/2)*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c -
 Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 7*d^5*(6*b*Sqrt[(-a^2 + b^2)*E^(2*c)] - 2*
a^2*d*E^c*Sqrt[x] + b^2*d*E^c*Sqrt[x])*x^(5/2)*PolyLog[2, -((a*E^(2*c + d*S
qrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 210*b*d^4*Sqrt[(-a^2 + b^
2)*E^(2*c)]*x^2*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 +
b^2)*E^(2*c)]))] - 84*a^2*d^5*E^c*x^(5/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x
]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 42*b^2*d^5*E^c*x^(5/2)*PolyLog
[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 210*
b*d^4*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^2*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(
b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 84*a^2*d^5*E^c*x^(5/2)*PolyLog[3, -
((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 42*b^2*d^
5*E^c*x^(5/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^
2)*E^(2*c)]))] + 840*b*d^3*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^(3/2)*PolyLog[4, -(
(a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 420*a^2*d^
4*E^c*x^2*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E
^(2*c)]))] - 210*b^2*d^4*E^c*x^2*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^
c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 840*b*d^3*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^
(3/2)*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*
c)]))] - 420*a^2*d^4*E^c*x^2*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c +
Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 210*b^2*d^4*E^c*x^2*PolyLog[4, -((a*E^(2*c
+ d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 2520*b*d^2*Sqrt[(-a^
2 + b^2)*E^(2*c)]*x*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^
2 + b^2)*E^(2*c)]))] - 1680*a^2*d^3*E^c*x^(3/2)*PolyLog[5, -((a*E^(2*c + d*
Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 840*b^2*d^3*E^c*x^(3/2)*
PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))]
```

```
 - 2520*b*d^2*Sqrt[(-a^2 + b^2)*E^(2*c)]*x*PolyLog[5, -((a*E^(2*c + d*Sqrt[
x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 1680*a^2*d^3*E^c*x^(3/2)*Poly
Log[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 8
40*b^2*d^3*E^c*x^(3/2)*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(
-a^2 + b^2)*E^(2*c)]))] + 5040*b*d*Sqrt[(-a^2 + b^2)*E^(2*c)]*Sqrt[x]*PolyL
og[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 50
40*a^2*d^2*E^c*x*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 +
 b^2)*E^(2*c)]))] - 2520*b^2*d^2*E^c*x*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))
/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 5040*b*d*Sqrt[(-a^2 + b^2)*E^(2*c
)]*Sqrt[x]*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*
E^(2*c)]))] - 5040*a^2*d^2*E^c*x*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^
c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 2520*b^2*d^2*E^c*x*PolyLog[6, -((a*E^(2
*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 5040*b*Sqrt[(-a^2
 + b^2)*E^(2*c)]*PolyLog[7, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 +
 b^2)*E^(2*c)]))] - 10080*a^2*d*E^c*Sqrt[x]*PolyLog[7, -((a*E^(2*c + d*Sqrt
[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 5040*b^2*d*E^c*Sqrt[x]*PolyL
og[7, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 50
40*b*Sqrt[(-a^2 + b^2)*E^(2*c)]*PolyLog[7, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c
 + Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 10080*a^2*d*E^c*Sqrt[x]*PolyLog[7, -((a*
E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 5040*b^2*d*E^
c*Sqrt[x]*PolyLog[7, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E
^(2*c)]))] + 10080*a^2*E^c*PolyLog[8, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sq
rt[(-a^2 + b^2)*E^(2*c)]))] - 5040*b^2*E^c*PolyLog[8, -((a*E^(2*c + d*Sqrt[
x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 10080*a^2*E^c*PolyLog[8, -((a
*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 5040*b^2*E^c
*PolyLog[8, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))
]))/(d^7*E^c*Sqrt[(-a^2 + b^2)*E^(2*c)]))*Sech[c + d*Sqrt[x]]^2)/(a^2*(a^2
- b^2)*d*(1 + E^(2*c))*(a + b*Sech[c + d*Sqrt[x]])^2) + (2*(b + a*Cosh[c +
d*Sqrt[x]])*Sech[c]*Sech[c + d*Sqrt[x]]^2*(b^3*x^(7/2)*Sinh[c] - a*b^2*x^(7
/2)*Sinh[d*Sqrt[x]]))/(a^2*(-a + b)*(a + b)*d*(a + b*Sech[c + d*Sqrt[x]])^2
)
```

Maple [F]

$$\int \frac{x^3}{\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}dx$$

[In] int(x^3/(a+b*sech(c+d*x^(1/2)))^2,x)

[Out] int(x^3/(a+b*sech(c+d*x^(1/2)))^2,x)

Fricas [F]

$$\int \frac{x^3}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^3}{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^3/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(x^3/(b^2*sech(d*sqrt(x) + c)^2 + 2*a*b*sech(d*sqrt(x) + c) + a^2),
 x)
```

Sympy [F]

$$\int \frac{x^3}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^3}{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(x**3/(a+b*sech(c+d*x**(1/2)))**2,x)
[Out] Integral(x**3/(a + b*sech(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{x^3}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^3}{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^3/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^3}{\left(a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(x^3/(a + b/cosh(c + d*x^(1/2)))^2,x)
[Out] int(x^3/(a + b/cosh(c + d*x^(1/2)))^2, x)
```

3.48 $\int \frac{x^2}{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 2123

$$\int \frac{x^2}{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\frac{2b^2x^{5/2}}{a^2\left(a^2-b^2\right)d}+\frac{x^3}{3a^2}-\frac{10b^2x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}$$

$$+\frac{2b^3x^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}$$

$$-\frac{4bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}-\frac{10b^2x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}$$

$$-\frac{2b^3x^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}$$

$$+\frac{4bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$-\frac{40b^2x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}$$

$$+\frac{10b^3x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}$$

$$-\frac{20bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$-\frac{40b^2x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}$$

$$-\frac{10b^3x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}$$

$$+\frac{20bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$+\frac{120b^2x\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}$$

$$-\frac{40b^3x^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}$$

$$+\frac{80bx^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$

$$+\frac{120b^2x\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}$$

$$+\frac{40b^3x^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}$$

$$80bx^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{\ }\right)$$

```
[Out] 2*b^2*x^(5/2)*sinh(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*cosh(c+d*x^(1/2)))+1/3*x
^3/a^2+10*b^3*x^2*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(
-a^2+b^2)^(3/2)/d^2-40*b^2*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b
^2)^(1/2)))/a^2/(a^2-b^2)/d^3-10*b^3*x^2*polylog(2,-a*exp(c+d*x^(1/2))/(b+(
-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+120*b^2*x*polylog(3,-a*exp(c+d*x
^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^4-40*b^3*x^(3/2)*polylog(3,-a
*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+120*b^2*x*
polylog(3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^4+40*b^
3*x^(3/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2
)^(3/2)/d^3+120*b^3*x*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a
^2/(-a^2+b^2)^(3/2)/d^4-120*b^3*x*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^
2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^4-4*b*x^(5/2)*ln(1+a*exp(c+d*x^(1/2))/(b-
(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)+4*b*x^(5/2)*ln(1+a*exp(c+d*x^(1/2
))/(b+(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)-20*b*x^2*polylog(2,-a*exp(c
+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+20*b*x^2*polylog
(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+80*b*
x^(3/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b
^2)^(1/2)-80*b*x^(3/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/
a^2/d^3/(-a^2+b^2)^(1/2)-240*b*x*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2
)^(1/2)))/a^2/d^4/(-a^2+b^2)^(1/2)+240*b*x*polylog(4,-a*exp(c+d*x^(1/2))/(b
+(-a^2+b^2)^(1/2)))/a^2/d^4/(-a^2+b^2)^(1/2)-240*b^2*polylog(4,-a*exp(c+d*x
^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^5-240*b^2*polylog(4,-
a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^5-240*b^3*
polylog(5,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^
(3/2)/d^5+240*b^3*polylog(5,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/
2)/a^2/(-a^2+b^2)^(3/2)/d^5+480*b*polylog(5,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^
2)^(1/2)))*x^(1/2)/a^2/d^5/(-a^2+b^2)^(1/2)-480*b*polylog(5,-a*exp(c+d*x^(1
/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^5/(-a^2+b^2)^(1/2)-10*b^2*x^2*ln(1
+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^2+2*b^3*x^(5/2)*l
n(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-10*b^2*
x^2*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^2-2*b^3*x
^(5/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d
-40*b^2*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^
2-b^2)/d^3+240*b^2*polylog(5,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/
(a^2-b^2)/d^6+240*b^2*polylog(5,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a
^2/(a^2-b^2)/d^6+240*b^3*polylog(6,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2))
)/a^2/(-a^2+b^2)^(3/2)/d^6-240*b^3*polylog(6,-a*exp(c+d*x^(1/2))/(b+(-a^2+b
^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^6-480*b*polylog(6,-a*exp(c+d*x^(1/2))/(b
-(-a^2+b^2)^(1/2)))/a^2/d^6/(-a^2+b^2)^(1/2)+480*b*polylog(6,-a*exp(c+d*x^(
1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^6/(-a^2+b^2)^(1/2)+2*b^2*x^(5/2)/a^2/(a^2
-b^2)/d
```

Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 2123, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules

used = {5544, 4276, 3405, 3401, 2296, 2221, 2611, 6744, 2320, 6724, 5681}

$$\int \frac{x^2}{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{2x^{5/2}\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}}+1\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d} - \frac{2x^{5/2}\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}}+1\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d}$$
$$+ \frac{10x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}$$
$$- \frac{10x^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}$$
$$- \frac{40x^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3}$$
$$+ \frac{40x^{3/2}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3}$$
$$+ \frac{120x\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4}$$
$$- \frac{120x\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4}$$
$$- \frac{240\sqrt{x}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^5}$$
$$+ \frac{240\sqrt{x}\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^5}$$
$$+ \frac{240\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^6}$$
$$- \frac{240\operatorname{PolyLog}\left(6,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^6} + \frac{2x^{5/2}b^2}{a^2\left(a^2-b^2\right)d}$$
$$- \frac{10x^2\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2} - \frac{10x^2\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2}$$
$$- \frac{40x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3}$$
$$- \frac{40x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3}$$
$$+ \frac{120x\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4}$$
$$+ \frac{120x\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4}$$
$$240\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^2$$

```
[In] Int[x^2/(a + b*Sech[c + d*Sqrt[x]])^2,x]

[Out] (2*b^2*x^(5/2))/(a^2*(a^2 - b^2)*d) + x^3/(3*a^2) - (10*b^2*x^2*Log[1 + (a*
E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) + (2*b^3*
x^(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 +
 b^2)^(3/2)*d) - (4*b*x^(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2
+ b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - (10*b^2*x^2*Log[1 + (a*E^(c + d*Sqrt[x
]))/(b + Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) - (2*b^3*x^(5/2)*Log[1 +
 (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d)
+ (4*b*x^(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*
Sqrt[-a^2 + b^2]*d) - (40*b^2*x^(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b
 - Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^3) + (10*b^3*x^2*PolyLog[2, -((a
*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2)
- (20*b*x^2*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a
^2*Sqrt[-a^2 + b^2]*d^2) - (40*b^2*x^(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]
))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^3) - (10*b^3*x^2*PolyLog[2,
 -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*
d^2) + (20*b*x^2*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))
])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (120*b^2*x*PolyLog[3, -((a*E^(c + d*Sqrt[x]
))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^4) - (40*b^3*x^(3/2)*PolyLo
g[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3
/2)*d^3) + (80*b*x^(3/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2
+ b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^3) + (120*b^2*x*PolyLog[3, -((a*E^(c + d
*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^4) + (40*b^3*x^(3/2
)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 +
 b^2)^(3/2)*d^3) - (80*b*x^(3/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sq
rt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^3) - (240*b^2*Sqrt[x]*PolyLog[4,
 -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^5) +
(120*b^3*x*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^
2*(-a^2 + b^2)^(3/2)*d^4) - (240*b*x*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b
- Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^4) - (240*b^2*Sqrt[x]*PolyLo
g[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^5
) - (120*b^3*x*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])
/(a^2*(-a^2 + b^2)^(3/2)*d^4) + (240*b*x*PolyLog[4, -((a*E^(c + d*Sqrt[x]))
/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^4) + (240*b^2*PolyLog[5,
 -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^6) -
(240*b^3*Sqrt[x]*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))
])/(a^2*(-a^2 + b^2)^(3/2)*d^5) + (480*b*Sqrt[x]*PolyLog[5, -((a*E^(c + d*S
qrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^5) + (240*b^2*Po
lyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)
*d^6) + (240*b^3*Sqrt[x]*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2
+ b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^5) - (480*b*Sqrt[x]*PolyLog[5, -((a*E^
(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^5) + (24
0*b^3*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a
^2 + b^2)^(3/2)*d^6) - (480*b*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[
```

```
-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^6) - (240*b^3*PolyLog[6, -((a*E^(c
+ d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^6) + (480
*b*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-
a^2 + b^2]*d^6) + (2*b^2*x^(5/2)*Sinh[c + d*Sqrt[x]])/(a*(a^2 - b^2)*d*(b +
 a*Cosh[c + d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^5}{(a+b\text{sech}(c+dx))^2}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{x^5}{a^2} + \frac{b^2x^5}{a^2(b+a\cosh(c+dx))^2} - \frac{2bx^5}{a^2(b+a\cosh(c+dx))}\right)\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^3}{3a^2} - \frac{(4b)\text{Subst}\left(\int \frac{x^5}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{x^5}{(b+a\cosh(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}$$

$$= \frac{x^3}{3a^2} + \frac{2b^2x^{5/2}\sinh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)} - \frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^5}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2} - \frac{(2b^3)\,\text{Subst}\left(\int \frac{x^5}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)} - \frac{(10b^2)\,\text{Subst}\left(\int \frac{x^4\sinh(c+dx)}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}$$

$$= \frac{2b^2x^{5/2}}{a^2\left(a^2-b^2\right)d} + \frac{x^3}{3a^2} + \frac{2b^2x^{5/2}\sinh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)} - \frac{(4b^3)\,\text{Subst}\left(\int \frac{e^{c+dx}x^5}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)} - \frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^5}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}} + \frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^5}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}} - \frac{(10b^2)\,\text{Subst}\left(\int \frac{e^{c+dx}x^4}{b-\sqrt{-a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d} - \frac{(10b^2)\,\text{Subst}\left(\int \frac{e^{c+dx}x^4}{b+\sqrt{-a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}$$

$$= \frac{2b^2x^{5/2}}{a^2(a^2-b^2)d} + \frac{x^3}{3a^2} - \frac{10b^2x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$- \frac{4bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{10b^2x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{4bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2b^2x^{5/2}\sinh\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{(4b^3)\,\text{Subst}\left(\int\frac{e^{c+dx}x^5}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx,x,\sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$
$$- \frac{(4b^3)\,\text{Subst}\left(\int\frac{e^{c+dx}x^5}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx,x,\sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$
$$+ \frac{(40b^2)\,\text{Subst}\left(\int x^3\log\left(1+\frac{ae^{c+dx}}{b-\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{(40b^2)\,\text{Subst}\left(\int x^3\log\left(1+\frac{ae^{c+dx}}{b+\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{(20b)\text{Subst}\left(\int x^4\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$- \frac{(20b)\text{Subst}\left(\int x^4\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$
\begin{aligned}
&= \frac{2b^2x^{5/2}}{a^2(a^2-b^2)d} + \frac{x^3}{3a^2} - \frac{10b^2x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2} \\
&+ \frac{2b^3x^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} - \frac{4bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&- \frac{10b^2x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2b^3x^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
&+ \frac{4bx^{5/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{40b^2x^{3/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3} \\
&- \frac{20bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{40b^2x^{3/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3} \\
&+ \frac{20bx^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{2b^2x^{5/2}\sinh\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)} \\
&+ \frac{(120b^2)\,\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx}}{b-\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^3} \\
&+ \frac{(120b^2)\,\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx}}{b+\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^3} \\
&+ \frac{(80b)\mathrm{Subst}\left(\int x^3\,\mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&- \frac{(80b)\mathrm{Subst}\left(\int x^3\,\mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&- \frac{(10b^3)\,\mathrm{Subst}\left(\int x^4\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
&+ \frac{(10b^3)\,\mathrm{Subst}\left(\int x^4\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}
\end{aligned}
$$

= Too large to display

Mathematica [A] (verified)

Time = 9.72 (sec) , antiderivative size = 2247, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Result too large to show}$$

```
[In] Integrate[x^2/(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (x^3*(b + a*Cosh[c + d*Sqrt[x]])^2*Sech[c + d*Sqrt[x]]^2)/(3*a^2*(a + b*Sec
h[c + d*Sqrt[x]])^2) + (2*b*E^c*(b + a*Cosh[c + d*Sqrt[x]])^2*(2*b*E^c*x^(5
/2) - ((1 + E^(2*c))*(5*b*d^4*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^2*Log[1 + (a*E^(
2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] + 2*a^2*d^5*E^c*x^(
5/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]
- b^2*d^5*E^c*x^(5/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 +
 b^2)*E^(2*c)])] + 5*b*d^4*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^2*Log[1 + (a*E^(2*c
 + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] - 2*a^2*d^5*E^c*x^(5/2
)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] + b
^2*d^5*E^c*x^(5/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^
2)*E^(2*c)])] + 5*d^3*(4*b*Sqrt[(-a^2 + b^2)*E^(2*c)] + 2*a^2*d*E^c*Sqrt[x]
 - b^2*d*E^c*Sqrt[x])*x^(3/2)*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c -
 Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 5*d^3*(4*b*Sqrt[(-a^2 + b^2)*E^(2*c)] - 2*
a^2*d*E^c*Sqrt[x] + b^2*d*E^c*Sqrt[x])*x^(3/2)*PolyLog[2, -((a*E^(2*c + d*S
qrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 60*b*d^2*Sqrt[(-a^2 + b^2
)*E^(2*c)]*x*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2
)*E^(2*c)]))] - 40*a^2*d^3*E^c*x^(3/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))
/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 20*b^2*d^3*E^c*x^(3/2)*PolyLog[3,
 -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 60*b*d^
2*Sqrt[(-a^2 + b^2)*E^(2*c)]*x*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c
+ Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 40*a^2*d^3*E^c*x^(3/2)*PolyLog[3, -((a*E^
(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 20*b^2*d^3*E^c*
x^(3/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(
2*c)]))] + 120*b*d*Sqrt[(-a^2 + b^2)*E^(2*c)]*Sqrt[x]*PolyLog[4, -((a*E^(2*
c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 120*a^2*d^2*E^c*x*
PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))]
 - 60*b^2*d^2*E^c*x*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^
2 + b^2)*E^(2*c)]))] + 120*b*d*Sqrt[(-a^2 + b^2)*E^(2*c)]*Sqrt[x]*PolyLog[4
, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 120*a^
2*d^2*E^c*x*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)
*E^(2*c)]))] + 60*b^2*d^2*E^c*x*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c
 + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 120*b*Sqrt[(-a^2 + b^2)*E^(2*c)]*PolyLog
[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 240*
a^2*d*E^c*Sqrt[x]*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2
+ b^2)*E^(2*c)]))] + 120*b^2*d*E^c*Sqrt[x]*PolyLog[5, -((a*E^(2*c + d*Sqrt[
x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 120*b*Sqrt[(-a^2 + b^2)*E^(2*
```

```
c)]*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)
]))] + 240*a^2*d*E^c*Sqrt[x]*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c +
Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 120*b^2*d*E^c*Sqrt[x]*PolyLog[5, -((a*E^(2*
c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 240*a^2*E^c*PolyLo
g[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 120
*b^2*E^c*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^
(2*c)]))] - 240*a^2*E^c*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[
(-a^2 + b^2)*E^(2*c)]))] + 120*b^2*E^c*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))
/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))]))/(d^5*E^c*Sqrt[(-a^2 + b^2)*E^(2*c
)]))*Sech[c + d*Sqrt[x]]^2)/(a^2*(a^2 - b^2)*d*(1 + E^(2*c))*(a + b*Sech[c
+ d*Sqrt[x]])^2) + (2*(b + a*Cosh[c + d*Sqrt[x]])*Sech[c]*Sech[c + d*Sqrt[x
]]^2*(b^3*x^(5/2)*Sinh[c] - a*b^2*x^(5/2)*Sinh[d*Sqrt[x]]))/(a^2*(-a + b)*(
a + b)*d*(a + b*Sech[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int \frac{x^2}{\left(a+b\ \mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(x^2/(a+b*sech(c+d*x^(1/2)))^2,x)
[Out] int(x^2/(a+b*sech(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x^2}{\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^2}{\left(b\,\mathrm{sech}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^2/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(x^2/(b^2*sech(d*sqrt(x) + c)^2 + 2*a*b*sech(d*sqrt(x) + c) + a^2),
 x)
```

Sympy [F]

$$\int \frac{x^2}{\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^2}{\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(x**2/(a+b*sech(c+d*x**(1/2)))**2,x)
[Out] Integral(x**2/(a + b*sech(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{x^2}{\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^2}{\left(b\,\mathrm{sech}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^2/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*sech(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^2}{\left(a+\frac{b}{\cosh\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(x^2/(a + b/cosh(c + d*x^(1/2)))^2,x)
```

```
[Out] int(x^2/(a + b/cosh(c + d*x^(1/2)))^2, x)
```

3.49 $\int \frac{x}{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1395

$$\int \frac{x}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{2b^2x^{3/2}}{a^2\left(a^2-b^2\right)d}+\frac{x^2}{2a^2}-\frac{6b^2x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}$$

$$+\frac{2b^3x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}$$

$$-\frac{4bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}-\frac{6b^2x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}$$

$$-\frac{2b^3x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}$$

$$+\frac{4bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$-\frac{12b^2\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}$$

$$+\frac{6b^3x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}$$

$$-\frac{12bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$-\frac{12b^2\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}$$

$$-\frac{6b^3x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}$$

$$+\frac{12bx\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$+\frac{12b^2\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}$$

$$-\frac{12b^3\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}$$

$$+\frac{24b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$

$$+\frac{12b^2\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}$$

$$+\frac{12b^3\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}$$

$$24b\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{\cdots}\right)$$

```
[Out] 1/2*x^2/a^2+2*b^2*x^(3/2)*sinh(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*cosh(c+d*x^(
1/2)))-12*b^3*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a
^2/(-a^2+b^2)^(3/2)/d^3+12*b^3*polylog(3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^
(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^3+24*b*polylog(3,-a*exp(c+d*x^(1/2))
/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^3/(-a^2+b^2)^(1/2)-24*b*polylog(3,-a*e
xp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^3/(-a^2+b^2)^(1/2)-6*b^
2*x*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^2+2*b^3*x
^(3/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d
-6*b^2*x*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^2-2*
b^3*x^(3/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3
/2)/d+6*b^3*x*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2
+b^2)^(3/2)/d^2-6*b^3*x*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))
/a^2/(-a^2+b^2)^(3/2)/d^2-4*b*x^(3/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)
^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)+4*b*x^(3/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^
2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)-12*b*x*polylog(2,-a*exp(c+d*x^(1/2))/
(b-(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+12*b*x*polylog(2,-a*exp(c+d*
x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)-12*b^2*polylog(2,-a
*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^3-12*b^2*po
lylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^3
-12*b^3*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^
(3/2)/d^4-24*b*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^4/
(-a^2+b^2)^(1/2)+24*b*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a
^2/d^4/(-a^2+b^2)^(1/2)+12*b^2*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^
(1/2)))/a^2/(a^2-b^2)/d^4+12*b^2*polylog(3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2
)^(1/2)))/a^2/(a^2-b^2)/d^4+12*b^3*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b
^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^4+2*b^2*x^(3/2)/a^2/(a^2-b^2)/d
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 1395, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules

used = {5544, 4276, 3405, 3401, 2296, 2221, 2611, 6744, 2320, 6724, 5681}

$$\int \frac{x}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{2x^{3/2}\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}}+1\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d} - \frac{2x^{3/2}\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}}+1\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d}$$
$$+\frac{6x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}$$
$$-\frac{6x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}$$
$$-\frac{12\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3}$$
$$+\frac{12\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3}$$
$$+\frac{12\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4}$$
$$-\frac{12\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4}+\frac{2x^{3/2}b^2}{a^2\left(a^2-b^2\right)d}$$
$$-\frac{6x\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2}-\frac{6x\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2}$$
$$-\frac{12\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3}$$
$$-\frac{12\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3}$$
$$+\frac{12\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4}$$
$$+\frac{12\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4}$$
$$+\frac{2x^{3/2}\sinh\left(c+d\sqrt{x}\right)b^2}{a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}$$
$$-\frac{4x^{3/2}\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}}+1\right)b}{a^2\sqrt{b^2-a^2}d}+\frac{4x^{3/2}\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}}+1\right)b}{a^2\sqrt{b^2-a^2}d}$$
$$-\frac{12x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2}$$
$$+\frac{12x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2}$$
$$+\frac{24\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^3}$$

```
[In] Int[x/(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*b^2*x^(3/2))/(a^2*(a^2 - b^2)*d) + x^2/(2*a^2) - (6*b^2*x*Log[1 + (a*E^(
c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) + (2*b^3*x^(
3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^
2)^(3/2)*d) - (4*b*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b
^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - (6*b^2*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b
 + Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) - (2*b^3*x^(3/2)*Log[1 + (a*E^
(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + (4*b
*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-
a^2 + b^2]*d) - (12*b^2*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqr
t[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^3) + (6*b^3*x*PolyLog[2, -((a*E^(c + d
*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (12*b*x
*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^
2 + b^2]*d^2) - (12*b^2*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqr
t[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^3) - (6*b^3*x*PolyLog[2, -((a*E^(c + d
*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (12*b*x
*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^
2 + b^2]*d^2) + (12*b^2*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 +
 b^2]))])/(a^2*(a^2 - b^2)*d^4) - (12*b^3*Sqrt[x]*PolyLog[3, -((a*E^(c + d*
Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + (24*b*Sq
rt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqr
t[-a^2 + b^2]*d^3) + (12*b^2*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-
a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^4) + (12*b^3*Sqrt[x]*PolyLog[3, -((a*E^(c
 + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3) - (24
*b*Sqrt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^
2*Sqrt[-a^2 + b^2]*d^3) + (12*b^3*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - S
qrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^4) - (24*b*PolyLog[4, -((a*E^
(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^4) - (12
*b^3*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^
2 + b^2)^(3/2)*d^4) + (24*b*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a
^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^4) + (2*b^2*x^(3/2)*Sinh[c + d*Sqrt[x
]])/(a*(a^2 - b^2)*d*(b + a*Cosh[c + d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
```

```
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
```

```
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^3}{(a+b\operatorname{sech}(c+dx))^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^3}{a^2}+\frac{b^2x^3}{a^2(b+a\cosh(c+dx))^2}-\frac{2bx^3}{a^2(b+a\cosh(c+dx))}\right)dx, x, \sqrt{x}\right)\\
&= \frac{x^2}{2a^2}-\frac{(4b)\text{Subst}\left(\int \frac{x^3}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^3}{(b+a\cosh(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= \frac{x^2}{2a^2}+\frac{2b^2x^{3/2}\sinh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}-\frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^3}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&\quad-\frac{(2b^3)\,\text{Subst}\left(\int \frac{x^3}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}-\frac{(6b^2)\,\text{Subst}\left(\int \frac{x^2\sinh(c+dx)}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}
\end{aligned}$$

$$= \frac{2b^2x^{3/2}}{a^2(a^2-b^2)d} + \frac{x^2}{2a^2} + \frac{2b^2x^{3/2}\sinh\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}$$
$$- \frac{(4b^3)\,\text{Subst}\left(\int \frac{e^{c+dx}x^3}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)}$$
$$- \frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^3}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$+ \frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^3}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$- \frac{(6b^2)\,\text{Subst}\left(\int \frac{e^{c+dx}x^2}{b-\sqrt{-a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(a^2-b^2)d}$$
$$- \frac{(6b^2)\,\text{Subst}\left(\int \frac{e^{c+dx}x^2}{b+\sqrt{-a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(a^2-b^2)d}$$

$$= \frac{2b^2x^{3/2}}{a^2(a^2-b^2)d} + \frac{x^2}{2a^2} - \frac{6b^2x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$- \frac{4bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{6b^2x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{4bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2b^2x^{3/2}\sinh\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{(4b^3)\,\text{Subst}\left(\int \frac{e^{c+dx}x^3}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$
$$- \frac{(4b^3)\,\text{Subst}\left(\int \frac{e^{c+dx}x^3}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$
$$+ \frac{(12b^2)\,\text{Subst}\left(\int x\log\left(1+\frac{ae^{c+dx}}{b-\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{(12b^2)\,\text{Subst}\left(\int x\log\left(1+\frac{ae^{c+dx}}{b+\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{(12b)\text{Subst}\left(\int x^2\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$- \frac{(12b)\text{Subst}\left(\int x^2\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$= \frac{2b^2x^{3/2}}{a^2(a^2-b^2)d} + \frac{x^2}{2a^2} - \frac{6b^2x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2}$$

$$+ \frac{2b^3x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} - \frac{4bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$- \frac{6b^2x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2b^3x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

$$+ \frac{4bx^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{12b^2\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3}$$

$$- \frac{12bx\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{12b^2\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3}$$

$$+ \frac{12bx\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{2b^2x^{3/2}\sinh\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}$$

$$+ \frac{(12b^2)\,\mathrm{Subst}\left(\int \mathrm{PolyLog}\left(2,-\frac{ae^{c+dx}}{b-\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^3}$$

$$+ \frac{(12b^2)\,\mathrm{Subst}\left(\int \mathrm{PolyLog}\left(2,-\frac{ae^{c+dx}}{b+\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^3}$$

$$+ \frac{(24b)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$- \frac{(24b)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$- \frac{(6b^3)\,\mathrm{Subst}\left(\int x^2\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

$$+ \frac{(6b^3)\,\mathrm{Subst}\left(\int x^2\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

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Mathematica [A] (verified)

Time = 8.95 (sec) , antiderivative size = 1393, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

$$= \frac{\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)\operatorname{sech}^2\left(c+d\sqrt{x}\right)\left(x^2\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)+\frac{4be^c\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)\left(2be^cx^{3/2}+\frac{e^{-c}\left(1+e^{2c}\right)\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[x/(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((b + a*Cosh[c + d*Sqrt[x]])*Sech[c + d*Sqrt[x]]^2*(x^2*(b + a*Cosh[c + d*S
qrt[x]]) + (4*b*E^c*(b + a*Cosh[c + d*Sqrt[x]])*(2*b*E^c*x^(3/2) + ((1 + E^
(2*c))*(-3*b*d^2*Sqrt[(-a^2 + b^2)*E^(2*c)]*x*Log[1 + (a*E^(2*c + d*Sqrt[x]
))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] - 2*a^2*d^3*E^c*x^(3/2)*Log[1 + (a
*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] + b^2*d^3*E^c*x
^(3/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])
] - 3*b*d^2*Sqrt[(-a^2 + b^2)*E^(2*c)]*x*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b
*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] + 2*a^2*d^3*E^c*x^(3/2)*Log[1 + (a*E^(2
*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] - b^2*d^3*E^c*x^(3/2
)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] + 3
*d*(-2*b*Sqrt[(-a^2 + b^2)*E^(2*c)] - 2*a^2*d*E^c*Sqrt[x] + b^2*d*E^c*Sqrt[
x])*Sqrt[x]*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)
*E^(2*c)]))] - 3*d*(2*b*Sqrt[(-a^2 + b^2)*E^(2*c)] - 2*a^2*d*E^c*Sqrt[x] +
b^2*d*E^c*Sqrt[x])*Sqrt[x]*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sq
rt[(-a^2 + b^2)*E^(2*c)]))] + 6*b*Sqrt[(-a^2 + b^2)*E^(2*c)]*PolyLog[3, -((
a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 12*a^2*d*E^
c*Sqrt[x]*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E
^(2*c)]))] - 6*b^2*d*E^c*Sqrt[x]*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^
c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 6*b*Sqrt[(-a^2 + b^2)*E^(2*c)]*PolyLog[
3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 12*a^
2*d*E^c*Sqrt[x]*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 +
b^2)*E^(2*c)]))] + 6*b^2*d*E^c*Sqrt[x]*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))
/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 12*a^2*E^c*PolyLog[4, -((a*E^(2*c
 + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 6*b^2*E^c*PolyLog[4
, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 12*a^2
*E^c*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c
```

```
)]))] - 6*b^2*E^c*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2
+ b^2)*E^(2*c)]))]))/(d^3*E^c*Sqrt[(-a^2 + b^2)*E^(2*c)])))/((a^2 - b^2)*d*
(1 + E^(2*c))) + (4*b^2*x^(3/2)*Sech[c]*(-(b*Sinh[c]) + a*Sinh[d*Sqrt[x]]))
/((a - b)*(a + b)*d)))/(2*a^2*(a + b*Sech[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int \frac{x}{\left(a+b\ \mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(x/(a+b*sech(c+d*x^(1/2)))^2,x)
[Out] int(x/(a+b*sech(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x}{\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x}{\left(b\,\mathrm{sech}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(x/(b^2*sech(d*sqrt(x) + c)^2 + 2*a*b*sech(d*sqrt(x) + c) + a^2), x
)
```

Sympy [F]

$$\int \frac{x}{\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x}{\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(x/(a+b*sech(c+d*x**(1/2)))**2,x)
[Out] Integral(x/(a + b*sech(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{x}{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x}{\left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(x/(b*sech(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x}{\left(a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(x/(a + b/cosh(c + d*x^(1/2)))^2,x)
[Out] int(x/(a + b/cosh(c + d*x^(1/2)))^2, x)
```

3.50 $\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*sech(c+d*x^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Int[1/(x*(a + b*Sech[c + d*Sqrt[x]])^2),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Sech[c + d*Sqrt[x]])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 179.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x*(a + b*Sech[c + d*Sqrt[x]])^2),x]
[Out] Integrate[1/(x*(a + b*Sech[c + d*Sqrt[x]])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(1/x/(a+b*sech(c+d*x^(1/2)))^2,x)
[Out] int(1/x/(a+b*sech(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{1}{x\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x*sech(d*sqrt(x) + c)^2 + 2*a*b*x*sech(d*sqrt(x) + c) + a^2
*x), x)
```

Sympy [N/A]

Not integrable

Time = 4.71 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/x/(a+b*sech(c+d*x**(1/2)))**2,x)
[Out] Integral(1/(x*(a + b*sech(c + d*sqrt(x)))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 253, normalized size of antiderivative = 12.65

$$\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)^2 x}\,dx$$

```
[In] integrate(1/x/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] -4*(b^3*sqrt(x)*e^(d*sqrt(x) + c) + a*b^2*sqrt(x))/((a^5*d*e^(2*c) - a^3*b^
2*d*e^(2*c))*x*e^(2*d*sqrt(x)) + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x*e^(d*sqr
t(x)) + (a^5*d - a^3*b^2*d)*x) + log(x)/a^2 - integrate(2*(a*b^2*sqrt(x) +
(b^3*sqrt(x)*e^c + (2*a^2*b*d*e^c - b^3*d*e^c)*x)*e^(d*sqrt(x)))/((a^5*d*e^
(2*c) - a^3*b^2*d*e^(2*c))*x^2*e^(2*d*sqrt(x)) + 2*(a^4*b*d*e^c - a^2*b^3*d
*e^c)*x^2*e^(d*sqrt(x)) + (a^5*d - a^3*b^2*d)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)^2 x}\,dx$$

```
[In] integrate(1/x/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(1/((b*sech(d*sqrt(x) + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x\left(a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(1/(x*(a + b/cosh(c + d*x^(1/2)))^2),x)
[Out] int(1/(x*(a + b/cosh(c + d*x^(1/2)))^2), x)
```

3.51 $\int \frac{1}{x^2\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*sech(c+d*x^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Int[1/(x^2*(a + b*Sech[c + d*Sqrt[x]])^2),x]
[Out] Defer[Int][1/(x^2*(a + b*Sech[c + d*Sqrt[x]])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 91.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x^2*(a + b*Sech[c + d*Sqrt[x]])^2),x]
[Out] Integrate[1/(x^2*(a + b*Sech[c + d*Sqrt[x]])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(1/x^2/(a+b*sech(c+d*x^(1/2)))^2,x)
[Out] int(1/x^2/(a+b*sech(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^2*sech(d*sqrt(x) + c)^2 + 2*a*b*x^2*sech(d*sqrt(x) + c) +
 a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 9.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/x**2/(a+b*sech(c+d*x**(1/2)))**2,x)
[Out] Integral(1/(x**2*(a + b*sech(c + d*sqrt(x)))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 324, normalized size of antiderivative = 16.20

$$\int \frac{1}{x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] -(4*a*b^2*sqrt(x) + (a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*x*e^(2*d*sqrt(x)) + (
a^3*d - a*b^2*d)*x + 2*(2*b^3*sqrt(x)*e^c + (a^2*b*d*e^c - b^3*d*e^c)*x)*e^
(d*sqrt(x)))/((a^5*d*e^(2*c) - a^3*b^2*d*e^(2*c))*x^2*e^(2*d*sqrt(x)) + 2*(
a^4*b*d*e^c - a^2*b^3*d*e^c)*x^2*e^(d*sqrt(x)) + (a^5*d - a^3*b^2*d)*x^2) -
 integrate(2*(3*a*b^2*sqrt(x) + (3*b^3*sqrt(x)*e^c + (2*a^2*b*d*e^c - b^3*d
*e^c)*x)*e^(d*sqrt(x)))/((a^5*d*e^(2*c) - a^3*b^2*d*e^(2*c))*x^3*e^(2*d*sqr
t(x)) + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^3*e^(d*sqrt(x)) + (a^5*d - a^3*b^
2*d)*x^3), x)
```

Giac [N/A]

Not integrable

Time = 3.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^2\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2 \left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^2\,{\left(a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}\right)}^2} \,d x$$

```
[In] int(1/(x^2*(a + b/cosh(c + d*x^(1/2)))^2),x)
[Out] int(1/(x^2*(a + b/cosh(c + d*x^(1/2)))^2), x)
```

3.52 $\int x^{3/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 254

$$\begin{aligned}\int x^{3/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=&\ \frac{2}{5}ax^{5/2}+\frac{4bx^2\arctan\left(e^{c+d\sqrt{x}}\right)}{d}\\&-\frac{8ibx^{3/2}\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{8ibx^{3/2}\operatorname{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}\\&+\frac{24ibx\operatorname{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{24ibx\operatorname{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}\\&-\frac{48ib\sqrt{x}\operatorname{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{48ib\sqrt{x}\operatorname{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4}\\&+\frac{48ib\operatorname{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5}-\frac{48ib\operatorname{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5}\end{aligned}$$

```
[Out] 2/5*a*x^(5/2)+4*b*x^2*arctan(exp(c+d*x^(1/2)))/d-8*I*b*x^(3/2)*polylog(2,-I
*exp(c+d*x^(1/2)))/d^2+8*I*b*x^(3/2)*polylog(2,I*exp(c+d*x^(1/2)))/d^2+24*I
*b*x*polylog(3,-I*exp(c+d*x^(1/2)))/d^3-24*I*b*x*polylog(3,I*exp(c+d*x^(1/2
)))/d^3+48*I*b*polylog(5,-I*exp(c+d*x^(1/2)))/d^5-48*I*b*polylog(5,I*exp(c+
d*x^(1/2)))/d^5-48*I*b*polylog(4,-I*exp(c+d*x^(1/2)))*x^(1/2)/d^4+48*I*b*po
lylog(4,I*exp(c+d*x^(1/2)))*x^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {14, 5544, 4265, 2611, 6744, 2320, 6724}

$$\int x^{3/2}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{2}{5}ax^{5/2}+\frac{4bx^2\arctan\left(e^{c+d\sqrt{x}}\right)}{d}+\frac{48ib\operatorname{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5}-\frac{48ib\operatorname{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5}-\frac{48ib\sqrt{x}\operatorname{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{48ib\sqrt{x}\operatorname{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{24ibx\operatorname{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{24ibx\operatorname{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{8ibx^{3/2}\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{8ibx^{3/2}\operatorname{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}$$

```
[In] Int[x^(3/2)*(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (2*a*x^(5/2))/5 + (4*b*x^2*ArcTan[E^(c + d*Sqrt[x])])/d - ((8*I)*b*x^(3/2)*
PolyLog[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((8*I)*b*x^(3/2)*PolyLog[2, I*E^(
c + d*Sqrt[x])])/d^2 + ((24*I)*b*x*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3
- ((24*I)*b*x*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 - ((48*I)*b*Sqrt[x]*Poly
Log[4, (-I)*E^(c + d*Sqrt[x])])/d^4 + ((48*I)*b*Sqrt[x]*PolyLog[4, I*E^(c +
 d*Sqrt[x])])/d^4 + ((48*I)*b*PolyLog[5, (-I)*E^(c + d*Sqrt[x])])/d^5 - ((4
8*I)*b*PolyLog[5, I*E^(c + d*Sqrt[x])])/d^5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(ax^{3/2}+bx^{3/2}\operatorname{sech}\left(c+d\sqrt{x}\right)\right)\,dx\\
&= \frac{2}{5}ax^{5/2}+b\int x^{3/2}\operatorname{sech}\left(c+d\sqrt{x}\right)\,dx\\
&= \frac{2}{5}ax^{5/2}+(2b)\text{Subst}\left(\int x^4\operatorname{sech}(c+dx)\,dx,x,\sqrt{x}\right)\\
&= \frac{2}{5}ax^{5/2}+\frac{4bx^2\arctan\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{(8ib)\text{Subst}\left(\int x^3\log\left(1-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d}\\
&\quad+\frac{(8ib)\text{Subst}\left(\int x^3\log\left(1+ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d}
\end{aligned}
$$

$$= \frac{2}{5}ax^{5/2} + \frac{4bx^2 \arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{8ibx^{3/2}\,\text{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{8ibx^{3/2}\,\text{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{(24ib)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(2, -ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2} - \frac{(24ib)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(2, ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$

$$= \frac{2}{5}ax^{5/2} + \frac{4bx^2 \arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{8ibx^{3/2}\,\text{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{8ibx^{3/2}\,\text{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{24ibx\,\text{PolyLog}\left(3, -ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{24ibx\,\text{PolyLog}\left(3, ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{(48ib)\text{Subst}\left(\int x\,\text{PolyLog}\left(3, -ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^3} + \frac{(48ib)\text{Subst}\left(\int x\,\text{PolyLog}\left(3, ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^3}$$

$$= \frac{2}{5}ax^{5/2} + \frac{4bx^2 \arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{8ibx^{3/2}\,\text{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{8ibx^{3/2}\,\text{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{24ibx\,\text{PolyLog}\left(3, -ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{24ibx\,\text{PolyLog}\left(3, ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{48ib\sqrt{x}\,\text{PolyLog}\left(4, -ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{48ib\sqrt{x}\,\text{PolyLog}\left(4, ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{(48ib)\text{Subst}\left(\int \text{PolyLog}\left(4, -ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^4} - \frac{(48ib)\text{Subst}\left(\int \text{PolyLog}\left(4, ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^4}$$

$$= \frac{2}{5}ax^{5/2} + \frac{4bx^2 \arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{8ibx^{3/2}\,\text{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{8ibx^{3/2}\,\text{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{24ibx\,\text{PolyLog}\left(3, -ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{24ibx\,\text{PolyLog}\left(3, ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{48ib\sqrt{x}\,\text{PolyLog}\left(4, -ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{48ib\sqrt{x}\,\text{PolyLog}\left(4, ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{(48ib)\text{Subst}\left(\int \frac{\text{PolyLog}(4,-ix)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{d^5} - \frac{(48ib)\text{Subst}\left(\int \frac{\text{PolyLog}(4,ix)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{d^5}$$

$$= \frac{2}{5}ax^{5/2} + \frac{4bx^2\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{8ibx^{3/2}\operatorname{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{8ibx^{3/2}\operatorname{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{24ibx\operatorname{PolyLog}\left(3, -ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{24ibx\operatorname{PolyLog}\left(3, ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{48ib\sqrt{x}\operatorname{PolyLog}\left(4, -ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{48ib\sqrt{x}\operatorname{PolyLog}\left(4, ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{48ib\operatorname{PolyLog}\left(5, -ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{48ib\operatorname{PolyLog}\left(5, ie^{c+d\sqrt{x}}\right)}{d^5}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.13

$$\int x^{3/2}\left(a + b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)\,dx = \frac{2\left(ad^5x^{5/2} + 5ibd^4x^2\log\left(1 - ie^{c+d\sqrt{x}}\right) - 5ibd^4x^2\log\left(1 + ie^{c+d\sqrt{x}}\right) - 20ibd^3x^{3/2}\operatorname{Poly}\right.}{}$$

```
[In] Integrate[x^(3/2)*(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (2*(a*d^5*x^(5/2) + (5*I)*b*d^4*x^2*Log[1 - I*E^(c + d*Sqrt[x])] - (5*I)*b*
d^4*x^2*Log[1 + I*E^(c + d*Sqrt[x])] - (20*I)*b*d^3*x^(3/2)*PolyLog[2, (-I)
*E^(c + d*Sqrt[x])] + (20*I)*b*d^3*x^(3/2)*PolyLog[2, I*E^(c + d*Sqrt[x])]
+ (60*I)*b*d^2*x*PolyLog[3, (-I)*E^(c + d*Sqrt[x])] - (60*I)*b*d^2*x*PolyLo
g[3, I*E^(c + d*Sqrt[x])] - (120*I)*b*d*Sqrt[x]*PolyLog[4, (-I)*E^(c + d*Sq
rt[x])] + (120*I)*b*d*Sqrt[x]*PolyLog[4, I*E^(c + d*Sqrt[x])] + (120*I)*b*P
olyLog[5, (-I)*E^(c + d*Sqrt[x])] - (120*I)*b*PolyLog[5, I*E^(c + d*Sqrt[x]
)]))/(5*d^5)
```

Maple [F]

$$\int x^{\frac{3}{2}}\left(a + b\operatorname{sech}\left(c + d\sqrt{x}\right)\right)dx$$

```
[In] int(x^(3/2)*(a+b*sech(c+d*x^(1/2))),x)
```

```
[Out] int(x^(3/2)*(a+b*sech(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x^{3/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int \left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(b*x^(3/2)*sech(d*sqrt(x) + c) + a*x^(3/2), x)
```

Sympy [F]

$$\int x^{3/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int x^{\frac{3}{2}}\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx$$

```
[In] integrate(x**(3/2)*(a+b*sech(c+d*x**(1/2))),x)
[Out] Integral(x**(3/2)*(a + b*sech(c + d*sqrt(x))), x)
```

Maxima [F]

$$\int x^{3/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int \left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")
[Out] 2/5*a*x^(5/2) + 2*b*integrate(x^(3/2)*e^(d*sqrt(x) + c)/(e^(2*d*sqrt(x) + 2
*c) + 1), x)
```

Giac [F]

$$\int x^{3/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int \left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int x^{3/2}\left(a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x^(3/2)*(a + b/cosh(c + d*x^(1/2))),x)
[Out] int(x^(3/2)*(a + b/cosh(c + d*x^(1/2))), x)
```

3.53 $\int \sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{2}{3}ax^{3/2}+\frac{4bx\arctan\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{4ib\sqrt{x}\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{4ib\sqrt{x}\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{4ib\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{4ib\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}$$

```
[Out] 2/3*a*x^(3/2)+4*b*x*arctan(exp(c+d*x^(1/2)))/d+4*I*b*polylog(3,-I*exp(c+d*x
^(1/2)))/d^3-4*I*b*polylog(3,I*exp(c+d*x^(1/2)))/d^3-4*I*b*polylog(2,-I*exp
(c+d*x^(1/2)))*x^(1/2)/d^2+4*I*b*polylog(2,I*exp(c+d*x^(1/2)))*x^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {14, 5544, 4265, 2611, 2320, 6724}

$$\int \sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\frac{2}{3}ax^{3/2}+\frac{4bx\arctan\left(e^{c+d\sqrt{x}}\right)}{d}+\frac{4ib\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{4ib\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{4ib\sqrt{x}\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{4ib\sqrt{x}\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}$$

```
[In] Int[Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (2*a*x^(3/2))/3 + (4*b*x*ArcTan[E^(c + d*Sqrt[x])])/d - ((4*I)*b*Sqrt[x]*Po
lyLog[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((4*I)*b*Sqrt[x]*PolyLog[2, I*E^(c
+ d*Sqrt[x])])/d^2 + ((4*I)*b*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((4
*I)*b*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \int \left(a\sqrt{x} + b\sqrt{x}\text{sech}\left(c + d\sqrt{x}\right)\right) dx$$

$$= \frac{2}{3}ax^{3/2} + b\int \sqrt{x}\text{sech}\left(c + d\sqrt{x}\right) dx$$

$$= \frac{2}{3}ax^{3/2} + (2b)\text{Subst}\left(\int x^2\text{sech}(c + dx)\,dx, x, \sqrt{x}\right)$$

$$= \frac{2}{3}ax^{3/2} + \frac{4bx\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{(4ib)\text{Subst}\left(\int x\log\left(1 - ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d} + \frac{(4ib)\text{Subst}\left(\int x\log\left(1 + ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d}$$

$$= \frac{2}{3}ax^{3/2} + \frac{4bx\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{4ib\sqrt{x}\,\text{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{4ib\sqrt{x}\,\text{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{(4ib)\text{Subst}\left(\int \text{PolyLog}\left(2, -ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2} - \frac{(4ib)\text{Subst}\left(\int \text{PolyLog}\left(2, ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$

$$= \frac{2}{3}ax^{3/2} + \frac{4bx\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{4ib\sqrt{x}\,\text{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{4ib\sqrt{x}\,\text{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{(4ib)\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{d^3} - \frac{(4ib)\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{d^3}$$

$$= \frac{2}{3}ax^{3/2} + \frac{4bx\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{4ib\sqrt{x}\,\text{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{4ib\sqrt{x}\,\text{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{4ib\,\text{PolyLog}\left(3, -ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{4ib\,\text{PolyLog}\left(3, ie^{c+d\sqrt{x}}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.23

$$\int \sqrt{x}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)\,dx = \frac{2\left(ad^3x^{3/2}+3ibd^2x\log\left(1-ie^{c+d\sqrt{x}}\right)-3ibd^2x\log\left(1+ie^{c+d\sqrt{x}}\right)-6ibd\sqrt{x}\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)+6ibd\sqrt{x}\right.}{3d^3}$$

```
[In] Integrate[Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (2*(a*d^3*x^(3/2) + (3*I)*b*d^2*x*Log[1 - I*E^(c + d*Sqrt[x])] - (3*I)*b*d^
2*x*Log[1 + I*E^(c + d*Sqrt[x])] - (6*I)*b*d*Sqrt[x]*PolyLog[2, (-I)*E^(c +
 d*Sqrt[x])] + (6*I)*b*d*Sqrt[x]*PolyLog[2, I*E^(c + d*Sqrt[x])] + (6*I)*b*
PolyLog[3, (-I)*E^(c + d*Sqrt[x])] - (6*I)*b*PolyLog[3, I*E^(c + d*Sqrt[x])
]))/(3*d^3)
```

Maple [F]

$$\int \left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)\sqrt{x}dx$$

```
[In] int((a+b*sech(c+d*x^(1/2)))*x^(1/2),x)
```

```
[Out] int((a+b*sech(c+d*x^(1/2)))*x^(1/2),x)
```

Fricas [F]

$$\int \sqrt{x}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int \left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)\sqrt{x}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))*x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(b*sqrt(x)*sech(d*sqrt(x) + c) + a*sqrt(x), x)
```

Sympy [F]

$$\int \sqrt{x}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int \sqrt{x}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)\,dx$$

```
[In] integrate((a+b*sech(c+d*x**(1/2)))*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*(a + b*sech(c + d*sqrt(x))), x)
```

Maxima [F]

$$\int \sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int \left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)\sqrt{x}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))*x^(1/2),x, algorithm="maxima")
[Out] 2/3*a*x^(3/2) + 2*b*integrate(sqrt(x)*e^(d*sqrt(x) + c)/(e^(2*d*sqrt(x) + 2
*c) + 1), x)
```

Giac [F]

$$\int \sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int \left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)\sqrt{x}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))*x^(1/2),x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)\,dx=\int \sqrt{x}\left(a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x^(1/2)*(a + b/cosh(c + d*x^(1/2))),x)
[Out] int(x^(1/2)*(a + b/cosh(c + d*x^(1/2))), x)
```

3.54 $\int \frac{a+b\,\mathrm{sech}(c+d\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 26

$$\int \frac{a+b\,\mathrm{sech}(c+d\sqrt{x})}{\sqrt{x}}\,dx = 2a\sqrt{x}+\frac{2b\arctan\left(\sinh\left(c+d\sqrt{x}\right)\right)}{d}$$

```
[Out] 2*b*arctan(sinh(c+d*x^(1/2)))/d+2*a*x^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {14, 5544, 3855}

$$\int \frac{a+b\,\mathrm{sech}(c+d\sqrt{x})}{\sqrt{x}}\,dx = 2a\sqrt{x}+\frac{2b\arctan\left(\sinh\left(c+d\sqrt{x}\right)\right)}{d}$$

```
[In] Int[(a + b*Sech[c + d*Sqrt[x]])/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*b*ArcTan[Sinh[c + d*Sqrt[x]]])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a}{\sqrt{x}} + \frac{b\text{sech}\left(c + d\sqrt{x}\right)}{\sqrt{x}}\right) dx \\ &= 2a\sqrt{x} + b\int \frac{\text{sech}\left(c + d\sqrt{x}\right)}{\sqrt{x}}\, dx \\ &= 2a\sqrt{x} + (2b)\text{Subst}\left(\int \text{sech}(c + dx)\, dx, x, \sqrt{x}\right) \\ &= 2a\sqrt{x} + \frac{2b\arctan\left(\sinh\left(c + d\sqrt{x}\right)\right)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{sech}\left(c + d\sqrt{x}\right)}{\sqrt{x}}\, dx = 2a\sqrt{x} + \frac{2b\arctan\left(\sinh\left(c + d\sqrt{x}\right)\right)}{d}$$

```
[In] Integrate[(a + b*Sech[c + d*Sqrt[x]])/Sqrt[x],x]
[Out] 2*a*Sqrt[x] + (2*b*ArcTan[Sinh[c + d*Sqrt[x]]])/d
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{2b\arctan\left(\sinh\left(c+d\sqrt{x}\right)\right)}{d}+2a\sqrt{x}$	23
default	$\frac{2b\arctan\left(\sinh\left(c+d\sqrt{x}\right)\right)}{d}+2a\sqrt{x}$	23
parts	$\frac{2b\arctan\left(\sinh\left(c+d\sqrt{x}\right)\right)}{d}+2a\sqrt{x}$	23

```
[In] int((a+b*sech(c+d*x^(1/2)))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*b*arctan(sinh(c+d*x^(1/2)))/d+2*a*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{a+b\operatorname{sech}\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\left(ad\sqrt{x}+2\,b\arctan\left(\cosh\left(d\sqrt{x}+c\right)+\sinh\left(d\sqrt{x}+c\right)\right)\right)}{d}$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x^(1/2),x, algorithm="fricas")
[Out] 2*(a*d*sqrt(x) + 2*b*arctan(cosh(d*sqrt(x) + c) + sinh(d*sqrt(x) + c)))/d
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{a+b\operatorname{sech}\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = 2a\sqrt{x}+2b\left(\begin{cases}\sqrt{x}\operatorname{sech}\left(c\right) & \text{for } d=0\\ \frac{2\operatorname{atan}\left(\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\right)}{d} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((a+b*sech(c+d*x**(1/2)))/x**(1/2),x)
[Out] 2*a*sqrt(x) + 2*b*Piecewise((sqrt(x)*sech(c), Eq(d, 0)), (2*atan(tanh(c/2 +
 d*sqrt(x)/2))/d, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{a+b\operatorname{sech}\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,a\sqrt{x}+\frac{2\,b\arctan\left(\sinh\left(d\sqrt{x}+c\right)\right)}{d}$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x^(1/2),x, algorithm="maxima")
[Out] 2*a*sqrt(x) + 2*b*arctan(sinh(d*sqrt(x) + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{a+b\operatorname{sech}\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx=\frac{2\left(d\sqrt{x}+c\right)a}{d}+\frac{4\,b\arctan\left(e^{\left(d\sqrt{x}+c\right)}\right)}{d}$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*(d*sqrt(x) + c)*a/d + 4*b*arctan(e^(d*sqrt(x) + c))/d
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \frac{a+b\operatorname{sech}\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx=2\,a\,\sqrt{x}+\frac{4\,\mathrm{atan}\left(\frac{b\,\mathrm{e}^{d\,\sqrt{x}}\,\mathrm{e}^{c}\,\sqrt{d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{d^2}}$$

```
[In] int((a + b/cosh(c + d*x^(1/2)))/x^(1/2),x)
```

```
[Out] 2*a*x^(1/2) + (4*atan((b*exp(d*x^(1/2))*exp(c)*(d^2)^(1/2))/(d*(b^2)^(1/2))
)*(b^2)^(1/2))/(d^2)^(1/2)
```

3.55 $\int \frac{a+b\text{sech}(c+d\sqrt{x})}{x^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=-\frac{2a}{\sqrt{x}}+b\text{Int}\left(\frac{\text{sech}\left(c+d\sqrt{x}\right)}{x^{3/2}},x\right)$$

```
[Out] -2*a/x^(1/2)+b*Unintegrable(sech(c+d*x^(1/2))/x^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx$$

```
[In] Int[(a + b*Sech[c + d*Sqrt[x]])/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + b*Defer[Int][Sech[c + d*Sqrt[x]]/x^(3/2), x]
```

Rubi steps

$$\text{integral}=\int \left(\frac{a}{x^{3/2}}+\frac{b\text{sech}\left(c+d\sqrt{x}\right)}{x^{3/2}}\right)\,dx$$

$$=-\frac{2a}{\sqrt{x}}+b\int \frac{\text{sech}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 13.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx$$

```
[In] Integrate[(a + b*Sech[c + d*Sqrt[x]])/x^(3/2),x]
[Out] Integrate[(a + b*Sech[c + d*Sqrt[x]])/x^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{a+b\,\operatorname{sech}\left(c+d\sqrt{x}\right)}{x^{\frac{3}{2}}}dx$$

```
[In] int((a+b*sech(c+d*x^(1/2)))/x^(3/2),x)
[Out] int((a+b*sech(c+d*x^(1/2)))/x^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{b\,\operatorname{sech}\left(d\sqrt{x}+c\right)+a}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x^(3/2),x, algorithm="fricas")
[Out] integral((b*sqrt(x)*sech(d*sqrt(x) + c) + a*sqrt(x))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{a+b\,\operatorname{sech}\left(c+d\sqrt{x}\right)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sech(c+d*x**(1/2)))/x**(3/2),x)
[Out] Integral((a + b*sech(c + d*sqrt(x)))/x**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{a+b\operatorname{sech}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{b\operatorname{sech}\left(d\sqrt{x}+c\right)+a}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(e^(d*sqrt(x) + c)/(x^(3/2)*e^(2*d*sqrt(x) + 2*c) + x^(3/2)),
x) - 2*a/sqrt(x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\operatorname{sech}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{b\operatorname{sech}\left(d\sqrt{x}+c\right)+a}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*sqrt(x) + c) + a)/x^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\operatorname{sech}\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}}{x^{3/2}}\,dx$$

```
[In] int((a + b/cosh(c + d*x^(1/2)))/x^(3/2),x)
```

```
[Out] int((a + b/cosh(c + d*x^(1/2)))/x^(3/2), x)
```

3.56 $\int \frac{a+b\text{sech}(c+d\sqrt{x})}{x^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\text{sech}(c+d\sqrt{x})}{x^{5/2}}\,dx = -\frac{2a}{3x^{3/2}} + b\text{Int}\left(\frac{\text{sech}(c+d\sqrt{x})}{x^{5/2}},x\right)$$

```
[Out] -2/3*a/x^(3/2)+b*Unintegrable(sech(c+d*x^(1/2))/x^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{sech}(c+d\sqrt{x})}{x^{5/2}}\,dx = \int \frac{a+b\text{sech}(c+d\sqrt{x})}{x^{5/2}}\,dx$$

```
[In] Int[(a + b*Sech[c + d*Sqrt[x]])/x^(5/2),x]
[Out] (-2*a)/(3*x^(3/2)) + b*Defer[Int][Sech[c + d*Sqrt[x]]/x^(5/2), x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{a}{x^{5/2}} + \frac{b\text{sech}(c+d\sqrt{x})}{x^{5/2}}\right)dx \\ &= -\frac{2a}{3x^{3/2}} + b\int \frac{\text{sech}(c+d\sqrt{x})}{x^{5/2}}\,dx \end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 15.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx$$

```
[In] Integrate[(a + b*Sech[c + d*Sqrt[x]])/x^(5/2),x]
[Out] Integrate[(a + b*Sech[c + d*Sqrt[x]])/x^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{a+b\,\operatorname{sech}\left(c+d\sqrt{x}\right)}{x^{\frac{5}{2}}}dx$$

```
[In] int((a+b*sech(c+d*x^(1/2)))/x^(5/2),x)
[Out] int((a+b*sech(c+d*x^(1/2)))/x^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{b\,\operatorname{sech}\left(d\sqrt{x}+c\right)+a}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x^(5/2),x, algorithm="fricas")
[Out] integral((b*sqrt(x)*sech(d*sqrt(x) + c) + a*sqrt(x))/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 4.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{a+b\,\operatorname{sech}\left(c+d\sqrt{x}\right)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sech(c+d*x**(1/2)))/x**(5/2),x)
[Out] Integral((a + b*sech(c + d*sqrt(x)))/x**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx = \int \frac{b\,\text{sech}\left(d\sqrt{x}+c\right)+a}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(e^(d*sqrt(x) + c)/(x^(5/2)*e^(2*d*sqrt(x) + 2*c) + x^(5/2)),
x) - 2/3*a/x^(3/2)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx = \int \frac{b\,\text{sech}\left(d\sqrt{x}+c\right)+a}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*sqrt(x) + c) + a)/x^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{sech}\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx = \int \frac{a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}}{x^{5/2}}\,dx$$

```
[In] int((a + b/cosh(c + d*x^(1/2)))/x^(5/2),x)
```

```
[Out] int((a + b/cosh(c + d*x^(1/2)))/x^(5/2), x)
```

3.57 $\int x^{3/2}\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 407

$$\int x^{3/2}\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{2b^2x^2}{d}+\frac{2}{5}a^2x^{5/2}+\frac{8abx^2\arctan\left(e^{c+d\sqrt{x}}\right)}{d}-\frac{8b^2x^{3/2}\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{16iabx^{3/2}\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{16iabx^{3/2}\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}-\frac{12b^2x\,\text{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{48iabx\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{48iabx\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}+\frac{12b^2\sqrt{x}\,\text{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{96iab\sqrt{x}\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{96iab\sqrt{x}\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4}-\frac{6b^2\,\text{PolyLog}\left(4,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5}+\frac{96iab\,\text{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5}-\frac{96iab\,\text{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5}+\frac{2b^2x^2\tanh\left(c+d\sqrt{x}\right)}{d}$$

```
[Out] 2*b^2*x^2/d+2/5*a^2*x^(5/2)+8*a*b*x^2*arctan(exp(c+d*x^(1/2)))/d-8*b^2*x^(3
/2)*ln(1+exp(2*c+2*d*x^(1/2)))/d^2+16*I*a*b*x^(3/2)*polylog(2,I*exp(c+d*x^(
1/2)))/d^2-16*I*a*b*x^(3/2)*polylog(2,-I*exp(c+d*x^(1/2)))/d^2-12*b^2*x*pol
ylog(2,-exp(2*c+2*d*x^(1/2)))/d^3-48*I*a*b*x*polylog(3,I*exp(c+d*x^(1/2)))/
d^3+96*I*a*b*polylog(4,I*exp(c+d*x^(1/2)))*x^(1/2)/d^4-6*b^2*polylog(4,-exp
(2*c+2*d*x^(1/2)))/d^5+48*I*a*b*x*polylog(3,-I*exp(c+d*x^(1/2)))/d^3-96*I*a
*b*polylog(4,-I*exp(c+d*x^(1/2)))*x^(1/2)/d^4+12*b^2*polylog(3,-exp(2*c+2*d
```

```
*x^(1/2)))*x^(1/2)/d^4+96*I*a*b*polylog(5,-I*exp(c+d*x^(1/2)))/d^5-96*I*a*b
*polylog(5,I*exp(c+d*x^(1/2)))/d^5+2*b^2*x^2*tanh(c+d*x^(1/2))/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {5544, 4275, 4265, 2611, 6744, 2320, 6724, 4269, 3799, 2221}

$$\int x^{3/2}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{2}{5}a^2x^{5/2}+\frac{8abx^2\arctan\left(e^{c+d\sqrt{x}}\right)}{d}+\frac{96iab\operatorname{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5}-\frac{96iab\operatorname{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5}-\frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4}+\frac{48iabx\operatorname{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{48iabx\operatorname{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}-\frac{6b^2\operatorname{PolyLog}\left(4,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5}+\frac{12b^2\sqrt{x}\operatorname{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{12b^2x\operatorname{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}-\frac{8b^2x^{3/2}\log\left(e^{2\left(c+d\sqrt{x}\right)}+1\right)}{d^2}+\frac{2b^2x^2\tanh\left(c+d\sqrt{x}\right)}{d}+\frac{2b^2x^2}{d}$$

```
[In] Int[x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*b^2*x^2)/d + (2*a^2*x^(5/2))/5 + (8*a*b*x^2*ArcTan[E^(c + d*Sqrt[x])])/d
 - (8*b^2*x^(3/2)*Log[1 + E^(2*(c + d*Sqrt[x]))])/d^2 - ((16*I)*a*b*x^(3/2)
*PolyLog[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((16*I)*a*b*x^(3/2)*PolyLog[2, I
*E^(c + d*Sqrt[x])])/d^2 - (12*b^2*x*PolyLog[2, -E^(2*(c + d*Sqrt[x]))])/d^
3 + ((48*I)*a*b*x*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((48*I)*a*b*x*P
olyLog[3, I*E^(c + d*Sqrt[x])])/d^3 + (12*b^2*Sqrt[x]*PolyLog[3, -E^(2*(c +
 d*Sqrt[x]))])/d^4 - ((96*I)*a*b*Sqrt[x]*PolyLog[4, (-I)*E^(c + d*Sqrt[x])]
)/d^4 + ((96*I)*a*b*Sqrt[x]*PolyLog[4, I*E^(c + d*Sqrt[x])])/d^4 - (6*b^2*P
olyLog[4, -E^(2*(c + d*Sqrt[x]))])/d^5 + ((96*I)*a*b*PolyLog[5, (-I)*E^(c +
 d*Sqrt[x])])/d^5 - ((96*I)*a*b*PolyLog[5, I*E^(c + d*Sqrt[x])])/d^5 + (2*b
^2*x^2*Tanh[c + d*Sqrt[x]])/d
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^4(a+b\,\text{sech}(c+dx))^2\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(a^2x^4+2abx^4\text{sech}(c+dx)+b^2x^4\text{sech}^2(c+dx)\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{5}a^2x^{5/2}+(4ab)\text{Subst}\left(\int x^4\text{sech}(c+dx)\,dx, x, \sqrt{x}\right)\\
&\quad + \left(2b^2\right)\text{Subst}\left(\int x^4\text{sech}^2(c+dx)\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{5}a^2x^{5/2}+\frac{8abx^2\arctan\left(e^{c+d\sqrt{x}}\right)}{d}+\frac{2b^2x^2\tanh\left(c+d\sqrt{x}\right)}{d}\\
&\quad -\frac{(16iab)\text{Subst}\left(\int x^3\log\left(1-ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d}\\
&\quad +\frac{(16iab)\text{Subst}\left(\int x^3\log\left(1+ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d}\\
&\quad -\frac{\left(8b^2\right)\text{Subst}\left(\int x^3\tanh(c+dx)\,dx, x, \sqrt{x}\right)}{d}
\end{aligned}$$

$$= \frac{2b^2x^2}{d} + \frac{2}{5}a^2x^{5/2} + \frac{8abx^2\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{2b^2x^2\tanh\left(c+d\sqrt{x}\right)}{d} + \frac{(48iab)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(2, -ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2} - \frac{(48iab)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(2, ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^2} - \frac{(16b^2)\operatorname{Subst}\left(\int \frac{e^{2(c+dx)}x^3}{1+e^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{d}$$

$$= \frac{2b^2x^2}{d} + \frac{2}{5}a^2x^{5/2} + \frac{8abx^2\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{8b^2x^{3/2}\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{48iabx\operatorname{PolyLog}\left(3, -ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{48iabx\operatorname{PolyLog}\left(3, ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{2b^2x^2\tanh\left(c+d\sqrt{x}\right)}{d} - \frac{(96iab)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(3, -ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^3} + \frac{(96iab)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(3, ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^3} + \frac{(24b^2)\operatorname{Subst}\left(\int x^2\log\left(1+e^{2(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$

$$= \frac{2b^2x^2}{d} + \frac{2}{5}a^2x^{5/2} + \frac{8abx^2\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{8b^2x^{3/2}\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2, -ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2, ie^{c+d\sqrt{x}}\right)}{d^2} - \frac{12b^2x\operatorname{PolyLog}\left(2, -e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{48iabx\operatorname{PolyLog}\left(3, -ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{48iabx\operatorname{PolyLog}\left(3, ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4, -ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4, ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{2b^2x^2\tanh\left(c+d\sqrt{x}\right)}{d} + \frac{(96iab)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(4, -ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^4} - \frac{(96iab)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(4, ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d^4} + \frac{(24b^2)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^3}$$

$$= \frac{2b^2x^2}{d} + \frac{2}{5}a^2x^{5/2} + \frac{8abx^2\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{8b^2x^{3/2}\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{16iabx^{3/2}\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{16iabx^{3/2}\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} - \frac{12b^2x\,\text{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{48iabx\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{48iabx\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{12b^2\sqrt{x}\,\text{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} - \frac{96iab\sqrt{x}\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{96iab\sqrt{x}\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{2b^2x^2\tanh\left(c+d\sqrt{x}\right)}{d} + \frac{(96iab)\text{Subst}\left(\int\frac{\text{PolyLog}(4,-ix)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^5} - \frac{(96iab)\text{Subst}\left(\int\frac{\text{PolyLog}(4,ix)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^5} - \frac{(12b^2)\,\text{Subst}\left(\int\text{PolyLog}\left(3,-e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$

$$= \frac{2b^2x^2}{d} + \frac{2}{5}a^2x^{5/2} + \frac{8abx^2\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{8b^2x^{3/2}\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{16iabx^{3/2}\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{16iabx^{3/2}\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} - \frac{12b^2x\,\text{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{48iabx\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{48iabx\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{12b^2\sqrt{x}\,\text{PolyLog}\left(3,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^4} - \frac{96iab\sqrt{x}\,\text{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{96iab\sqrt{x}\,\text{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{96iab\,\text{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{96iab\,\text{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} + \frac{2b^2x^2\tanh\left(c+d\sqrt{x}\right)}{d} - \frac{(6b^2)\,\text{Subst}\left(\int\frac{\text{PolyLog}(3,-x)}{x}\,dx,x,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^5}$$

$$= \frac{2b^2x^2}{d} + \frac{2}{5}a^2x^{5/2} + \frac{8abx^2\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{8b^2x^{3/2}\log\left(1+e^{2(c+d\sqrt{x})}\right)}{d^2} - \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} - \frac{12b^2x\operatorname{PolyLog}\left(2,-e^{2(c+d\sqrt{x})}\right)}{d^3} + \frac{48iabx\operatorname{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{48iabx\operatorname{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{12b^2\sqrt{x}\operatorname{PolyLog}\left(3,-e^{2(c+d\sqrt{x})}\right)}{d^4} - \frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,-ie^{c+d\sqrt{x}}\right)}{d^4} + \frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,ie^{c+d\sqrt{x}}\right)}{d^4} - \frac{6b^2\operatorname{PolyLog}\left(4,-e^{2(c+d\sqrt{x})}\right)}{d^5} + \frac{96iab\operatorname{PolyLog}\left(5,-ie^{c+d\sqrt{x}}\right)}{d^5} - \frac{96iab\operatorname{PolyLog}\left(5,ie^{c+d\sqrt{x}}\right)}{d^5} + \frac{2b^2x^2\tanh\left(c+d\sqrt{x}\right)}{d}$$

Mathematica [A] (verified)

Time = 5.89 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.22

$$\int x^{3/2}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx = \frac{2\cosh\left(c+d\sqrt{x}\right)\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2\left(\frac{10b^2e^{2c}x^2\cosh\left(c+d\sqrt{x}\right)}{d(1+e^{2c})}+a^2x^{5/2}\cosh\left(c+d\right.\right.}{}$$

```
[In] Integrate[x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*Cosh[c + d*Sqrt[x]]*(a + b*Sech[c + d*Sqrt[x]])^2*((10*b^2*E^(2*c)*x^2*C
osh[c + d*Sqrt[x]])/(d*(1 + E^(2*c))) + a^2*x^(5/2)*Cosh[c + d*Sqrt[x]] + (
(5*I)*b*Cosh[c + d*Sqrt[x]]*(2*a*d^4*x^2*Log[1 - I*E^(c + d*Sqrt[x])] - 2*a
*d^4*x^2*Log[1 + I*E^(c + d*Sqrt[x])] + (4*I)*b*d^3*x^(3/2)*Log[1 + E^(2*(c
 + d*Sqrt[x]))] - 8*a*d^3*x^(3/2)*PolyLog[2, (-I)*E^(c + d*Sqrt[x])] + 8*a*
d^3*x^(3/2)*PolyLog[2, I*E^(c + d*Sqrt[x])] + (6*I)*b*d^2*x*PolyLog[2, -E^(
2*(c + d*Sqrt[x]))] + 24*a*d^2*x*PolyLog[3, (-I)*E^(c + d*Sqrt[x])] - 24*a*
d^2*x*PolyLog[3, I*E^(c + d*Sqrt[x])] - (6*I)*b*d*Sqrt[x]*PolyLog[3, -E^(2*
(c + d*Sqrt[x]))] - 48*a*d*Sqrt[x]*PolyLog[4, (-I)*E^(c + d*Sqrt[x])] + 48*
a*d*Sqrt[x]*PolyLog[4, I*E^(c + d*Sqrt[x])] + (3*I)*b*PolyLog[4, -E^(2*(c +
 d*Sqrt[x]))] + 48*a*PolyLog[5, (-I)*E^(c + d*Sqrt[x])] - 48*a*PolyLog[5, I
*E^(c + d*Sqrt[x])]))/d^5 + (5*b^2*x^2*Sech[c]*Sinh[d*Sqrt[x]])/d))/(5*(b +
 a*Cosh[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int x^{\frac{3}{2}} \left(a + b \,\mathrm{sech}\left(c + d\sqrt{x}\right)\right)^2 dx$$

```
[In] int(x^(3/2)*(a+b*sech(c+d*x^(1/2)))^2,x)
```

```
[Out] int(x^(3/2)*(a+b*sech(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x^{3/2} \left(a + b\mathrm{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int \left(b \,\mathrm{sech}\left(d\sqrt{x} + c\right) + a\right)^2 x^{\frac{3}{2}} \, dx$$

```
[In] integrate(x^(3/2)*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^(3/2)*sech(d*sqrt(x) + c)^2 + 2*a*b*x^(3/2)*sech(d*sqrt(x) +
 c) + a^2*x^(3/2), x)
```

Sympy [F]

$$\int x^{3/2} \left(a + b\mathrm{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int x^{\frac{3}{2}} \left(a + b \,\mathrm{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx$$

```
[In] integrate(x**(3/2)*(a+b*sech(c+d*x**(1/2)))**2,x)
```

```
[Out] Integral(x**(3/2)*(a + b*sech(c + d*sqrt(x)))**2, x)
```

Maxima [F]

$$\int x^{3/2} \left(a + b\mathrm{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int \left(b \,\mathrm{sech}\left(d\sqrt{x} + c\right) + a\right)^2 x^{\frac{3}{2}} \, dx$$

```
[In] integrate(x^(3/2)*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 2/5*(a^2*d*x^(5/2)*e^(2*d*sqrt(x) + 2*c) + a^2*d*x^(5/2) - 10*b^2*x^2)/(d*e
^(2*d*sqrt(x) + 2*c) + d) + integrate(4*(a*b*d*x^(5/2)*e^(d*sqrt(x) + c) +
2*b^2*x^2)/(d*x*e^(2*d*sqrt(x) + 2*c) + d*x), x)
```

Giac [F]

$$\int x^{3/2} \left(a + b\text{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int \left(b\,\text{sech}\left(d\sqrt{x} + c\right) + a\right)^2 x^{\frac{3}{2}} \, dx$$

```
[In] integrate(x^(3/2)*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)^2*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2} \left(a + b\text{sech}\left(c + d\sqrt{x}\right)\right)^2 \, dx = \int x^{3/2} \left(a + \frac{b}{\cosh\left(c + d\,\sqrt{x}\right)}\right)^2 \, dx$$

```
[In] int(x^(3/2)*(a + b/cosh(c + d*x^(1/2)))^2,x)
[Out] int(x^(3/2)*(a + b/cosh(c + d*x^(1/2)))^2, x)
```

3.58 $\int \sqrt{x}\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 229

$$\begin{aligned}\int \sqrt{x}\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx &= \frac{2b^2x}{d}+\frac{2}{3}a^2x^{3/2}+\frac{8abx\arctan\left(e^{c+d\sqrt{x}}\right)}{d}\\ &\quad-\frac{4b^2\sqrt{x}\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2}\\ &\quad-\frac{8iab\sqrt{x}\,\mathrm{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}\\ &\quad+\frac{8iab\sqrt{x}\,\mathrm{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}\\ &\quad-\frac{2b^2\,\mathrm{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}\\ &\quad+\frac{8iab\,\mathrm{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}\\ &\quad-\frac{8iab\,\mathrm{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}+\frac{2b^2x\tanh\left(c+d\sqrt{x}\right)}{d}\end{aligned}$$

```
[Out] 2*b^2*x/d+2/3*a^2*x^(3/2)+8*a*b*x*arctan(exp(c+d*x^(1/2)))/d-2*b^2*polylog(
2,-exp(2*c+2*d*x^(1/2)))/d^3+8*I*a*b*polylog(3,-I*exp(c+d*x^(1/2)))/d^3-8*I
*a*b*polylog(3,I*exp(c+d*x^(1/2)))/d^3-4*b^2*ln(1+exp(2*c+2*d*x^(1/2)))*x^(
1/2)/d^2-8*I*a*b*polylog(2,-I*exp(c+d*x^(1/2)))*x^(1/2)/d^2+8*I*a*b*polylog
(2,I*exp(c+d*x^(1/2)))*x^(1/2)/d^2+2*b^2*x*tanh(c+d*x^(1/2))/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5544, 4275, 4265, 2611, 2320, 6724, 4269, 3799, 2221, 2317, 2438}

$$\int \sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{2}{3}a^2x^{3/2}+\frac{8abx\arctan\left(e^{c+d\sqrt{x}}\right)}{d}+\frac{8iab\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{8iab\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3}-\frac{8iab\sqrt{x}\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2}+\frac{8iab\sqrt{x}\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2}-\frac{2b^2\,\text{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}-\frac{4b^2\sqrt{x}\log\left(e^{2\left(c+d\sqrt{x}\right)}+1\right)}{d^2}+\frac{2b^2x\tanh\left(c+d\sqrt{x}\right)}{d}+\frac{2b^2x}{d}$$

```
[In] Int[Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*b^2*x)/d + (2*a^2*x^(3/2))/3 + (8*a*b*x*ArcTan[E^(c + d*Sqrt[x])])/d - (
4*b^2*Sqrt[x]*Log[1 + E^(2*(c + d*Sqrt[x]))])/d^2 - ((8*I)*a*b*Sqrt[x]*Poly
Log[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((8*I)*a*b*Sqrt[x]*PolyLog[2, I*E^(c
+ d*Sqrt[x])])/d^2 - (2*b^2*PolyLog[2, -E^(2*(c + d*Sqrt[x]))])/d^3 + ((8*I
)*a*b*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((8*I)*a*b*PolyLog[3, I*E^(
c + d*Sqrt[x])])/d^3 + (2*b^2*x*Tanh[c + d*Sqrt[x]])/d
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int x^2(a+b\,\text{sech}(c+dx))^2\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(a^2x^2+2abx^2\text{sech}(c+dx)+b^2x^2\text{sech}^2(c+dx)\right)\,dx, x, \sqrt{x}\right)$$

$$= \frac{2}{3}a^2x^{3/2}+(4ab)\text{Subst}\left(\int x^2\text{sech}(c+dx)\,dx, x, \sqrt{x}\right) + \left(2b^2\right)\text{Subst}\left(\int x^2\text{sech}^2(c+dx)\,dx, x, \sqrt{x}\right)$$

$$= \frac{2}{3}a^2x^{3/2}+\frac{8abx\arctan\left(e^{c+d\sqrt{x}}\right)}{d}+\frac{2b^2x\tanh\left(c+d\sqrt{x}\right)}{d} - \frac{(8iab)\text{Subst}\left(\int x\log\left(1-ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d} + \frac{(8iab)\text{Subst}\left(\int x\log\left(1+ie^{c+dx}\right)\,dx, x, \sqrt{x}\right)}{d} - \frac{\left(4b^2\right)\text{Subst}\left(\int x\tanh(c+dx)\,dx, x, \sqrt{x}\right)}{d}$$

$$= \frac{2b^2x}{d} + \frac{2}{3}a^2x^{3/2} + \frac{8abx\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{8iab\sqrt{x}\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{8iab\sqrt{x}\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{2b^2x\tanh\left(c+d\sqrt{x}\right)}{d} + \frac{(8iab)\text{Subst}\left(\int \text{PolyLog}\left(2,-ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^2} - \frac{(8iab)\text{Subst}\left(\int \text{PolyLog}\left(2,ie^{c+dx}\right)\,dx,x,\sqrt{x}\right)}{d^2} - \frac{(8b^2)\,\text{Subst}\left(\int \frac{e^{2(c+dx)}x}{1+e^{2(c+dx)}}\,dx,x,\sqrt{x}\right)}{d}$$

$$= \frac{2b^2x}{d} + \frac{2}{3}a^2x^{3/2} + \frac{8abx\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{4b^2\sqrt{x}\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{8iab\sqrt{x}\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{8iab\sqrt{x}\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{2b^2x\tanh\left(c+d\sqrt{x}\right)}{d} + \frac{(8iab)\text{Subst}\left(\int \frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^3} - \frac{(8iab)\text{Subst}\left(\int \frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{d^3} + \frac{(4b^2)\,\text{Subst}\left(\int \log\left(1+e^{2(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2}$$

$$= \frac{2b^2x}{d} + \frac{2}{3}a^2x^{3/2} + \frac{8abx\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{4b^2\sqrt{x}\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{8iab\sqrt{x}\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{8iab\sqrt{x}\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{8iab\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{8iab\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{2b^2x\tanh\left(c+d\sqrt{x}\right)}{d} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3}$$

$$= \frac{2b^2x}{d} + \frac{2}{3}a^2x^{3/2} + \frac{8abx\arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{4b^2\sqrt{x}\log\left(1+e^{2\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{8iab\sqrt{x}\,\text{PolyLog}\left(2,-ie^{c+d\sqrt{x}}\right)}{d^2} + \frac{8iab\sqrt{x}\,\text{PolyLog}\left(2,ie^{c+d\sqrt{x}}\right)}{d^2} - \frac{2b^2\,\text{PolyLog}\left(2,-e^{2\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{8iab\,\text{PolyLog}\left(3,-ie^{c+d\sqrt{x}}\right)}{d^3} - \frac{8iab\,\text{PolyLog}\left(3,ie^{c+d\sqrt{x}}\right)}{d^3} + \frac{2b^2x\tanh\left(c+d\sqrt{x}\right)}{d}$$

Mathematica [A] (verified)

Time = 3.35 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.50

$$\int \sqrt{x}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2 \, dx$$
$$= \frac{2\cosh\left(c+d\sqrt{x}\right)\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2\left(a^2x^{3/2}\cosh\left(c+d\sqrt{x}\right)+\frac{3b\cosh\left(c+d\sqrt{x}\right)\left(2be^{2c}x-2b\left(1+e^{2c}\right)x+\frac{b\left(1+e^{2c}\right)}{}\right.}{}\right.}{}$$

```
[In] Integrate[Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*Cosh[c + d*Sqrt[x]]*(a + b*Sech[c + d*Sqrt[x]])^2*(a^2*x^(3/2)*Cosh[c +
d*Sqrt[x]] + (3*b*Cosh[c + d*Sqrt[x]]*(2*b*E^(2*c)*x - 2*b*(1 + E^(2*c))*x
+ (b*(1 + E^(2*c))*(2*d^2*x - 2*d*Sqrt[x]*Log[1 + E^(2*(c + d*Sqrt[x]))] -
PolyLog[2, -E^(2*(c + d*Sqrt[x]))]))/d^2 + ((2*I)*a*(1 + E^(2*c))*(d^2*x*Lo
g[1 - I*E^(c + d*Sqrt[x])] - d^2*x*Log[1 + I*E^(c + d*Sqrt[x])] - 2*d*Sqrt[
x]*PolyLog[2, (-I)*E^(c + d*Sqrt[x])] + 2*d*Sqrt[x]*PolyLog[2, I*E^(c + d*S
qrt[x])] + 2*PolyLog[3, (-I)*E^(c + d*Sqrt[x])] - 2*PolyLog[3, I*E^(c + d*S
qrt[x])]))/d^2))/(d*(1 + E^(2*c))) + (3*b^2*x*Sech[c]*Sinh[d*Sqrt[x]])/d))/
(3*(b + a*Cosh[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int \left(a+b\,\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^{2}\sqrt{x}dx$$

```
[In] int((a+b*sech(c+d*x^(1/2)))^2*x^(1/2),x)
```

```
[Out] int((a+b*sech(c+d*x^(1/2)))^2*x^(1/2),x)
```

Fricas [F]

$$\int \sqrt{x}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2 \, dx = \int \left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)^{2}\sqrt{x}\, dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(b^2*sqrt(x)*sech(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*sech(d*sqrt(x) +
 c) + a^2*sqrt(x), x)
```

Sympy [F]

$$\int \sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int \sqrt{x}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx$$

```
[In] integrate((a+b*sech(c+d*x**(1/2)))**2*x**(1/2),x)
[Out] Integral(sqrt(x)*(a + b*sech(c + d*sqrt(x)))**2, x)
```

Maxima [F]

$$\int \sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int \left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)^2\sqrt{x}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2*x^(1/2),x, algorithm="maxima")
[Out] 2/3*(a^2*d*x^(3/2)*e^(2*d*sqrt(x) + 2*c) + a^2*d*x^(3/2) - 6*b^2*x)/(d*e^(2
*d*sqrt(x) + 2*c) + d) + integrate(4*(a*b*d*x^(3/2)*e^(d*sqrt(x) + c) + b^2
*x)/(d*x*e^(2*d*sqrt(x) + 2*c) + d*x), x)
```

Giac [F]

$$\int \sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int \left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)^2\sqrt{x}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2*x^(1/2),x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)^2*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2\,dx=\int \sqrt{x}\left(a+\frac{b}{\cosh\left(c+d\sqrt{x}\right)}\right)^2\,dx$$

```
[In] int(x^(1/2)*(a + b/cosh(c + d*x^(1/2)))^2,x)
[Out] int(x^(1/2)*(a + b/cosh(c + d*x^(1/2)))^2, x)
```

3.59 $\int \frac{\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx=2a^2\sqrt{x}+\frac{4ab\arctan\left(\sinh\left(c+d\sqrt{x}\right)\right)}{d}+\frac{2b^2\tanh\left(c+d\sqrt{x}\right)}{d}$$

```
[Out] 4*a*b*arctan(sinh(c+d*x^(1/2)))/d+2*a^2*x^(1/2)+2*b^2*tanh(c+d*x^(1/2))/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5544, 3858, 3855, 3852, 8}

$$\int \frac{\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx=2a^2\sqrt{x}+\frac{4ab\arctan\left(\sinh\left(c+d\sqrt{x}\right)\right)}{d}+\frac{2b^2\tanh\left(c+d\sqrt{x}\right)}{d}$$

```
[In] Int[(a + b*Sech[c + d*Sqrt[x]])^2/Sqrt[x],x]
```

```
[Out] 2*a^2*Sqrt[x] + (4*a*b*ArcTan[Sinh[c + d*Sqrt[x]]])/d + (2*b^2*Tanh[c + d*S
qrt[x]])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int (a+b\text{sech}(c+dx))^2\,dx, x, \sqrt{x}\right) \\
&= 2a^2\sqrt{x}+(4ab)\text{Subst}\left(\int \text{sech}(c+dx)\,dx, x, \sqrt{x}\right)+\left(2b^2\right)\text{Subst}\left(\int \text{sech}^2(c+dx)\,dx, x, \sqrt{x}\right) \\
&= 2a^2\sqrt{x}+\frac{4ab\arctan\left(\sinh\left(c+d\sqrt{x}\right)\right)}{d}+\frac{(2ib^2)\,\text{Subst}\left(\int 1\,dx, x, -i\tanh\left(c+d\sqrt{x}\right)\right)}{d} \\
&= 2a^2\sqrt{x}+\frac{4ab\arctan\left(\sinh\left(c+d\sqrt{x}\right)\right)}{d}+\frac{2b^2\tanh\left(c+d\sqrt{x}\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = 2a^2\sqrt{x}+\frac{4ab\arctan\left(\sinh\left(c+d\sqrt{x}\right)\right)}{d}+\frac{2b^2\tanh\left(c+d\sqrt{x}\right)}{d}$$

```
[In] Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/Sqrt[x],x]
[Out] 2*a^2*Sqrt[x] + (4*a*b*ArcTan[Sinh[c + d*Sqrt[x]]])/d + (2*b^2*Tanh[c + d*S
qrt[x]])/d
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

method	result	size
parts	$\frac{4ab\arctan(\sinh(c+d\sqrt{x}))}{d}+2a^2\sqrt{x}+\frac{2b^2\tanh(c+d\sqrt{x})}{d}$	42
derivativedivides	$\frac{2a^2(c+d\sqrt{x})+8ab\arctan\left(e^{c+d\sqrt{x}}\right)+2b^2\tanh(c+d\sqrt{x})}{d}$	43
default	$\frac{2a^2(c+d\sqrt{x})+8ab\arctan\left(e^{c+d\sqrt{x}}\right)+2b^2\tanh(c+d\sqrt{x})}{d}$	43

```
[In] int((a+b*sech(c+d*x^(1/2)))^2/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 4*a*b*arctan(sinh(c+d*x^(1/2)))/d+2*a^2*x^(1/2)+2*b^2*tanh(c+d*x^(1/2))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(41) = 82.

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 4.13

$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{\sqrt{x}}\,dx$$
$$= \frac{2\left(a^2d\sqrt{x}\cosh\left(d\sqrt{x}+c\right)^2+2\,a^2d\sqrt{x}\cosh\left(d\sqrt{x}+c\right)\sinh\left(d\sqrt{x}+c\right)+a^2d\sqrt{x}\sinh\left(d\sqrt{x}+c\right)^2+a^2d\sqrt{x}\right.}{d\cosh\left(d\sqrt{x}+c\right)^2+}$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="fricas")
[Out] 2*(a^2*d*sqrt(x)*cosh(d*sqrt(x) + c)^2 + 2*a^2*d*sqrt(x)*cosh(d*sqrt(x) + c
)*sinh(d*sqrt(x) + c) + a^2*d*sqrt(x)*sinh(d*sqrt(x) + c)^2 + a^2*d*sqrt(x)
 - 2*b^2 + 4*(a*b*cosh(d*sqrt(x) + c)^2 + 2*a*b*cosh(d*sqrt(x) + c)*sinh(d*
sqrt(x) + c) + a*b*sinh(d*sqrt(x) + c)^2 + a*b)*arctan(cosh(d*sqrt(x) + c)
+ sinh(d*sqrt(x) + c)))/(d*cosh(d*sqrt(x) + c)^2 + 2*d*cosh(d*sqrt(x) + c)*
sinh(d*sqrt(x) + c) + d*sinh(d*sqrt(x) + c)^2 + d)
```

Sympy [F]

$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{\sqrt{x}}\,dx = \int \frac{(a+b\operatorname{sech}\left(c+d\sqrt{x}\right))^2}{\sqrt{x}}\,dx$$

```
[In] integrate((a+b*sech(c+d*x**(1/2)))**2/x**(1/2),x)
[Out] Integral((a + b*sech(c + d*sqrt(x)))**2/sqrt(x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = 2\,a^2\sqrt{x}+\frac{4\,ab\arctan\left(\sinh\left(d\sqrt{x}+c\right)\right)}{d}+\frac{4\,b^2}{d\left(e^{\left(-2\,d\sqrt{x}-2\,c\right)}+1\right)}$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*a^2*sqrt(x) + 4*a*b*arctan(sinh(d*sqrt(x) + c))/d + 4*b^2/(d*(e^(-2*d*sqr
t(x) - 2*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.17

$$\int \frac{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = \frac{2\left(d\sqrt{x}+c\right)a^2}{d}+\frac{8\,ab\arctan\left(e^{\left(d\sqrt{x}+c\right)}\right)}{d}-\frac{4\,b^2}{d\left(e^{\left(2\,d\sqrt{x}+2\,c\right)}+1\right)}$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*(d*sqrt(x) + c)*a^2/d + 8*a*b*arctan(e^(d*sqrt(x) + c))/d - 4*b^2/(d*(e^(
2*d*sqrt(x) + 2*c) + 1))
```

Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

$$\int \frac{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = 2\,a^2\,\sqrt{x}+\frac{8\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{d\,\sqrt{x}}\,\mathrm{e}^{c}\,\sqrt{d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{d^2}}-\frac{4\,b^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,\sqrt{x}}+1\right)}$$

```
[In] int((a + b/cosh(c + d*x^(1/2)))^2/x^(1/2),x)
```

```
[Out] 2*a^2*x^(1/2) + (8*atan((a*b*exp(d*x^(1/2))*exp(c)*(d^2)^(1/2))/(d*(a^2*b^2
)^(1/2)))*(a^2*b^2)^(1/2))/(d^2)^(1/2) - (4*b^2)/(d*(exp(2*c + 2*d*x^(1/2))
 + 1))
```

3.60 $\int \frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\text{Int}\left(\frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*sech(c+d*x^(1/2)))^2/x^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx$$

```
[In] Int[(a + b*Sech[c + d*Sqrt[x]])^2/x^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Sech[c + d*Sqrt[x]])^2/x^(3/2), x]
```

Rubi steps

$$\text{integral}=\int \frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 36.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx$$

```
[In] Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/x^(3/2),x]
[Out] Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/x^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\,\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{3}{2}}}dx$$

```
[In] int((a+b*sech(c+d*x^(1/2)))^2/x^(3/2),x)
[Out] int((a+b*sech(c+d*x^(1/2)))^2/x^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(b\,\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="fricas")
[Out] integral((b^2*sqrt(x)*sech(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*sech(d*sqrt(x)
+ c) + a^2*sqrt(x))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sech(c+d*x**(1/2)))**2/x**(3/2),x)
[Out] Integral((a + b*sech(c + d*sqrt(x)))**2/x**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 106, normalized size of antiderivative = 4.82

$$\int \frac{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="maxima")
[Out] -2*(a^2*d*sqrt(x)*e^(2*d*sqrt(x) + 2*c) + a^2*d*sqrt(x) + 2*b^2)/(d*x*e^(2*
d*sqrt(x) + 2*c) + d*x) + integrate(4*(a*b*d*x*e^(d*sqrt(x) + c) - b^2*sqrt
(x))/(d*x^(5/2)*e^(2*d*sqrt(x) + 2*c) + d*x^(5/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)^2/x^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}\right)^2}{x^{3/2}}\,dx$$

```
[In] int((a + b/cosh(c + d*x^(1/2)))^2/x^(3/2),x)
[Out] int((a + b/cosh(c + d*x^(1/2)))^2/x^(3/2), x)
```

3.61 $\int \frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\text{Int}\left(\frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*sech(c+d*x^(1/2)))^2/x^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx$$

```
[In] Int[(a + b*Sech[c + d*Sqrt[x]])^2/x^(5/2),x]
[Out] Defer[Int][(a + b*Sech[c + d*Sqrt[x]])^2/x^(5/2), x]
```

Rubi steps

$$\text{integral}=\int \frac{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 34.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx$$

```
[In] Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/x^(5/2),x]
[Out] Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/x^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\,\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^{2}}{x^{\frac{5}{2}}}dx$$

```
[In] int((a+b*sech(c+d*x^(1/2)))^2/x^(5/2),x)
[Out] int((a+b*sech(c+d*x^(1/2)))^2/x^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(b\,\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)^{2}}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="fricas")
[Out] integral((b^2*sqrt(x)*sech(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*sech(d*sqrt(x)
+ c) + a^2*sqrt(x))/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 5.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\text{sech}(c+d\sqrt{x}))^2}{x^{5/2}}\,dx=\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sech(c+d*x**(1/2)))**2/x**(5/2),x)
[Out] Integral((a + b*sech(c + d*sqrt(x)))**2/x**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\text{sech}(c+d\sqrt{x}))^2}{x^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\text{sech}(c+d\sqrt{x}))^2}{x^{5/2}}\,dx=\int \frac{(b\operatorname{sech}(d\sqrt{x}+c)+a)^2}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*sech(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="giac")
[Out] integrate((b*sech(d*sqrt(x) + c) + a)^2/x^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\text{sech}(c+d\sqrt{x}))^2}{x^{5/2}}\,dx=\int \frac{\left(a+\frac{b}{\cosh(c+d\sqrt{x})}\right)^2}{x^{5/2}}\,dx$$

```
[In] int((a + b/cosh(c + d*x^(1/2)))^2/x^(5/2),x)
[Out] int((a + b/cosh(c + d*x^(1/2)))^2/x^(5/2), x)
```

3.62 $\int \frac{x^{3/2}}{a+b\,\text{sech}\left(c+d\sqrt{x}\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 601

$$\begin{aligned}\int \frac{x^{3/2}}{a+b\,\text{sech}\left(c+d\sqrt{x}\right)}\,dx =& \frac{2x^{5/2}}{5a}-\frac{2bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}\\&+\frac{2bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{8bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}\\&+\frac{8bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{24bx\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}\\&-\frac{24bx\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{48b\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}\\&+\frac{48b\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}\\&+\frac{48b\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}-\frac{48b\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}\end{aligned}$$

```
[Out] 2/5*x^(5/2)/a-2*b*x^2*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a/d/(-a
^2+b^2)^(1/2)+2*b*x^2*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d/(-a
^2+b^2)^(1/2)-8*b*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)
))/a/d^2/(-a^2+b^2)^(1/2)+8*b*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^
2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)+24*b*x*polylog(3,-a*exp(c+d*x^(1/2))/
(b-(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)-24*b*x*polylog(3,-a*exp(c+d*x^
(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)+48*b*polylog(5,-a*exp(c
+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a/d^5/(-a^2+b^2)^(1/2)-48*b*polylog(5,-a*
exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d^5/(-a^2+b^2)^(1/2)-48*b*polylog(
```

```
4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^4/(-a^2+b^2)^(1/2)+
48*b*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^4/(-a^
2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5544, 4276, 3401, 2296, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{x^{3/2}}{a+b\text{sech}\left(c+d\sqrt{x}\right)}\,dx = \frac{48b\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} - \frac{48b\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} - \frac{48b\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{48b\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{24bx\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{24bx\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{8bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{8bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{2bx^2\log\left(\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}+1\right)}{ad\sqrt{b^2-a^2}} + \frac{2bx^2\log\left(\frac{ae^{c+d\sqrt{x}}}{\sqrt{b^2-a^2}+b}+1\right)}{ad\sqrt{b^2-a^2}} + \frac{2x^{5/2}}{5a}$$

```
[In] Int[x^(3/2)/(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (2*x^(5/2))/(5*a) - (2*b*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 +
 b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (2*b*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b
 + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (8*b*x^(3/2)*PolyLog[2, -((
a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (
8*b*x^(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a
*Sqrt[-a^2 + b^2]*d^2) + (24*b*x*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sq
rt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (24*b*x*PolyLog[3, -((a*E^(c
+ d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (48*b*Sq
rt[x]*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[
-a^2 + b^2]*d^4) + (48*b*Sqrt[x]*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sq
rt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (48*b*PolyLog[5, -((a*E^(c +
d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5) - (48*b*Poly
Log[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2
]*d^5)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^4}{a+b\text{sech}(c+dx)}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^4}{a}-\frac{bx^4}{a(b+a\cosh(c+dx))}\right)dx, x, \sqrt{x}\right)\\
&= \frac{2x^{5/2}}{5a}-\frac{(2b)\text{Subst}\left(\int \frac{x^4}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{2x^{5/2}}{5a}-\frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^4}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{2x^{5/2}}{5a}-\frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^4}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}\\
&\quad+\frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^4}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}\\
&= \frac{2x^{5/2}}{5a}-\frac{2bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{2bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}\\
&\quad+\frac{(8b)\text{Subst}\left(\int x^3\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}\\
&\quad-\frac{(8b)\text{Subst}\left(\int x^3\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}
\end{aligned}$$

$$=\frac{2x^{5/2}}{5a}-\frac{2bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{2bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{8bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{8bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{(24b)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}-\frac{(24b)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$=\frac{2x^{5/2}}{5a}-\frac{2bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{2bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{8bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{8bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{24bx\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{24bx\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{(48b)\text{Subst}\left(\int x\,\text{PolyLog}\left(3,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}+\frac{(48b)\text{Subst}\left(\int x\,\text{PolyLog}\left(3,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$=\frac{2x^{5/2}}{5a}-\frac{2bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{2bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{8bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{8bx^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{24bx\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{24bx\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{48b\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}+\frac{48b\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}+\frac{(48b)\text{Subst}\left(\int\text{PolyLog}\left(4,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4}-\frac{(48b)\text{Subst}\left(\int\text{PolyLog}\left(4,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4}$$

$$= \frac{2x^{5/2}}{5a} - \frac{2bx^2 \log \left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^2 \log \left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{8bx^{3/2} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{8bx^{3/2} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{24bx \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{24bx \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{48b\sqrt{x} \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{48b\sqrt{x} \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{(48b)\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(4, \frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x} \, dx, x, e^{c+d\sqrt{x}}\right)}{a\sqrt{-a^2+b^2}d^5} - \frac{(48b)\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(4, -\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x} \, dx, x, e^{c+d\sqrt{x}}\right)}{a\sqrt{-a^2+b^2}d^5}$$

$$= \frac{2x^{5/2}}{5a} - \frac{2bx^2 \log \left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^2 \log \left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{8bx^{3/2} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{8bx^{3/2} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{24bx \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{24bx \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{48b\sqrt{x} \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{48b\sqrt{x} \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{48b \operatorname{PolyLog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} - \frac{48b \operatorname{PolyLog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.77

$$\int \frac{x^{3/2}}{a + b\text{sech}\left(c + d\sqrt{x}\right)} \, dx = \frac{2\left(\sqrt{-a^2+b^2}d^5x^{5/2} - 5bd^4x^2 \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right) + 5bd^4x^2 \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b}}\right.\right.}{}$$

```
[In] Integrate[x^(3/2)/(a + b*Sech[c + d*Sqrt[x]]),x]
[Out] (2*(Sqrt[-a^2 + b^2]*d^5*x^(5/2) - 5*b*d^4*x^2*Log[1 + (a*E^(c + d*Sqrt[x])
)/(b - Sqrt[-a^2 + b^2])] + 5*b*d^4*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b +
```

```
Sqrt[-a^2 + b^2])] - 20*b*d^3*x^(3/2)*PolyLog[2, (a*E^(c + d*Sqrt[x]))/(-b
+ Sqrt[-a^2 + b^2])] + 20*b*d^3*x^(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/
(b + Sqrt[-a^2 + b^2]))] + 60*b*d^2*x*PolyLog[3, (a*E^(c + d*Sqrt[x]))/(-b
+ Sqrt[-a^2 + b^2])] - 60*b*d^2*x*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + S
qrt[-a^2 + b^2]))] - 120*b*d*Sqrt[x]*PolyLog[4, (a*E^(c + d*Sqrt[x]))/(-b +
 Sqrt[-a^2 + b^2])] + 120*b*d*Sqrt[x]*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b
 + Sqrt[-a^2 + b^2]))] + 120*b*PolyLog[5, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[
-a^2 + b^2])] - 120*b*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b
^2]))]))/(5*a*Sqrt[-a^2 + b^2]*d^5)
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)}dx$$

```
[In] int(x^(3/2)/(a+b*sech(c+d*x^(1/2))),x)
[Out] int(x^(3/2)/(a+b*sech(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x^{3/2}}{a+b\mathrm{sech}\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{\frac{3}{2}}}{b\,\mathrm{sech}\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^(3/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(x^(3/2)/(b*sech(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x^{3/2}}{a+b\mathrm{sech}\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{\frac{3}{2}}}{a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)}\,dx$$

```
[In] integrate(x**(3/2)/(a+b*sech(c+d*x**(1/2))),x)
[Out] Integral(x**(3/2)/(a + b*sech(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{3/2}}{a+b\text{sech}\left(c+d\sqrt{x}\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^(3/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{x^{3/2}}{a+b\text{sech}\left(c+d\sqrt{x}\right)}\,dx = \int \frac{x^{\frac{3}{2}}}{b\,\text{sech}\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^(3/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(b*sech(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{a+b\text{sech}\left(c+d\sqrt{x}\right)}\,dx = \int \frac{x^{3/2}}{a+\frac{b}{\cosh\left(c+d\sqrt{x}\right)}}\,dx$$

```
[In] int(x^(3/2)/(a + b/cosh(c + d*x^(1/2))),x)
```

```
[Out] int(x^(3/2)/(a + b/cosh(c + d*x^(1/2))), x)
```

3.63 $\int \frac{\sqrt{x}}{a+b\,\text{sech}(c+d\sqrt{x})}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 361

$$\begin{aligned}\int \frac{\sqrt{x}}{a+b\,\text{sech}\left(c+d\sqrt{x}\right)}\,dx &= \frac{2x^{3/2}}{3a}-\frac{2bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d}+\frac{2bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d}\\
&\quad-\frac{4b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^2}\\
&\quad+\frac{4b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^2}\\
&\quad+\frac{4b\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^3}-\frac{4b\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,d^3}\end{aligned}$$

```
[Out] 2/3*x^(3/2)/a-2*b*x*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a/d/(-a^2
+b^2)^(1/2)+2*b*x*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d/(-a^2+b
^2)^(1/2)+4*b*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a/d^3/(-a
^2+b^2)^(1/2)-4*b*polylog(3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d^3
/(-a^2+b^2)^(1/2)-4*b*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x
^(1/2)/a/d^2/(-a^2+b^2)^(1/2)+4*b*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^
2)^(1/2)))*x^(1/2)/a/d^2/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5544, 4276, 3401, 2296, 2221, 2611, 2320, 6724}

$$\int \frac{\sqrt{x}}{a+b\text{sech}\left(c+d\sqrt{x}\right)}\,dx = \frac{4b\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}-\frac{4b\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}-\frac{4b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}+\frac{4b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}-\frac{2bx\log\left(\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}+1\right)}{ad\sqrt{b^2-a^2}}+\frac{2bx\log\left(\frac{ae^{c+d\sqrt{x}}}{\sqrt{b^2-a^2}+b}+1\right)}{ad\sqrt{b^2-a^2}}+\frac{2x^{3/2}}{3a}$$

```
[In] Int[Sqrt[x]/(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (2*x^(3/2))/(3*a) - (2*b*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b
^2])])/(a*Sqrt[-a^2 + b^2]*d) + (2*b*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + S
qrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (4*b*Sqrt[x]*PolyLog[2, -((a*E^
(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (4*b*
Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqr
t[-a^2 + b^2]*d^2) + (4*b*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2
 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (4*b*PolyLog[3, -((a*E^(c + d*Sqrt[x
]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^2}{a + b\text{sech}(c + dx)}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{x^2}{a} - \frac{bx^2}{a(b+a\cosh(c+dx))}\right) dx, x, \sqrt{x}\right)$$

$$= \frac{2x^{3/2}}{3a} - \frac{(2b)\text{Subst}\left(\int \frac{x^2}{b+a\cosh(c+dx)} dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{2x^{3/2}}{3a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{a+2be^{c+dx}+ae^{2(c+dx)}} dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{2x^{3/2}}{3a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}} + \frac{(4b)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}$$

$$= \frac{2x^{3/2}}{3a} - \frac{2bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{(4b)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d} - \frac{(4b)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}$$

$$= \frac{2x^{3/2}}{3a} - \frac{2bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{(4b)\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{(4b)\text{Subst}\left(\int \text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$=\frac{2x^{3/2}}{3a}-\frac{2bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{2bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{4b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{4b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{(4b)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{(4b)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$=\frac{2x^{3/2}}{3a}-\frac{2bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{2bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{4b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{4b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{4b\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{4b\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.79

$$\int\frac{\sqrt{x}}{a+b\operatorname{sech}\left(c+d\sqrt{x}\right)}\,dx=\frac{2\left(\sqrt{-a^2+b^2}d^3x^{3/2}-3bd^2x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)+3bd^2x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)-6bd\sqrt{x}\operatorname{PolyLog}\left(2,\frac{ae^{c-}}{-b+\sqrt{}}\right.\right.}{3a\sqrt{-a}}$$

```
[In] Integrate[Sqrt[x]/(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (2*(Sqrt[-a^2 + b^2]*d^3*x^(3/2) - 3*b*d^2*x*Log[1 + (a*E^(c + d*Sqrt[x]))/
(b - Sqrt[-a^2 + b^2])] + 3*b*d^2*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt
[-a^2 + b^2])] - 6*b*d*Sqrt[x]*PolyLog[2, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[
-a^2 + b^2])] + 6*b*d*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[
-a^2 + b^2]))] + 6*b*PolyLog[3, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[-a^2 + b^2
])] - 6*b*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))]))/(3*
a*Sqrt[-a^2 + b^2]*d^3)
```

Maple [F]

$$\int \frac{\sqrt{x}}{a + b\,\mathrm{sech}\left(c + d\sqrt{x}\right)}dx$$

```
[In] int(x^(1/2)/(a+b*sech(c+d*x^(1/2))),x)
[Out] int(x^(1/2)/(a+b*sech(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{\sqrt{x}}{a + b\mathrm{sech}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{\sqrt{x}}{b\,\mathrm{sech}\left(d\sqrt{x} + c\right) + a}\,dx$$

```
[In] integrate(x^(1/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(sqrt(x)/(b*sech(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{a + b\mathrm{sech}\left(c + d\sqrt{x}\right)}\,dx = \int \frac{\sqrt{x}}{a + b\,\mathrm{sech}\left(c + d\sqrt{x}\right)}\,dx$$

```
[In] integrate(x**(1/2)/(a+b*sech(c+d*x**(1/2))),x)
[Out] Integral(sqrt(x)/(a + b*sech(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x}}{a + b\mathrm{sech}\left(c + d\sqrt{x}\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^(1/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{\sqrt{x}}{a + b\text{sech}\left(c + d\sqrt{x}\right)} \, dx = \int \frac{\sqrt{x}}{b\,\text{sech}\left(d\sqrt{x} + c\right) + a} \, dx$$

```
[In] integrate(x^(1/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(sqrt(x)/(b*sech(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{a + b\text{sech}\left(c + d\sqrt{x}\right)} \, dx = \int \frac{\sqrt{x}}{a + \frac{b}{\cosh\left(c + d\sqrt{x}\right)}} \, dx$$

```
[In] int(x^(1/2)/(a + b/cosh(c + d*x^(1/2))),x)
[Out] int(x^(1/2)/(a + b/cosh(c + d*x^(1/2))), x)
```

3.64 $\int \frac{1}{\sqrt{x}\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \frac{1}{\sqrt{x}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\frac{2\sqrt{x}}{a}-\frac{4b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}$$

```
[Out] -4*b*arctan((a-b)^(1/2)*tanh(1/2*c+1/2*d*x^(1/2))/(a+b)^(1/2))/a/d/(a-b)^(1
/2)/(a+b)^(1/2)+2*x^(1/2)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5544, 3868, 2738, 214}

$$\int \frac{1}{\sqrt{x}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\frac{2\sqrt{x}}{a}-\frac{4b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

```
[In] Int[1/(Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]])),x]
```

```
[Out] (2*Sqrt[x])/a - (4*b*ArcTan[(Sqrt[a - b]*Tanh[(c + d*Sqrt[x])/2])/Sqrt[a +
b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{1}{a+b\text{sech}(c+dx)}\,dx, x, \sqrt{x}\right)\\
&= \frac{2\sqrt{x}}{a} - \frac{2\text{Subst}\left(\int \frac{1}{1+\frac{a\cosh(c+dx)}{b}}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{2\sqrt{x}}{a} + \frac{(4i)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, i\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{ad}\\
&= \frac{2\sqrt{x}}{a} - \frac{4b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx = \frac{2\left(\frac{c}{d}+\sqrt{x}+\frac{2b\arctan\left(\frac{(-a+b)\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}\right)}{a}$$

```
[In] Integrate[1/(Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]])),x]
[Out] (2*(c/d + Sqrt[x] + (2*b*ArcTan[((-a + b)*Tanh[(c + d*Sqrt[x])/2])/Sqrt[a^2
 - b^2]])/(Sqrt[a^2 - b^2]*d)))/a
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{-\frac{2\ln\left(\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)-1\right)}{a}+\frac{2\ln\left(\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+1\right)}{a}-\frac{4b\arctan\left(\frac{(a-b)\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}}{d}$	90
default	$\frac{-\frac{2\ln\left(\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)-1\right)}{a}+\frac{2\ln\left(\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+1\right)}{a}-\frac{4b\arctan\left(\frac{(a-b)\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}}{d}$	90

```
[In] int(1/(a+b*sech(c+d*x^(1/2)))/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/a*ln(tanh(1/2*c+1/2*d*x^(1/2))-1)+1/a*ln(tanh(1/2*c+1/2*d*x^(1/2))+
1)-2*b/a/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*c+1/2*d*x^(1/2))/((a+b)*
(a-b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(55) = 110.

Time = 0.30 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.74

$$\int \frac{1}{\sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$$
$$=\left[\frac{2\left((a^2-b^2)d\sqrt{x}-\sqrt{-a^2+b^2}b\log\left(\frac{ab+\left(b^2+\sqrt{-a^2+b^2}b\right)\cosh\left(d\sqrt{x}+c\right)+\left(a^2-b^2-\sqrt{-a^2+b^2}b\right)\sinh\left(d\sqrt{x}+c\right)+\sqrt{-a^2+b^2}a}{a\cosh\left(d\sqrt{x}+c\right)+b}\right)\right.}{(a^3-ab^2)d}\right.$$

```
[In] integrate(1/(a+b*sech(c+d*x^(1/2)))/x^(1/2),x, algorithm="fricas")
```

```
[Out] [2*((a^2 - b^2)*d*sqrt(x) - sqrt(-a^2 + b^2)*b*log((a*b + (b^2 + sqrt(-a^2
+ b^2)*b)*cosh(d*sqrt(x) + c) + (a^2 - b^2 - sqrt(-a^2 + b^2)*b)*sinh(d*sqr
t(x) + c) + sqrt(-a^2 + b^2)*a)/(a*cosh(d*sqrt(x) + c) + b)))/((a^3 - a*b^2
)*d), 2*((a^2 - b^2)*d*sqrt(x) + 2*sqrt(a^2 - b^2)*b*arctan(-(sqrt(a^2 - b^
2)*a*cosh(d*sqrt(x) + c) + sqrt(a^2 - b^2)*a*sinh(d*sqrt(x) + c) + sqrt(a^2
 - b^2)*b)/(a^2 - b^2)))/((a^3 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\,(a + b\mathrm{sech}\,(c + d\sqrt{x}))}\,dx = \int \frac{1}{\sqrt{x}\,(a + b\,\mathrm{sech}\,(c + d\sqrt{x}))}\,dx$$

```
[In] integrate(1/(a+b*sech(c+d*x**(1/2)))/x**(1/2),x)
[Out] Integral(1/(sqrt(x)*(a + b*sech(c + d*sqrt(x)))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x}\,(a + b\mathrm{sech}\,(c + d\sqrt{x}))}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*sech(c+d*x^(1/2)))/x^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{x}\,(a + b\mathrm{sech}\,(c + d\sqrt{x}))}\,dx = -\frac{4\,b\arctan\left(\frac{ae^{\left(d\sqrt{x}+c\right)}+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2 - b^2}ad} + \frac{2\,(d\sqrt{x} + c)}{ad}$$

```
[In] integrate(1/(a+b*sech(c+d*x^(1/2)))/x^(1/2),x, algorithm="giac")
[Out] -4*b*arctan((a*e^(d*sqrt(x) + c) + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a*d
) + 2*(d*sqrt(x) + c)/(a*d)
```

Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.28

$$\int \frac{1}{\sqrt{x}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\frac{2\sqrt{x}}{a}+\frac{2\,b\,\ln\left(\frac{2\,b\,\mathrm{e}^{d\,\sqrt{x}}\,\mathrm{e}^{c}}{a^2\,\sqrt{x}}-\frac{2\,b\,\left(a+b\,\mathrm{e}^{d\,\sqrt{x}}\,\mathrm{e}^{c}\right)}{a^2\,\sqrt{x}\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a\,d\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{2\,b\,\ln\left(\frac{2\,b\,\mathrm{e}^{d\,\sqrt{x}}\,\mathrm{e}^{c}}{a^2\,\sqrt{x}}+\frac{2\,b\,\left(a+b\,\mathrm{e}^{d\,\sqrt{x}}\,\mathrm{e}^{c}\right)}{a^2\,\sqrt{x}\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a\,d\,\sqrt{a+b}\,\sqrt{b-a}}$$

```
[In] int(1/(x^(1/2)*(a + b/cosh(c + d*x^(1/2)))),x)
```

```
[Out] (2*x^(1/2))/a + (2*b*log((2*b*exp(d*x^(1/2))*exp(c))/(a^2*x^(1/2)) - (2*b*(
a + b*exp(d*x^(1/2))*exp(c)))/(a^2*x^(1/2)*(a + b)^(1/2)*(b - a)^(1/2))))/(
a*d*(a + b)^(1/2)*(b - a)^(1/2)) - (2*b*log((2*b*exp(d*x^(1/2))*exp(c))/(a^
2*x^(1/2)) + (2*b*(a + b*exp(d*x^(1/2))*exp(c)))/(a^2*x^(1/2)*(a + b)^(1/2)
*(b - a)^(1/2))))/(a*d*(a + b)^(1/2)*(b - a)^(1/2))
```

3.65 $\int \frac{1}{x^{3/2}\left(a+b\,\mathbf{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{3/2}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\operatorname{Int}\left(\frac{1}{x^{3/2}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x^(3/2)/(a+b*sech(c+d*x^(1/2))),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^{3/2}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{3/2}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Int[1/(x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])),x]
```

```
[Out] Defer[Int][1/(x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^{3/2}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{3/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{3/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Integrate[1/(x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])),x]
[Out] Integrate[1/(x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{3}{2}}\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)}dx$$

```
[In] int(1/x^(3/2)/(a+b*sech(c+d*x^(1/2))),x)
[Out] int(1/x^(3/2)/(a+b*sech(c+d*x^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{3/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(sqrt(x)/(b*x^2*sech(d*sqrt(x) + c) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{\frac{3}{2}}\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] integrate(1/x**(3/2)/(a+b*sech(c+d*x**(1/2))),x)
[Out] Integral(1/(x**(3/2)*(a + b*sech(c + d*sqrt(x)))), x)
```

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.00

$$\int \frac{1}{x^{3/2} \left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] -2*b*integrate(e^(d*sqrt(x) + c)/(a^2*x^(3/2)*e^(2*d*sqrt(x) + 2*c) + 2*a*b
*x^(3/2)*e^(d*sqrt(x) + c) + a^2*x^(3/2)), x) - 2/(a*sqrt(x))
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2} \left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sech(d*sqrt(x) + c) + a)*x^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2} \left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{3/2}\,\left(a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}\right)}\,dx$$

```
[In] int(1/(x^(3/2)*(a + b/cosh(c + d*x^(1/2)))),x)
```

```
[Out] int(1/(x^(3/2)*(a + b/cosh(c + d*x^(1/2)))), x)
```

3.66 $\int \frac{1}{x^{5/2}\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{5/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\text{Int}\left(\frac{1}{x^{5/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x^(5/2)/(a+b*sech(c+d*x^(1/2))),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^{5/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{5/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Int[1/(x^(5/2)*(a + b*Sech[c + d*Sqrt[x]])),x]
[Out] Defer[Int][1/(x^(5/2)*(a + b*Sech[c + d*Sqrt[x]])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^{5/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{5/2} \left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx = \int \frac{1}{x^{5/2} \left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Integrate[1/(x^(5/2)*(a + b*Sech[c + d*Sqrt[x]])),x]
[Out] Integrate[1/(x^(5/2)*(a + b*Sech[c + d*Sqrt[x]])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{5}{2}} \left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)}dx$$

```
[In] int(1/x^(5/2)/(a+b*sech(c+d*x^(1/2))),x)
[Out] int(1/x^(5/2)/(a+b*sech(c+d*x^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{5/2} \left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx = \int \frac{1}{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(sqrt(x)/(b*x^3*sech(d*sqrt(x) + c) + a*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 6.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2} \left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx = \int \frac{1}{x^{\frac{5}{2}} \left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] integrate(1/x**(5/2)/(a+b*sech(c+d*x**(1/2))),x)
[Out] Integral(1/(x**(5/2)*(a + b*sech(c + d*sqrt(x)))), x)
```

Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.00

$$\int \frac{1}{x^{5/2}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")
[Out] -2*b*integrate(e^(d*sqrt(x) + c)/(a^2*x^(5/2)*e^(2*d*sqrt(x) + 2*c) + 2*a*b
*x^(5/2)*e^(d*sqrt(x) + c) + a^2*x^(5/2)), x) - 2/3/(a*x^(3/2))
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(1/((b*sech(d*sqrt(x) + c) + a)*x^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{5/2}\,\left(a+\frac{b}{\cosh\left(c+d\,\sqrt{x}\right)}\right)}\,dx$$

```
[In] int(1/(x^(5/2)*(a + b/cosh(c + d*x^(1/2)))),x)
[Out] int(1/(x^(5/2)*(a + b/cosh(c + d*x^(1/2)))), x)
```

3.67 $\int \frac{x^{3/2}}{\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 1755

$$\int \frac{x^{3/2}}{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\frac{2b^2x^2}{a^2(a^2-b^2)\,d}+\frac{2x^{5/2}}{5a^2}$$
$$-\frac{8b^2x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^2}+\frac{2b^3x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d}$$
$$-\frac{4bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}-\frac{8b^2x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^2}$$
$$-\frac{2b^3x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d}+\frac{4bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$-\frac{24b^2x\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^3}+\frac{8b^3x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2}$$
$$-\frac{16bx^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}-\frac{24b^2x\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^3}$$
$$-\frac{8b^3x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2}+\frac{16bx^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+\frac{48b^2\sqrt{x}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^4}-\frac{24b^3x\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3}$$
$$+\frac{48bx\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}+\frac{48b^2\sqrt{x}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^4}$$
$$+\frac{24b^3x\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3}-\frac{48bx\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$-\frac{48b^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^5}+\frac{48b^3\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^4}$$
$$-\frac{96b\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^4}-\frac{48b^2\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^5}$$
$$-\frac{48b^3\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^4}$$
$$+\frac{96b\sqrt{x}\operatorname{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^4}-\frac{48b^3\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^5}$$
$$+\frac{96b\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^5}+\frac{48b^3\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^5}$$
$$-\frac{96b\operatorname{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^5}+\frac{2b^2x^2\sinh\left(c+d\sqrt{x}\right)}{a(a^2-b^2)\,d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}$$

```
[Out] 2*b^2*x^2*sinh(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*cosh(c+d*x^(1/2)))+2/5*x^(5/
2)/a^2-4*b*x^2*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^
2)^(1/2)+4*b*x^2*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+
b^2)^(1/2)-16*b*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))
/a^2/d^2/(-a^2+b^2)^(1/2)+16*b*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a
^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+48*b*x*polylog(3,-a*exp(c+d*x^(1/2
))/(b-(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-48*b*x*polylog(3,-a*exp(c
+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)+48*b^2*polylog(3
,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^4+48*b^2
*polylog(3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/
d^4+48*b^3*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/
(-a^2+b^2)^(3/2)/d^4-48*b^3*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/
2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^4-96*b*polylog(4,-a*exp(c+d*x^(1/2))/(b
-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^4/(-a^2+b^2)^(1/2)+96*b*polylog(4,-a*exp(
c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^4/(-a^2+b^2)^(1/2)-8*b^2*x
^(3/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^2+2*b^
3*x^2*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-
8*b^2*x^(3/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d
^2-2*b^3*x^2*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(
3/2)/d-24*b^2*x*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^
2-b^2)/d^3+8*b^3*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2))
)/a^2/(-a^2+b^2)^(3/2)/d^2-24*b^2*x*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+
b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-8*b^3*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/
(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-24*b^3*x*polylog(3,-a*exp(c+
d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+24*b^3*x*polylog(
3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3-48*b^2
*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^5-48*b
^2*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^5-48
*b^3*polylog(5,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/
2)/d^5+48*b^3*polylog(5,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2
+b^2)^(3/2)/d^5+96*b*polylog(5,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^
2/d^5/(-a^2+b^2)^(1/2)-96*b*polylog(5,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/
2)))/a^2/d^5/(-a^2+b^2)^(1/2)+2*b^2*x^2/a^2/(a^2-b^2)/d
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 1755, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules

used = {5544, 4276, 3405, 3401, 2296, 2221, 2611, 6744, 2320, 6724, 5681}

$$\int \frac{x^{3/2}}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{2x^2\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}}+1\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d} - \frac{2x^2\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}}+1\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d} + \frac{8x^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2} - \frac{8x^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2} - \frac{24x\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3} + \frac{24x\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3} + \frac{48\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4} - \frac{48\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4} - \frac{48\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^5} + \frac{48\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^5} + \frac{2x^2b^2}{a^2\left(a^2-b^2\right)d} - \frac{8x^{3/2}\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2} - \frac{8x^{3/2}\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2} - \frac{24x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3} - \frac{24x\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3} + \frac{48\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4} + \frac{48\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4} - \frac{48\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^5} - \frac{48\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^5} + \frac{2x^2\sinh\left(c+d\sqrt{x}\right)b^2}{a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)} - \frac{4x^2\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}}+1\right)b}{a^2\sqrt{b^2-a^2}d} + \frac{4x^2\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}}+1\right)b}{a^2\sqrt{b^2-a^2}d} - \frac{16x^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2} + \frac{16x^{3/2}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2} + \frac{48x\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^3} - \frac{48x\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^3} - \frac{96\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^4} + \frac{96\sqrt{x}\,\text{PolyLog}\left(4,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^4} + \frac{96\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^5} - \frac{96\,\text{PolyLog}\left(5,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^5} + \frac{2x^{5/2}}{5a^2}$$

```
[In] Int[x^(3/2)/(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*b^2*x^2)/(a^2*(a^2 - b^2)*d) + (2*x^(5/2))/(5*a^2) - (8*b^2*x^(3/2)*Log[
1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) +
(2*b^3*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^
2 + b^2)^(3/2)*d) - (4*b*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 +
 b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - (8*b^2*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt
[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) - (2*b^3*x^2*Log[1 + (
a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) +
(4*b*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-
a^2 + b^2]*d) - (24*b^2*x*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2
 + b^2]))])/(a^2*(a^2 - b^2)*d^3) + (8*b^3*x^(3/2)*PolyLog[2, -((a*E^(c + d
*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (16*b*x
^(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sq
rt[-a^2 + b^2]*d^2) - (24*b^2*x*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqr
t[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^3) - (8*b^3*x^(3/2)*PolyLog[2, -((a*E^
(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (
16*b*x^(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(
a^2*Sqrt[-a^2 + b^2]*d^2) + (48*b^2*Sqrt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x
]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^4) - (24*b^3*x*PolyLog[3,
-((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d
^3) + (48*b*x*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/
(a^2*Sqrt[-a^2 + b^2]*d^3) + (48*b^2*Sqrt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[
x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^4) + (24*b^3*x*PolyLog[3,
 -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*
d^3) - (48*b*x*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])
/(a^2*Sqrt[-a^2 + b^2]*d^3) - (48*b^2*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b
 - Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^5) + (48*b^3*Sqrt[x]*PolyLog[4,
-((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d
^4) - (96*b*Sqrt[x]*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2
]))])/(a^2*Sqrt[-a^2 + b^2]*d^4) - (48*b^2*PolyLog[4, -((a*E^(c + d*Sqrt[x]
))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^5) - (48*b^3*Sqrt[x]*PolyLo
g[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3
/2)*d^4) + (96*b*Sqrt[x]*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2
+ b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^4) - (48*b^3*PolyLog[5, -((a*E^(c + d*Sq
rt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^5) + (96*b*Poly
Log[5, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b
^2]*d^5) + (48*b^3*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]
))])/(a^2*(-a^2 + b^2)^(3/2)*d^5) - (96*b*PolyLog[5, -((a*E^(c + d*Sqrt[x])
)/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^5) + (2*b^2*x^2*Sinh[c
+ d*Sqrt[x]])/(a*(a^2 - b^2)*d*(b + a*Cosh[c + d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^4}{(a+b\text{sech}(c+dx))^2}\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(\frac{x^4}{a^2} + \frac{b^2x^4}{a^2(b+a\cosh(c+dx))^2} - \frac{2bx^4}{a^2(b+a\cosh(c+dx))}\right)\,dx, x, \sqrt{x}\right)$$

$$= \frac{2x^{5/2}}{5a^2} - \frac{(4b)\text{Subst}\left(\int \frac{x^4}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{x^4}{(b+a\cosh(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}$$

$$= \frac{2x^{5/2}}{5a^2} + \frac{2b^2x^2\sinh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)} - \frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^4}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}$$
$$- \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{x^4}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)} - \frac{\left(8b^2\right)\text{Subst}\left(\int \frac{x^3\sinh(c+dx)}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}$$

$$= \frac{2b^2x^2}{a^2\left(a^2-b^2\right)d} + \frac{2x^{5/2}}{5a^2} + \frac{2b^2x^2\sinh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}$$
$$- \frac{\left(4b^3\right)\text{Subst}\left(\int \frac{e^{c+dx}x^4}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}$$
$$- \frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^4}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$+ \frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^4}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$- \frac{\left(8b^2\right)\text{Subst}\left(\int \frac{e^{c+dx}x^3}{b-\sqrt{-a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}$$
$$- \frac{\left(8b^2\right)\text{Subst}\left(\int \frac{e^{c+dx}x^3}{b+\sqrt{-a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}$$

$$= \frac{2b^2x^2}{a^2\left(a^2-b^2\right)d} + \frac{2x^{5/2}}{5a^2} - \frac{8b^2x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}$$

$$- \frac{4bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{8b^2x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}$$

$$+ \frac{4bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2b^2x^2\sinh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}$$

$$+ \frac{\left(4b^3\right)\text{Subst}\left(\int\frac{e^{c+dx}x^4}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\left(-a^2+b^2\right)^{3/2}}$$

$$- \frac{\left(4b^3\right)\text{Subst}\left(\int\frac{e^{c+dx}x^4}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\left(-a^2+b^2\right)^{3/2}}$$

$$+ \frac{\left(24b^2\right)\text{Subst}\left(\int x^2\log\left(1+\frac{ae^{c+dx}}{b-\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)d^2}$$

$$+ \frac{\left(24b^2\right)\text{Subst}\left(\int x^2\log\left(1+\frac{ae^{c+dx}}{b+\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)d^2}$$

$$+ \frac{(16b)\text{Subst}\left(\int x^3\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$- \frac{(16b)\text{Subst}\left(\int x^3\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$
\begin{aligned}
&= \frac{2b^2x^2}{a^2\left(a^2-b^2\right)d}+\frac{2x^{5/2}}{5a^2}-\frac{8b^2x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^2} \\
&+\frac{2b^3x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}-\frac{4bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&-\frac{8b^2x^{3/2}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}-\frac{2b^3x^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d} \\
&+\frac{4bx^2\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}-\frac{24b^2x\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^3} \\
&-\frac{16bx^{3/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}-\frac{24b^2x\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(a^2-b^2\right)d^3} \\
&+\frac{16bx^{3/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}+\frac{2b^2x^2\sinh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)} \\
&+\frac{\left(48b^2\right)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx}}{b-\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\left(a^2-b^2\right)d^3} \\
&+\frac{\left(48b^2\right)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx}}{b+\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\left(a^2-b^2\right)d^3} \\
&+\frac{(48b)\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&-\frac{(48b)\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&-\frac{\left(8b^3\right)\mathrm{Subst}\left(\int x^3\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d} \\
&+\frac{\left(8b^3\right)\mathrm{Subst}\left(\int x^3\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}
\end{aligned}
$$

$= \text{Too large to display}$

Mathematica [A] (verified)

Time = 8.27 (sec) , antiderivative size = 1769, normalized size of antiderivative = 1.01

$$\int \frac{x^{3/2}}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{2\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)\text{sech}^2\left(c+d\sqrt{x}\right)\left(x^{5/2}\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)+\frac{5be}{}\right.}{}$$

```
[In] Integrate[x^(3/2)/(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*(b + a*Cosh[c + d*Sqrt[x]])*Sech[c + d*Sqrt[x]]^2*(x^(5/2)*(b + a*Cosh[c
 + d*Sqrt[x]]) + (5*b*E^c*(b + a*Cosh[c + d*Sqrt[x]])*(2*b*E^c*x^2 - ((1 +
E^(2*c))*(4*b*d^3*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^(3/2)*Log[1 + (a*E^(2*c + d*
Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] + 2*a^2*d^4*E^c*x^2*Log[1 +
 (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] - b^2*d^4*E^
c*x^2*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]
 + 4*b*d^3*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^(3/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]
))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] - 2*a^2*d^4*E^c*x^2*Log[1 + (a*E^(
2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] + b^2*d^4*E^c*x^2*L
og[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] + 4*d^
2*(3*b*Sqrt[(-a^2 + b^2)*E^(2*c)] + 2*a^2*d*E^c*Sqrt[x] - b^2*d*E^c*Sqrt[x]
)*x*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)
]))] + 4*d^2*(3*b*Sqrt[(-a^2 + b^2)*E^(2*c)] - 2*a^2*d*E^c*Sqrt[x] + b^2*d*
E^c*Sqrt[x])*x*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b
^2)*E^(2*c)]))] - 24*b*d*Sqrt[(-a^2 + b^2)*E^(2*c)]*Sqrt[x]*PolyLog[3, -((a
*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 24*a^2*d^2*E
^c*x*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c
)]))] + 12*b^2*d^2*E^c*x*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt
[(-a^2 + b^2)*E^(2*c)]))] - 24*b*d*Sqrt[(-a^2 + b^2)*E^(2*c)]*Sqrt[x]*PolyL
og[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 24
*a^2*d^2*E^c*x*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b
^2)*E^(2*c)]))] - 12*b^2*d^2*E^c*x*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*
E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 24*b*Sqrt[(-a^2 + b^2)*E^(2*c)]*PolyL
og[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 48
*a^2*d*E^c*Sqrt[x]*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2
 + b^2)*E^(2*c)]))] - 24*b^2*d*E^c*Sqrt[x]*PolyLog[4, -((a*E^(2*c + d*Sqrt[
x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 24*b*Sqrt[(-a^2 + b^2)*E^(2*c
)]*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]
))] - 48*a^2*d*E^c*Sqrt[x]*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sq
```

```
rt[(-a^2 + b^2)*E^(2*c)]))] + 24*b^2*d*E^c*Sqrt[x]*PolyLog[4, -((a*E^(2*c +
 d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 48*a^2*E^c*PolyLog[5,
 -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 24*b^2*
E^c*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)
]))] + 48*a^2*E^c*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2
+ b^2)*E^(2*c)]))] - 24*b^2*E^c*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c
 + Sqrt[(-a^2 + b^2)*E^(2*c)]))]))/(d^4*E^c*Sqrt[(-a^2 + b^2)*E^(2*c)])))/(
(a^2 - b^2)*d*(1 + E^(2*c))) + (5*b^2*x^2*Sech[c]*(-(b*Sinh[c]) + a*Sinh[d*
Sqrt[x]]))/((a - b)*(a + b)*d)))/(5*a^2*(a + b*Sech[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{\left(a+b\ \mathrm{sech}\left(c+d\sqrt{x}\right)\right)^{2}}dx$$

```
[In] int(x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x)
```

```
[Out] int(x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x^{3/2}}{\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^{\frac{3}{2}}}{\left(b\,\mathrm{sech}\left(d\sqrt{x}+c\right)+a\right)^{2}}\,dx$$

```
[In] integrate(x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] integral(x^(3/2)/(b^2*sech(d*sqrt(x) + c)^2 + 2*a*b*sech(d*sqrt(x) + c) + a
^2), x)
```

Sympy [F]

$$\int \frac{x^{3/2}}{\left(a+b\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^{\frac{3}{2}}}{\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^{2}}\,dx$$

```
[In] integrate(x**(3/2)/(a+b*sech(c+d*x**(1/2)))**2,x)
```

```
[Out] Integral(x**(3/2)/(a + b*sech(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{3/2}}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{x^{3/2}}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^{\frac{3}{2}}}{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(b*sech(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^{3/2}}{\left(a+\frac{b}{\cosh\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(x^(3/2)/(a + b/cosh(c + d*x^(1/2)))^2,x)
```

```
[Out] int(x^(3/2)/(a + b/cosh(c + d*x^(1/2)))^2, x)
```

3.68 $\int \frac{\sqrt{x}}{\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 1027

$$
\begin{aligned}
\int \frac{\sqrt{x}}{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx &= \frac{2b^2x}{a^2(a^2-b^2)\,d}+\frac{2x^{3/2}}{3a^2}-\frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^2} \\
&+\frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d}-\frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&-\frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^2}-\frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d} \\
&+\frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}-\frac{4b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^3} \\
&+\frac{4b^3\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2} \\
&-\frac{8b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&-\frac{4b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^3} \\
&-\frac{4b^3\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2} \\
&+\frac{8b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&-\frac{4b^3\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3} \\
&+\frac{8b\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} \\
&+\frac{4b^3\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3} \\
&-\frac{8b\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} \\
&+\frac{2b^2x\sinh\left(c+d\sqrt{x}\right)}{a(a^2-b^2)\,d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}
\end{aligned}
$$

```
[Out] 2*b^2*x/a^2/(a^2-b^2)/d+2/3*x^(3/2)/a^2+2*b^3*x*ln(1+a*exp(c+d*x^(1/2))/(b-
(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-2*b^3*x*ln(1+a*exp(c+d*x^(1/2))/(
b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-4*b^2*polylog(2,-a*exp(c+d*x^(1
/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-4*b^2*polylog(2,-a*exp(c+d*x^(
1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-4*b^3*polylog(3,-a*exp(c+d*x^
(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+4*b^3*polylog(3,-a*ex
p(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+2*b^2*x*sinh(
c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*cosh(c+d*x^(1/2)))-4*b*x*ln(1+a*exp(c+d*x^(
1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)+4*b*x*ln(1+a*exp(c+d*x^(
1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)+8*b*polylog(3,-a*exp(c+d
*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-8*b*polylog(3,-a*e
xp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-4*b^2*ln(1+a
*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^2-4*b^2*ln(
1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^2+4*b^3*
polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^
(3/2)/d^2-4*b^3*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)
/a^2/(-a^2+b^2)^(3/2)/d^2-8*b*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(
1/2)))*x^(1/2)/a^2/d^2/(-a^2+b^2)^(1/2)+8*b*polylog(2,-a*exp(c+d*x^(1/2))/(
b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^2/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 1027, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules

used = {5544, 4276, 3405, 3401, 2296, 2221, 2611, 2320, 6724, 5681, 2317, 2438}

$$\int \frac{\sqrt{x}}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{2x\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}}+1\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d} - \frac{2x\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}}+1\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d}$$
$$+\frac{4\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}$$
$$-\frac{4\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}$$
$$-\frac{4\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3}$$
$$+\frac{4\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3}+\frac{2xb^2}{a^2\left(a^2-b^2\right)d}$$
$$-\frac{4\sqrt{x}\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2}-\frac{4\sqrt{x}\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2}$$
$$-\frac{4\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3}$$
$$-\frac{4\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3}$$
$$+\frac{2x\sinh\left(c+d\sqrt{x}\right)b^2}{a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}$$
$$-\frac{4x\log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}}+1\right)b}{a^2\sqrt{b^2-a^2}d}+\frac{4x\log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}}+1\right)b}{a^2\sqrt{b^2-a^2}d}$$
$$-\frac{8\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2}$$
$$+\frac{8\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2}$$
$$+\frac{8\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^3}$$
$$-\frac{8\,\text{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^3}+\frac{2x^{3/2}}{3a^2}$$

```
[In] Int[Sqrt[x]/(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*b^2*x)/(a^2*(a^2 - b^2)*d) + (2*x^(3/2))/(3*a^2) - (4*b^2*Sqrt[x]*Log[1
+ (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) + (2
*b^3*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 +
b^2)^(3/2)*d) - (4*b*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])
])/(a^2*Sqrt[-a^2 + b^2]*d) - (4*b^2*Sqrt[x]*Log[1 + (a*E^(c + d*Sqrt[x]))/
(b + Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) - (2*b^3*x*Log[1 + (a*E^(c +
 d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + (4*b*x*L
og[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]
*d) - (4*b^2*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(
a^2*(a^2 - b^2)*d^3) + (4*b^3*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b
 - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (8*b*Sqrt[x]*PolyLog
[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]
*d^2) - (4*b^2*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])
/(a^2*(a^2 - b^2)*d^3) - (4*b^3*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/
(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (8*b*Sqrt[x]*PolyL
og[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^
2]*d^2) - (4*b^3*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))
])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + (8*b*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(
b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^3) + (4*b^3*PolyLog[3, -((
a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3)
 - (8*b*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*S
qrt[-a^2 + b^2]*d^3) + (2*b^2*x*Sinh[c + d*Sqrt[x]])/(a*(a^2 - b^2)*d*(b +
a*Cosh[c + d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^2}{(a+b\text{sech}(c+dx))^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^2}{a^2}+\frac{b^2x^2}{a^2(b+a\cosh(c+dx))^2}-\frac{2bx^2}{a^2(b+a\cosh(c+dx))}\right)dx, x, \sqrt{x}\right)\\
&= \frac{2x^{3/2}}{3a^2}-\frac{(4b)\text{Subst}\left(\int \frac{x^2}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^2}{(b+a\cosh(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= \frac{2x^{3/2}}{3a^2}+\frac{2b^2x\sinh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}-\frac{(8b)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&\quad-\frac{(2b^3)\,\text{Subst}\left(\int \frac{x^2}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}-\frac{(4b^2)\,\text{Subst}\left(\int \frac{x\sinh(c+dx)}{b+a\cosh(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}
\end{aligned}
$$

$$= \frac{2b^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{2b^2x\sinh\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}$$
$$- \frac{(4b^3)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)}$$
$$- \frac{(8b)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$+ \frac{(8b)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$- \frac{(4b^2)\operatorname{Subst}\left(\int \frac{e^{c+dx}x}{b-\sqrt{-a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(a^2-b^2)d}$$
$$- \frac{(4b^2)\operatorname{Subst}\left(\int \frac{e^{c+dx}x}{b+\sqrt{-a^2+b^2}+ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(a^2-b^2)d}$$

$$= \frac{2b^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} - \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$- \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2b^2x\sinh\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{(4b^3)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$
$$- \frac{(4b^3)\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$
$$+ \frac{(4b^2)\operatorname{Subst}\left(\int \log\left(1+\frac{ae^{c+dx}}{b-\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{(4b^2)\operatorname{Subst}\left(\int \log\left(1+\frac{ae^{c+dx}}{b+\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{(8b)\operatorname{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$- \frac{(8b)\operatorname{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$= \frac{2b^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} - \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2}$$

$$+ \frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} - \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$- \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

$$+ \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{8b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$+ \frac{8b\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{2b^2x\sinh\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}$$

$$+ \frac{(4b^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{ax}{b-\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{a^2(a^2-b^2)d^3}$$

$$+ \frac{(4b^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{c+d\sqrt{x}}\right)}{a^2(a^2-b^2)d^3}$$

$$+ \frac{(8b)\text{Subst}\left(\int\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$- \frac{(8b)\text{Subst}\left(\int\text{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$- \frac{(4b^3)\,\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

$$+ \frac{(4b^3)\,\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

$$
\begin{aligned}
&= \frac{2b^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} - \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2} \\
&+ \frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} - \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&- \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
&+ \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{4b^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3} \\
&+ \frac{4b^3\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} - \frac{8b\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&- \frac{4b^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{4b^3\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} \\
&+ \frac{8b\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{2b^2x\sinh\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)} \\
&+ \frac{(8b)\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{a^2\sqrt{-a^2+b^2}d^3} \\
&- \frac{(8b)\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{c+d\sqrt{x}}\right)}{a^2\sqrt{-a^2+b^2}d^3} \\
&- \frac{(4b^3)\,\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d^2} \\
&+ \frac{(4b^3)\,\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d^2}
\end{aligned}
$$

$$= \frac{2b^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} - \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2}$$

$$+ \frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} - \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$- \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

$$+ \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{4b^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3}$$

$$+ \frac{4b^3\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} - \frac{8b\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$- \frac{4b^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{4b^3\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$

$$+ \frac{8b\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{8b\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$

$$- \frac{8b\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} + \frac{2b^2x\sinh\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}$$

$$- \frac{(4b^3)\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{c+d\sqrt{x}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$

$$+ \frac{(4b^3)\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{c+d\sqrt{x}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$

$$= \frac{2b^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} - \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} - \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$- \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2b^3x\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{4bx\log\left(1+\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{4b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$+ \frac{4b^3\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} - \frac{8b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$- \frac{4b^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{4b^3\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$+ \frac{8b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{4b^3\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$+ \frac{8b\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} + \frac{4b^3\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$- \frac{8b\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} + \frac{2b^2x\sinh\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)}$$

Mathematica [A] (verified)

Time = 5.15 (sec) , antiderivative size = 986, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{x}}{\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

$$= \frac{2\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)\operatorname{sech}^2\left(c+d\sqrt{x}\right)\left(x^{3/2}\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)+\frac{3be^c\left(b+a\cosh\left(c+d\sqrt{x}\right)\right)\left(2be^cx-\frac{e^{-c}\left(1+e^{2c}\right.}{}\right.}{}\right.}{}$$

```
[In] Integrate[Sqrt[x]/(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*(b + a*Cosh[c + d*Sqrt[x]])*Sech[c + d*Sqrt[x]]^2*(x^(3/2)*(b + a*Cosh[c
 + d*Sqrt[x]]) + (3*b*E^c*(b + a*Cosh[c + d*Sqrt[x]])*(2*b*E^c*x - ((1 + E^
(2*c))*(2*b*d*Sqrt[(-a^2 + b^2)*E^(2*c)]*Sqrt[x]*Log[1 + (a*E^(2*c + d*Sqrt
[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] + 2*a^2*d^2*E^c*x*Log[1 + (a*E^
(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] - b^2*d^2*E^c*x*Lo
g[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] + 2*b*d
*Sqrt[(-a^2 + b^2)*E^(2*c)]*Sqrt[x]*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c
+ Sqrt[(-a^2 + b^2)*E^(2*c)])] - 2*a^2*d^2*E^c*x*Log[1 + (a*E^(2*c + d*Sqrt
[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] + b^2*d^2*E^c*x*Log[1 + (a*E^(2
*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] + 2*(b*Sqrt[(-a^2 +
b^2)*E^(2*c)] + 2*a^2*d*E^c*Sqrt[x] - b^2*d*E^c*Sqrt[x])*PolyLog[2, -((a*E^
(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 2*(b*Sqrt[(-a^2
 + b^2)*E^(2*c)] - 2*a^2*d*E^c*Sqrt[x] + b^2*d*E^c*Sqrt[x])*PolyLog[2, -((a
*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 4*a^2*E^c*Po
lyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] +
 2*b^2*E^c*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*
E^(2*c)]))] + 4*a^2*E^c*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[
(-a^2 + b^2)*E^(2*c)]))] - 2*b^2*E^c*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(
b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))]))/(d^2*E^c*Sqrt[(-a^2 + b^2)*E^(2*c)]
)))/((a^2 - b^2)*d*(1 + E^(2*c))) + (3*b^2*x*Sech[c]*(b*Sinh[c] - a*Sinh[d*
Sqrt[x]]))/((-a^2 + b^2)*d)))/(3*a^2*(a + b*Sech[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int \frac{\sqrt{x}}{\left(a + b \operatorname{sech}\left(c + d\sqrt{x}\right)\right)^2} dx$$

```
[In] int(x^(1/2)/(a+b*sech(c+d*x^(1/2)))^2,x)
```

```
[Out] int(x^(1/2)/(a+b*sech(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{\sqrt{x}}{\left(a + b\text{sech}\left(c + d\sqrt{x}\right)\right)^2} dx = \int \frac{\sqrt{x}}{\left(b \operatorname{sech}\left(d\sqrt{x} + c\right) + a\right)^2} dx$$

```
[In] integrate(x^(1/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)/(b^2*sech(d*sqrt(x) + c)^2 + 2*a*b*sech(d*sqrt(x) + c) + a
^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{\sqrt{x}}{\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(x**(1/2)/(a+b*sech(c+d*x**(1/2)))**2,x)
[Out] Integral(sqrt(x)/(a + b*sech(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x}}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^(1/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more det
ails)Is
```

Giac [F]

$$\int \frac{\sqrt{x}}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{\sqrt{x}}{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^(1/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(sqrt(x)/(b*sech(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{\sqrt{x}}{\left(a+\frac{b}{\cosh\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(x^(1/2)/(a + b/cosh(c + d*x^(1/2)))^2,x)
[Out] int(x^(1/2)/(a + b/cosh(c + d*x^(1/2)))^2, x)
```

3.69 $\int \frac{1}{\sqrt{x}\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{1}{\sqrt{x}\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\frac{2\sqrt{x}}{a^2}-\frac{4b(2a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d}+\frac{2b^2\tanh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)}$$

```
[Out] -4*b*(2*a^2-b^2)*arctan((a-b)^(1/2)*tanh(1/2*c+1/2*d*x^(1/2))/(a+b)^(1/2))/
a^2/(a-b)^(3/2)/(a+b)^(3/2)/d+2*x^(1/2)/a^2+2*b^2*tanh(c+d*x^(1/2))/a/(a^2-
b^2)/d/(a+b*sech(c+d*x^(1/2)))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5544, 3870, 4004, 3916, 2738, 214}

$$\int \frac{1}{\sqrt{x}\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=-\frac{4b(2a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{2b^2\tanh\left(c+d\sqrt{x}\right)}{ad\left(a^2-b^2\right)\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)}+\frac{2\sqrt{x}}{a^2}$$

```
[In] Int[1/(Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]])^2),x]
```

```
[Out] (2*Sqrt[x])/a^2 - (4*b*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tanh[(c + d*Sqrt[x
])/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (2*b^2*Tanh[c +
d*Sqrt[x]])/(a*(a^2 - b^2)*d*(a + b*Sech[c + d*Sqrt[x]]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{1}{(a+b\text{sech}(c+dx))^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2b^2\tanh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)} - \frac{2\text{Subst}\left(\int \frac{-a^2+b^2+ab\operatorname{sech}(c+dx)}{a+b\operatorname{sech}(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)}$$

$$= \frac{2\sqrt{x}}{a^2} + \frac{2b^2\tanh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)} - \frac{\left(2b\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{\operatorname{sech}(c+dx)}{a+b\operatorname{sech}(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}$$

$$= \frac{2\sqrt{x}}{a^2} + \frac{2b^2\tanh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)} - \frac{\left(2\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a\cosh(c+dx)}{b}}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}$$

$$= \frac{2\sqrt{x}}{a^2} + \frac{2b^2\tanh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)} + \frac{\left(4i\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, i\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{a^2\left(a^2-b^2\right)d}$$

$$= \frac{2\sqrt{x}}{a^2} - \frac{4b\left(2a^2-b^2\right)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{2b^2\tanh\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.83

$$\int \frac{1}{\sqrt{x}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{2\left(a\left(\left(a^2-b^2\right)^{3/2}\left(c+d\sqrt{x}\right)+\left(4a^2b-2b^3\right)\arctan\left(\frac{(-a+b)\tanh\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a^2-b^2}}\right)\right)\cosh\left(c+d\sqrt{x}\right)+b\left(\left(a^2-b^2\right)^{3}\right.\right.}{a^2(a-b)(a+b)\sqrt{a^2-b^2}d\left(b+\right.}$$

```
[In] Integrate[1/(Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]])^2),x]
```

```
[Out] (2*(a*((a^2 - b^2)^(3/2)*(c + d*Sqrt[x]) + (4*a^2*b - 2*b^3)*ArcTan[((-a +
b)*Tanh[(c + d*Sqrt[x])/2])/Sqrt[a^2 - b^2]])*Cosh[c + d*Sqrt[x]] + b*((a^2
 - b^2)^(3/2)*(c + d*Sqrt[x]) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c
+ d*Sqrt[x])/2])/Sqrt[a^2 - b^2]] + a*b*Sqrt[a^2 - b^2]*Sinh[c + d*Sqrt[x]]
)))/(a^2*(a - b)*(a + b)*Sqrt[a^2 - b^2]*d*(b + a*Cosh[c + d*Sqrt[x]]))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{-\frac{2\ln\left(\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)-1\right)}{a^2}-\frac{4b\left(-\frac{ab\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{\left(a^2-b^2\right)\left(\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)^2 a-\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)^2 b+a+b\right)}+\frac{\left(2a^2-b^2\right)\arctan\left(\frac{(a-b)\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right)}{a^2}}{d}$
default	$\frac{-\frac{2\ln\left(\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)-1\right)}{a^2}-\frac{4b\left(-\frac{ab\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{\left(a^2-b^2\right)\left(\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)^2 a-\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)^2 b+a+b\right)}+\frac{\left(2a^2-b^2\right)\arctan\left(\frac{(a-b)\tanh\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right)}{a^2}}{d}$

```
[In] int(1/(a+b*sech(c+d*x^(1/2)))^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/a^2*ln(tanh(1/2*c+1/2*d*x^(1/2))-1)-2/a^2*b*(-a*b/(a^2-b^2)*tanh(1/
2*c+1/2*d*x^(1/2))/(tanh(1/2*c+1/2*d*x^(1/2))^2*a-tanh(1/2*c+1/2*d*x^(1/2))
^2*b+a+b)+(2*a^2-b^2)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2
*c+1/2*d*x^(1/2))/((a+b)*(a-b))^(1/2)))+1/a^2*ln(tanh(1/2*c+1/2*d*x^(1/2))+
1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. 2(110) = 220.

Time = 0.30 (sec) , antiderivative size = 1387, normalized size of antiderivative = 10.92

$$\int \frac{1}{\sqrt{x}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*sech(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-2*(2*a^3*b^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x)*cosh(d*sqrt(
x) + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x)*sinh(d*sqrt(x) + c)^2 - (a^
5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x) + 2*(a^2*b^3 - b^5 - (a^4*b - 2*a^2*b^3 +
b^5)*d*sqrt(x))*cosh(d*sqrt(x) + c) + ((2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*c
osh(d*sqrt(x) + c)^2 + (2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*sinh(d*sqrt(x) +
c)^2 + 2*(2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2)*cosh(d*sqrt(x) + c) + 2*((2*a^3
*b - a*b^3)*sqrt(-a^2 + b^2)*cosh(d*sqrt(x) + c) + (2*a^2*b^2 - b^4)*sqrt(-
a^2 + b^2))*sinh(d*sqrt(x) + c) + (2*a^3*b - a*b^3)*sqrt(-a^2 + b^2))*log((
a*b + (b^2 + sqrt(-a^2 + b^2)*b)*cosh(d*sqrt(x) + c) + (a^2 - b^2 - sqrt(-a
^2 + b^2)*b)*sinh(d*sqrt(x) + c) + sqrt(-a^2 + b^2)*a)/(a*cosh(d*sqrt(x) +
```

```
c) + b)) + 2*(a^2*b^3 - b^5 - (a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x)*cosh(d*sq
rt(x) + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*sqrt(x))*sinh(d*sqrt(x) + c))/((a^
7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*sqrt(x) + c)^2 + (a^7 - 2*a^5*b^2 + a^3*b
^4)*d*sinh(d*sqrt(x) + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cosh(d*sqrt
(x) + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d
*cosh(d*sqrt(x) + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*sqrt(x) + c)
), -2*(2*a^3*b^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x)*cosh(d*sqr
t(x) + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x)*sinh(d*sqrt(x) + c)^2 - (
a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x) - 2*((2*a^3*b - a*b^3)*sqrt(a^2 - b^2)*c
osh(d*sqrt(x) + c)^2 + (2*a^3*b - a*b^3)*sqrt(a^2 - b^2)*sinh(d*sqrt(x) + c
)^2 + 2*(2*a^2*b^2 - b^4)*sqrt(a^2 - b^2)*cosh(d*sqrt(x) + c) + 2*((2*a^3*b
 - a*b^3)*sqrt(a^2 - b^2)*cosh(d*sqrt(x) + c) + (2*a^2*b^2 - b^4)*sqrt(a^2
- b^2))*sinh(d*sqrt(x) + c) + (2*a^3*b - a*b^3)*sqrt(a^2 - b^2))*arctan(-(s
qrt(a^2 - b^2)*a*cosh(d*sqrt(x) + c) + sqrt(a^2 - b^2)*a*sinh(d*sqrt(x) + c
) + sqrt(a^2 - b^2)*b)/(a^2 - b^2)) + 2*(a^2*b^3 - b^5 - (a^4*b - 2*a^2*b^3
 + b^5)*d*sqrt(x))*cosh(d*sqrt(x) + c) + 2*(a^2*b^3 - b^5 - (a^5 - 2*a^3*b^
2 + a*b^4)*d*sqrt(x)*cosh(d*sqrt(x) + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*sqrt
(x))*sinh(d*sqrt(x) + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*sqrt(x) + c
)^2 + (a^7 - 2*a^5*b^2 + a^3*b^4)*d*sinh(d*sqrt(x) + c)^2 + 2*(a^6*b - 2*a^
4*b^3 + a^2*b^5)*d*cosh(d*sqrt(x) + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*
((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*sqrt(x) + c) + (a^6*b - 2*a^4*b^3 + a
^2*b^5)*d)*sinh(d*sqrt(x) + c))]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\sqrt{x}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/(a+b*sech(c+d*x**(1/2)))**2/x**(1/2),x)
[Out] Integral(1/(sqrt(x)*(a + b*sech(c + d*sqrt(x)))**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*sech(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{4\left(2a^2b-b^3\right)\arctan\left(\frac{ae^{\left(d\sqrt{x}+c\right)}+b}{\sqrt{a^2-b^2}}\right)}{\left(a^4d-a^2b^2d\right)\sqrt{a^2-b^2}} - \frac{4\left(b^3e^{\left(d\sqrt{x}+c\right)}+ab^2\right)}{\left(a^4d-a^2b^2d\right)\left(ae^{\left(2d\sqrt{x}+2c\right)}+2be^{\left(d\sqrt{x}+c\right)}+a\right)} + \frac{2\left(d\sqrt{x}+c\right)}{a^2d}$$

```
[In] integrate(1/(a+b*sech(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] -4*(2*a^2*b - b^3)*arctan((a*e^(d*sqrt(x) + c) + b)/sqrt(a^2 - b^2))/((a^4*
d - a^2*b^2*d)*sqrt(a^2 - b^2)) - 4*(b^3*e^(d*sqrt(x) + c) + a*b^2)/((a^4*d
 - a^2*b^2*d)*(a*e^(2*d*sqrt(x) + 2*c) + 2*b*e^(d*sqrt(x) + c) + a)) + 2*(d
*sqrt(x) + c)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.71

$$\int \frac{1}{\sqrt{x}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{2\sqrt{x}}{a^2} - \frac{\frac{4b^2\sqrt{x}}{d\left(a^3\sqrt{x}-ab^2\sqrt{x}\right)}+\frac{4b^3\sqrt{x}\,e^{c+d\sqrt{x}}}{a\,d\left(a^3\sqrt{x}-ab^2\sqrt{x}\right)}}{a+2b\,e^{c+d\sqrt{x}}+a\,e^{2c+2d\sqrt{x}}} + \frac{\ln\left(\frac{2e^{c+d\sqrt{x}}\left(2a^2b-b^3\right)}{a^3\sqrt{x}\left(a^2-b^2\right)}-\frac{\left(4a^2b-2b^3\right)\left(a+b\,e^{c+d\sqrt{x}}\right)}{a^3\sqrt{x}\left(a+b\right)^{3/2}\left(b-a\right)^{3/2}}\right)\left(4a^2b-2b^3\right)}{a^2d\left(a+b\right)^{3/2}\left(b-a\right)^{3/2}} - \frac{2b\ln\left(\frac{2e^{c+d\sqrt{x}}\left(2a^2b-b^3\right)}{a^3\sqrt{x}\left(a^2-b^2\right)}+\frac{2b\left(a+b\,e^{c+d\sqrt{x}}\right)\left(2a^2-b^2\right)}{a^3\sqrt{x}\left(a+b\right)^{3/2}\left(b-a\right)^{3/2}}\right)\left(2a^2-b^2\right)}{a^2d\left(a+b\right)^{3/2}\left(b-a\right)^{3/2}}$$

```
[In] int(1/(x^(1/2)*(a + b/cosh(c + d*x^(1/2)))^2),x)
```

```
[Out] (2*x^(1/2))/a^2 - ((4*b^2*x^(1/2))/(d*(a^3*x^(1/2) - a*b^2*x^(1/2))) + (4*b
^3*x^(1/2)*exp(c + d*x^(1/2)))/(a*d*(a^3*x^(1/2) - a*b^2*x^(1/2))))/(a + 2*
b*exp(c + d*x^(1/2)) + a*exp(2*c + 2*d*x^(1/2))) + (log((2*exp(c + d*x^(1/2
))*(2*a^2*b - b^3))/(a^3*x^(1/2)*(a^2 - b^2)) - ((4*a^2*b - 2*b^3)*(a + b*e
xp(c + d*x^(1/2))))/(a^3*x^(1/2)*(a + b)^(3/2)*(b - a)^(3/2)))*(4*a^2*b - 2
```

```
*b^3))/(a^2*d*(a + b)^(3/2)*(b - a)^(3/2)) - (2*b*log((2*exp(c + d*x^(1/2))
*(2*a^2*b - b^3))/(a^3*x^(1/2)*(a^2 - b^2)) + (2*b*(a + b*exp(c + d*x^(1/2)
))*(2*a^2 - b^2))/(a^3*x^(1/2)*(a + b)^(3/2)*(b - a)^(3/2)))*(2*a^2 - b^2))
/(a^2*d*(a + b)^(3/2)*(b - a)^(3/2))
```

3.70 $\int \frac{1}{x^{3/2}\left(a+b\,\mathbf{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{3/2}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x^{3/2}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^{3/2}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{3/2}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Int[1/(x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])^2),x]
```

```
[Out] Defer[Int][1/(x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^{3/2}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 81.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{3/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{3/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])^2),x]
[Out] Integrate[1/(x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{3}{2}}\left(a+b\,\text{sech}\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(1/x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x)
[Out] int(1/x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{3/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\,\text{sech}\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(sqrt(x)/(b^2*x^2*sech(d*sqrt(x) + c)^2 + 2*a*b*x^2*sech(d*sqrt(x)
+ c) + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{\frac{3}{2}}\left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/x**(3/2)/(a+b*sech(c+d*x**(1/2)))**2,x)
[Out] Integral(1/(x**(3/2)*(a + b*sech(c + d*sqrt(x)))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 317, normalized size of antiderivative = 14.41

$$\int \frac{1}{x^{3/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] -2*(2*a*b^2 + (a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*sqrt(x)*e^(2*d*sqrt(x)) + 2
*(b^3*e^c + (a^2*b*d*e^c - b^3*d*e^c)*sqrt(x))*e^(d*sqrt(x)) + (a^3*d - a*b
^2*d)*sqrt(x))/((a^5*d*e^(2*c) - a^3*b^2*d*e^(2*c))*x*e^(2*d*sqrt(x)) + 2*(
a^4*b*d*e^c - a^2*b^3*d*e^c)*x*e^(d*sqrt(x)) + (a^5*d - a^3*b^2*d)*x) - int
egrate(2*(2*a*b^2*sqrt(x) + (2*b^3*sqrt(x)*e^c + (2*a^2*b*d*e^c - b^3*d*e^c
)*x)*e^(d*sqrt(x)))/((a^5*d*e^(2*c) - a^3*b^2*d*e^(2*c))*x^(5/2)*e^(2*d*sqr
t(x)) + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^(5/2)*e^(d*sqrt(x)) + (a^5*d - a^
3*b^2*d)*x^(5/2)), x)
```

Giac [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(1/((b*sech(d*sqrt(x) + c) + a)^2*x^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2} \left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{3/2}\,{\left(a+\frac{b}{\mathrm{cosh}\left(c+d\,\sqrt{x}\right)}\right)}^2} \,d x$$

```
[In] int(1/(x^(3/2)*(a + b/cosh(c + d*x^(1/2)))^2),x)
[Out] int(1/(x^(3/2)*(a + b/cosh(c + d*x^(1/2)))^2), x)
```

3.71 $\int \frac{1}{x^{5/2}\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{5/2}\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\mathrm{Int}\left(\frac{1}{x^{5/2}\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^(5/2)/(a+b*sech(c+d*x^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^{5/2}\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{5/2}\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Int[1/(x^(5/2)*(a + b*Sech[c + d*Sqrt[x]])^2),x]
```

```
[Out] Defer[Int][1/(x^(5/2)*(a + b*Sech[c + d*Sqrt[x]])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^{5/2}\left(a+b\,\mathrm{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 81.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{5/2} \left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2} \, dx=\int \frac{1}{x^{5/2} \left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2} \, dx$$

```
[In] Integrate[1/(x^(5/2)*(a + b*Sech[c + d*Sqrt[x]])^2),x]
[Out] Integrate[1/(x^(5/2)*(a + b*Sech[c + d*Sqrt[x]])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{5}{2}} \left(a+b \operatorname{sech}\left(c+d\sqrt{x}\right)\right)^{2}}dx$$

```
[In] int(1/x^(5/2)/(a+b*sech(c+d*x^(1/2)))^2,x)
[Out] int(1/x^(5/2)/(a+b*sech(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{5/2} \left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2} \, dx=\int \frac{1}{\left(b \operatorname{sech}\left(d \sqrt{x}+c\right)+a\right)^{2} x^{\frac{5}{2}}} \, dx$$

```
[In] integrate(1/x^(5/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(sqrt(x)/(b^2*x^3*sech(d*sqrt(x) + c)^2 + 2*a*b*x^3*sech(d*sqrt(x)
+ c) + a^2*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 50.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2} \left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2} \, dx=\int \frac{1}{x^{\frac{5}{2}} \left(a+b\operatorname{sech}\left(c+d\sqrt{x}\right)\right)^2} \, dx$$

```
[In] integrate(1/x**(5/2)/(a+b*sech(c+d*x**(1/2)))**2,x)
[Out] Integral(1/(x**(5/2)*(a + b*sech(c + d*sqrt(x)))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 324, normalized size of antiderivative = 14.73

$$\int \frac{1}{x^{5/2} \left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2} \, dx=\int \frac{1}{\left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)^2 x^{\frac{5}{2}}} \, dx$$

```
[In] integrate(1/x^(5/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] -2/3*(6*a*b^2 + (a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*sqrt(x)*e^(2*d*sqrt(x)) +
 2*(3*b^3*e^c + (a^2*b*d*e^c - b^3*d*e^c)*sqrt(x))*e^(d*sqrt(x)) + (a^3*d -
 a*b^2*d)*sqrt(x))/((a^5*d*e^(2*c) - a^3*b^2*d*e^(2*c))*x^2*e^(2*d*sqrt(x))
 + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^2*e^(d*sqrt(x)) + (a^5*d - a^3*b^2*d)*
x^2) - integrate(2*(4*a*b^2*sqrt(x) + (4*b^3*sqrt(x)*e^c + (2*a^2*b*d*e^c -
 b^3*d*e^c)*x)*e^(d*sqrt(x)))/((a^5*d*e^(2*c) - a^3*b^2*d*e^(2*c))*x^(7/2)*
e^(2*d*sqrt(x)) + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^(7/2)*e^(d*sqrt(x)) + (
a^5*d - a^3*b^2*d)*x^(7/2)), x)
```

Giac [N/A]

Not integrable

Time = 3.09 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^{5/2} \left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2} \, dx=\int \frac{1}{\left(b\operatorname{sech}\left(d\sqrt{x}+c\right)+a\right)^2 x^{\frac{5}{2}}} \, dx$$

```
[In] integrate(1/x^(5/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 2.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2}\left(a+b\text{sech}\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{5/2}\,\left(a+\frac{b}{\mathrm{cosh}\left(c+d\,\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(1/(x^(5/2)*(a + b/cosh(c + d*x^(1/2)))^2),x)
[Out] int(1/(x^(5/2)*(a + b/cosh(c + d*x^(1/2)))^2), x)
```

3.72 $\int (ex)^m (a+b\mathbf{sech}(c+dx^n))^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m (a+b\operatorname{sech}(c+dx^n))^p \, dx = x^{-m}(ex)^m \operatorname{Int}(x^m(a+b\operatorname{sech}(c+dx^n))^p, x)$$

```
[Out] (e*x)^m*Unintegrable(x^m*(a+b*sech(c+d*x^n))^p,x)/(x^m)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (a+b\operatorname{sech}(c+dx^n))^p \, dx = \int (ex)^m (a+b\operatorname{sech}(c+dx^n))^p \, dx$$

```
[In] Int[(e*x)^m*(a + b*Sech[c + d*x^n])^p,x]
[Out] ((e*x)^m*Defer[Int][x^m*(a + b*Sech[c + d*x^n])^p, x])/x^m
```

Rubi steps

$$\text{integral} = \left(x^{-m}(ex)^m\right) \int x^m(a+b\operatorname{sech}(c+dx^n))^p \, dx$$

Mathematica [N/A]

Not integrable

Time = 20.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\text{sech}(c+dx^n))^p\,dx = \int (ex)^m (a+b\text{sech}(c+dx^n))^p\,dx$$

```
[In] Integrate[(e*x)^m*(a + b*Sech[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^m*(a + b*Sech[c + d*x^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (ex)^m (a+b\,\text{sech}\,(c+d\,x^n))^p dx$$

```
[In] int((e*x)^m*(a+b*sech(c+d*x^n))^p,x)
[Out] int((e*x)^m*(a+b*sech(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a+b\text{sech}(c+dx^n))^p\,dx = \int (ex)^m (b\,\text{sech}\,(dx^n+c)+a)^p\,dx$$

```
[In] integrate((e*x)^m*(a+b*sech(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*(b*sech(d*x^n + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 44.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m (a+b\text{sech}(c+dx^n))^p\,dx = \int (ex)^m (a+b\,\text{sech}\,(c+dx^n))^p\,dx$$

```
[In] integrate((e*x)**m*(a+b*sech(c+d*x**n))**p,x)
[Out] Integral((e*x)**m*(a + b*sech(c + d*x**n))**p, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\text{sech}(c + dx^n))^p \, dx = \int (ex)^m (b\,\text{sech}\,(dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*sech(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*sech(d*x^n + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\text{sech}(c + dx^n))^p \, dx = \int (ex)^m (b\,\text{sech}\,(dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*sech(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*sech(d*x^n + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (ex)^m (a + b\text{sech}(c + dx^n))^p \, dx = \int \left(a + \frac{b}{\cosh\left(c + d\,x^n\right)}\right)^p (e\,x)^m \, dx$$

```
[In] int((a + b/cosh(c + d*x^n))^p*(e*x)^m,x)
[Out] int((a + b/cosh(c + d*x^n))^p*(e*x)^m, x)
```

3.73 $\int (ex)^{-1+n} (a + b\text{sech}(c + dx^n)) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int (ex)^{-1+n} (a + b\text{sech}(c + dx^n)) \, dx = \frac{a(ex)^n}{en} + \frac{bx^{-n}(ex)^n \arctan(\sinh(c + dx^n))}{den}$$

```
[Out] a*(e*x)^n/e/n+b*(e*x)^n*arctan(sinh(c+d*x^n))/d/e/n/(x^n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {14, 5548, 5544, 3855}

$$\int (ex)^{-1+n} (a + b\text{sech}(c + dx^n)) \, dx = \frac{a(ex)^n}{en} + \frac{bx^{-n}(ex)^n \arctan(\sinh(c + dx^n))}{den}$$

```
[In] Int[(e*x)^(-1 + n)*(a + b*Sech[c + d*x^n]),x]
```

```
[Out] (a*(e*x)^n)/(e*n) + (b*(e*x)^n*ArcTan[Sinh[c + d*x^n]])/(d*e*n*x^n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5548

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a(ex)^{-1+n} + b(ex)^{-1+n}\text{sech}(c+dx^n)\right)\,dx \\
&= \frac{a(ex)^n}{en} + b\int (ex)^{-1+n}\text{sech}(c+dx^n)\,dx \\
&= \frac{a(ex)^n}{en} + \frac{(bx^{-n}(ex)^n)\int x^{-1+n}\text{sech}(c+dx^n)\,dx}{e} \\
&= \frac{a(ex)^n}{en} + \frac{(bx^{-n}(ex)^n)\,\text{Subst}\left(\int \text{sech}(c+dx)\,dx, x, x^n\right)}{en} \\
&= \frac{a(ex)^n}{en} + \frac{bx^{-n}(ex)^n\arctan\left(\sinh\left(c+dx^n\right)\right)}{den}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int (ex)^{-1+n}\left(a+b\text{sech}(c+dx^n)\right)\,dx = \frac{x^{-n}(ex)^n\left(a(c+dx^n)+b\arctan\left(\sinh\left(c+dx^n\right)\right)\right)}{den}$$

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Sech[c + d*x^n]),x]
[Out] ((e*x)^n*(a*(c + d*x^n) + b*ArcTan[Sinh[c + d*x^n]]))/(d*e*n*x^n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.72 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.52

method	result
risch	$\frac{ax\,\mathrm{e}^{\frac{(-1+n)\left(-i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)\pi+i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^2\pi+i\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^2\pi-i\,\mathrm{csgn}(iex)^3\pi+2\ln(e)+2\ln(x)\right)}{2}}}{n}+\frac{2\arctan\left(\mathrm{e}^{c+d\,x}\right.}{}$

```
[In] int((e*x)^(-1+n)*(a+b*sech(c+d*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] a/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*csg
n(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*ln(
x)))+2*arctan(exp(c+d*x^n))/d/e*e^n/n*b*exp(1/2*I*Pi*csgn(I*e*x)*(-1+n)*(cs
gn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(44) = 88.

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.77

$$\int (ex)^{-1+n}\left(a+b\mathrm{sech}(c+dx^n)\right)\,dx$$
$$=\frac{ad\cosh\left((n-1)\log\left(e\right)\right)\cosh\left(n\log\left(x\right)\right)+ad\cosh\left(n\log\left(x\right)\right)\sinh\left((n-1)\log\left(e\right)\right)+2\left(b\cosh\left((n-1)\log\right.\right.}{}$$

```
[In] integrate((e*x)^(-1+n)*(a+b*sech(c+d*x^n)),x, algorithm="fricas")
```

```
[Out] (a*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + a*d*cosh(n*log(x))*sinh((n - 1)*
log(e)) + 2*(b*cosh((n - 1)*log(e)) + b*sinh((n - 1)*log(e)))*arctan(cosh(d
*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*
log(x)) + c)) + (a*d*cosh((n - 1)*log(e)) + a*d*sinh((n - 1)*log(e)))*sinh(
n*log(x)))/(d*n)
```

Sympy [F]

$$\int (ex)^{-1+n}\left(a+b\mathrm{sech}(c+dx^n)\right)\,dx=\int (ex)^{n-1}\left(a+b\,\mathrm{sech}\left(c+dx^n\right)\right)\,dx$$

```
[In] integrate((e*x)**(-1+n)*(a+b*sech(c+d*x**n)),x)
```

```
[Out] Integral((e*x)**(n - 1)*(a + b*sech(c + d*x**n)), x)
```

Maxima [F]

$$\int (ex)^{-1+n} (a + b\mathrm{sech}(c + dx^n)) \, dx = \int (b\,\mathrm{sech}\,(dx^n + c) + a)(ex)^{n-1} \, dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*sech(c+d*x^n)),x, algorithm="maxima")
```

```
[Out] 2*b*integrate((e*x)^(n - 1)/(e^(d*x^n + c) + e^(-d*x^n - c)), x) + (e*x)^n*
a/(e*n)
```

Giac [F]

$$\int (ex)^{-1+n} (a + b\mathrm{sech}(c + dx^n)) \, dx = \int (b\,\mathrm{sech}\,(dx^n + c) + a)(ex)^{n-1} \, dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*sech(c+d*x^n)),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x^n + c) + a)*(e*x)^(n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.50

$$\int (ex)^{-1+n} (a + b\mathrm{sech}(c + dx^n)) \, dx = \frac{2\,\mathrm{atan}\left(\frac{b\,x\,\mathrm{e}^{d\,x^n}\,\mathrm{e}^c\,{(e\,x)}^{n-1}\,\sqrt{d^2\,n^2\,x^{2\,n}}}{d\,n\,x^n\,\sqrt{b^2\,x^2\,{(e\,x)}^{2\,n-2}}}\right)\,\sqrt{b^2\,x^2\,{(e\,x)}^{2\,n-2}}}{\sqrt{d^2\,n^2\,x^{2\,n}}} + \frac{a\,x\,{(e\,x)}^{n-1}}{n}$$

```
[In] int((a + b/cosh(c + d*x^n))*(e*x)^(n - 1),x)
```

```
[Out] (2*atan((b*x*exp(d*x^n)*exp(c)*(e*x)^(n - 1)*(d^2*n^2*x^(2*n))^(1/2))/(d*n*
x^n*(b^2*x^2*(e*x)^(2*n - 2))^(1/2)))*(b^2*x^2*(e*x)^(2*n - 2))^(1/2))/(d^2
*n^2*x^(2*n))^(1/2) + (a*x*(e*x)^(n - 1))/n
```

3.74 $\int (ex)^{-1+2n}(a+b\text{sech}(c+dx^n))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int (ex)^{-1+2n}(a+b\text{sech}(c+dx^n))\,dx=\frac{a(ex)^{2n}}{2en}+\frac{2bx^{-n}(ex)^{2n}\arctan\left(e^{c+dx^n}\right)}{den}-\frac{ibx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-ie^{c+dx^n}\right)}{d^2en}+\frac{ibx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,ie^{c+dx^n}\right)}{d^2en}$$

```
[Out] 1/2*a*(e*x)^(2*n)/e/n+2*b*(e*x)^(2*n)*arctan(exp(c+d*x^n))/d/e/n/(x^n)-I*b*
(e*x)^(2*n)*polylog(2,-I*exp(c+d*x^n))/d^2/e/n/(x^(2*n))+I*b*(e*x)^(2*n)*po
lylog(2,I*exp(c+d*x^n))/d^2/e/n/(x^(2*n))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {14, 5548, 5544, 4265, 2317, 2438}

$$\int (ex)^{-1+2n}(a+b\text{sech}(c+dx^n))\,dx=\frac{a(ex)^{2n}}{2en}+\frac{2bx^{-n}(ex)^{2n}\arctan\left(e^{c+dx^n}\right)}{den}-\frac{ibx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-ie^{dx^n+c}\right)}{d^2en}+\frac{ibx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,ie^{dx^n+c}\right)}{d^2en}$$

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Sech[c + d*x^n]),x]
```

```
[Out] (a*(e*x)^(2*n))/(2*e*n) + (2*b*(e*x)^(2*n)*ArcTan[E^(c + d*x^n)])/(d*e*n*x^
n) - (I*b*(e*x)^(2*n)*PolyLog[2, (-I)*E^(c + d*x^n)])/(d^2*e*n*x^(2*n)) + (
I*b*(e*x)^(2*n)*PolyLog[2, I*E^(c + d*x^n)])/(d^2*e*n*x^(2*n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5548

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \int \left(a(ex)^{-1+2n} + b(ex)^{-1+2n}\text{sech}(c+dx^n)\right)\,dx$$

$$
\begin{aligned}
&= \frac{a(ex)^{2n}}{2en} + b\int (ex)^{-1+2n}\operatorname{sech}(c+dx^n)\,dx \\
&= \frac{a(ex)^{2n}}{2en} + \frac{(bx^{-2n}(ex)^{2n})\int x^{-1+2n}\operatorname{sech}(c+dx^n)\,dx}{e} \\
&= \frac{a(ex)^{2n}}{2en} + \frac{(bx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int x\operatorname{sech}(c+dx)\,dx,x,x^n\right)}{en} \\
&= \frac{a(ex)^{2n}}{2en} + \frac{2bx^{-n}(ex)^{2n}\arctan\left(e^{c+dx^n}\right)}{den} \\
&\quad - \frac{(ibx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int \log\left(1-ie^{c+dx}\right)\,dx,x,x^n\right)}{den} \\
&\quad + \frac{(ibx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int \log\left(1+ie^{c+dx}\right)\,dx,x,x^n\right)}{den} \\
&= \frac{a(ex)^{2n}}{2en} + \frac{2bx^{-n}(ex)^{2n}\arctan\left(e^{c+dx^n}\right)}{den} \\
&\quad - \frac{(ibx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{c+dx^n}\right)}{d^2en} \\
&\quad + \frac{(ibx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,x,e^{c+dx^n}\right)}{d^2en} \\
&= \frac{a(ex)^{2n}}{2en} + \frac{2bx^{-n}(ex)^{2n}\arctan\left(e^{c+dx^n}\right)}{den} \\
&\quad - \frac{ibx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-ie^{c+dx^n}\right)}{d^2en} + \frac{ibx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,ie^{c+dx^n}\right)}{d^2en}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.93

$$
\int (ex)^{-1+2n}\left(a+b\operatorname{sech}(c+dx^n)\right)\,dx
= \frac{x^{-2n}(ex)^{2n}\left(ad^2x^{2n}+2ibc\log\left(1-ie^{c+dx^n}\right)-b\pi\log\left(1-ie^{c+dx^n}\right)+2ibdx^n\log\left(1-ie^{c+dx^n}\right)-2ibc\log\left(1\right.\right.}{}
$$

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sech[c + d*x^n]),x]
```

```
[Out] ((e*x)^(2*n)*(a*d^2*x^(2*n) + (2*I)*b*c*Log[1 - I*E^(c + d*x^n)] - b*Pi*Log
[1 - I*E^(c + d*x^n)] + (2*I)*b*d*x^n*Log[1 - I*E^(c + d*x^n)] - (2*I)*b*c*
Log[1 + I*E^(c + d*x^n)] + b*Pi*Log[1 + I*E^(c + d*x^n)] - (2*I)*b*d*x^n*Lo
g[1 + I*E^(c + d*x^n)] - (2*I)*b*c*Log[Cot[((2*I)*c + Pi + (2*I)*d*x^n)/4]]
 + b*Pi*Log[Cot[((2*I)*c + Pi + (2*I)*d*x^n)/4]] - (2*I)*b*PolyLog[2, (-I)*
E^(c + d*x^n)] + (2*I)*b*PolyLog[2, I*E^(c + d*x^n)]))/(2*d^2*e*n*x^(2*n))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.70 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.73

method	result
risch	$\frac{a x \,\mathrm{e}^{\frac{(2n-1)\left(-i \,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)\pi+i \,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^{2}\pi+i \,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^{2}\pi-i \,\mathrm{csgn}(iex)^{3}\pi+2\ln(e)+2\ln(x)\right)}{2}}}{2n}+\frac{2b\,\mathrm{e}^{-i\pi n \,\mathrm{csgn}(ie)\,\mathrm{c}}}{}$

[In] int((e*x)^(2*n-1)*(a+b*sech(c+d*x^n)),x,method=_RETURNVERBOSE)

[Out] 1/2*a/n*x*exp(1/2*(2*n-1)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*ln(x)))+2*b*exp(-I*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(I*Pi*n*csgn(I*e)*csgn(I*e*x)^2)*exp(I*Pi*n*csgn(I*x)*csgn(I*e*x)^2)*exp(-I*Pi*n*csgn(I*e*x)^3)*exp(1/2*I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(-1/2*I*Pi*csgn(I*e)*csgn(I*e*x)^2)*exp(-1/2*I*Pi*csgn(I*x)*csgn(I*e*x)^2)*exp(1/2*I*Pi*csgn(I*e*x)^3)*(e^n)^2/e*exp(c)/n/d^2*(-1/2*(-exp(2*c))^(1/2)*x^n*d*(ln(1+exp(d*x^n)*(-exp(2*c))^(1/2))-ln(1-exp(d*x^n)*(-exp(2*c))^(1/2)))*exp(-2*c)-1/2*(-exp(2*c))^(1/2)*(dilog(1+exp(d*x^n)*(-exp(2*c))^(1/2))-dilog(1-exp(d*x^n)*(-exp(2*c))^(1/2)))*exp(-2*c))

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 664 vs. 2(124) = 248.

Time = 0.28 (sec) , antiderivative size = 664, normalized size of antiderivative = 4.92

$$\int (ex)^{-1+2n}\left(a+b\operatorname{sech}(c+dx^n)\right)\,dx=\text{Too large to display}$$

[In] integrate((e*x)^(-1+2*n)*(a+b*sech(c+d*x^n)),x, algorithm="fricas")

[Out] 1/2*(a*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + a*d^2*cosh(n*log(x))^2*sinh((2*n - 1)*log(e)) + (a*d^2*cosh((2*n - 1)*log(e)) + a*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x))^2 - 2*(-I*b*cosh((2*n - 1)*log(e)) - I*b*sinh((2*n - 1)*log(e)))*dilog(I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) - 2*(I*b*cosh((2*n - 1)*log(e)) + I*b*sinh((2*n - 1)*log(e)))*dilog(-I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) - 2*(I*b*c*cosh((2*n - 1)*log(e)) + I*b*c*sinh((2*n - 1)*log(e)))*log(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + I) - 2*(-I*b*c*cosh((2*n - 1)*log(e)) - I*b*c*sinh((2*n - 1)*log(e)))*log(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x))

```
 + d*sinh(n*log(x)) + c) - I) - 2*(I*b*d*cosh((2*n - 1)*log(e))*cosh(n*log(
x)) + I*b*c*cosh((2*n - 1)*log(e)) + (I*b*d*cosh(n*log(x)) + I*b*c)*sinh((2
*n - 1)*log(e)) + (I*b*d*cosh((2*n - 1)*log(e)) + I*b*d*sinh((2*n - 1)*log(
e)))*sinh(n*log(x)))*log(I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) +
I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 1) - 2*(-I*b*d*cosh((2*n
- 1)*log(e))*cosh(n*log(x)) - I*b*c*cosh((2*n - 1)*log(e)) + (-I*b*d*cosh(n
*log(x)) - I*b*c)*sinh((2*n - 1)*log(e)) + (-I*b*d*cosh((2*n - 1)*log(e)) -
 I*b*d*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*log(-I*cosh(d*cosh(n*log(x))
 + d*sinh(n*log(x)) + c) - I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)
+ 1) + 2*(a*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + a*d^2*cosh(n*log(x)
)*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))/(d^2*n)
```

Sympy [F]

$$\int (ex)^{-1+2n}(a+b\text{sech}(c+dx^n))\,dx=\int (ex)^{2n-1}\left(a+b\operatorname{sech}\left(c+dx^n\right)\right)dx$$

```
[In] integrate((e*x)**(-1+2*n)*(a+b*sech(c+d*x**n)),x)
[Out] Integral((e*x)**(2*n - 1)*(a + b*sech(c + d*x**n)), x)
```

Maxima [F]

$$\int (ex)^{-1+2n}(a+b\text{sech}(c+dx^n))\,dx=\int (b\operatorname{sech}\left(dx^n+c\right)+a)(ex)^{2n-1}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sech(c+d*x^n)),x, algorithm="maxima")
[Out] 2*b*integrate((e*x)^(2*n - 1)/(e^(d*x^n + c) + e^(-d*x^n - c)), x) + 1/2*(e
*x)^(2*n)*a/(e*n)
```

Giac [F]

$$\int (ex)^{-1+2n}(a+b\text{sech}(c+dx^n))\,dx=\int (b\operatorname{sech}\left(dx^n+c\right)+a)(ex)^{2n-1}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sech(c+d*x^n)),x, algorithm="giac")
[Out] integrate((b*sech(d*x^n + c) + a)*(e*x)^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}(a+b\operatorname{sech}(c+dx^n))\,dx=\int \left(a+\frac{b}{\cosh\left(c+d\,x^n\right)}\right)\,{\left(e\,x\right)}^{2\,n-1}\,dx$$

```
[In] int((a + b/cosh(c + d*x^n))*(e*x)^(2*n - 1),x)
[Out] int((a + b/cosh(c + d*x^n))*(e*x)^(2*n - 1), x)
```

3.75 $\int (ex)^{-1+3n}(a+b\,\mathrm{sech}(c+dx^n))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 217

$$\int (ex)^{-1+3n}(a+b\,\mathrm{sech}(c+dx^n))\,dx=\frac{a(ex)^{3n}}{3en}+\frac{2bx^{-n}(ex)^{3n}\arctan\left(e^{c+dx^n}\right)}{den}-\frac{2ibx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-ie^{c+dx^n}\right)}{d^2en}+\frac{2ibx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,ie^{c+dx^n}\right)}{d^2en}+\frac{2ibx^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,-ie^{c+dx^n}\right)}{d^3en}-\frac{2ibx^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,ie^{c+dx^n}\right)}{d^3en}$$

```
[Out] 1/3*a*(e*x)^(3*n)/e/n+2*b*(e*x)^(3*n)*arctan(exp(c+d*x^n))/d/e/n/(x^n)-2*I*
b*(e*x)^(3*n)*polylog(2,-I*exp(c+d*x^n))/d^2/e/n/(x^(2*n))+2*I*b*(e*x)^(3*n
)*polylog(2,I*exp(c+d*x^n))/d^2/e/n/(x^(2*n))+2*I*b*(e*x)^(3*n)*polylog(3,-
I*exp(c+d*x^n))/d^3/e/n/(x^(3*n))-2*I*b*(e*x)^(3*n)*polylog(3,I*exp(c+d*x^n
))/d^3/e/n/(x^(3*n))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used

$= \{14, 5548, 5544, 4265, 2611, 2320, 6724\}$

$$\int (ex)^{-1+3n}\left(a+b\operatorname{sech}(c+dx^n)\right)\,dx = \frac{a(ex)^{3n}}{3en} + \frac{2bx^{-n}(ex)^{3n}\arctan\left(e^{c+dx^n}\right)}{den} + \frac{2ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-ie^{dx^n+c}\right)}{d^3en} - \frac{2ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,ie^{dx^n+c}\right)}{d^3en} - \frac{2ibx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-ie^{dx^n+c}\right)}{d^2en} + \frac{2ibx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,ie^{dx^n+c}\right)}{d^2en}$$

```
[In] Int[(e*x)^(-1 + 3*n)*(a + b*Sech[c + d*x^n]),x]
```

```
[Out] (a*(e*x)^(3*n))/(3*e*n) + (2*b*(e*x)^(3*n)*ArcTan[E^(c + d*x^n)])/(d*e*n*x^
n) - ((2*I)*b*(e*x)^(3*n)*PolyLog[2, (-I)*E^(c + d*x^n)])/(d^2*e*n*x^(2*n))
 + ((2*I)*b*(e*x)^(3*n)*PolyLog[2, I*E^(c + d*x^n)])/(d^2*e*n*x^(2*n)) + ((
2*I)*b*(e*x)^(3*n)*PolyLog[3, (-I)*E^(c + d*x^n)])/(d^3*e*n*x^(3*n)) - ((2*
I)*b*(e*x)^(3*n)*PolyLog[3, I*E^(c + d*x^n)])/(d^3*e*n*x^(3*n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5548

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a(ex)^{-1+3n}+b(ex)^{-1+3n}\operatorname{sech}(c+dx^n)\right)\,dx\\
&= \frac{a(ex)^{3n}}{3en}+b\int (ex)^{-1+3n}\operatorname{sech}(c+dx^n)\,dx\\
&= \frac{a(ex)^{3n}}{3en}+\frac{\left(bx^{-3n}(ex)^{3n}\right)\int x^{-1+3n}\operatorname{sech}(c+dx^n)\,dx}{e}\\
&= \frac{a(ex)^{3n}}{3en}+\frac{\left(bx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x^2\operatorname{sech}(c+dx)\,dx,x,x^n\right)}{en}\\
&= \frac{a(ex)^{3n}}{3en}+\frac{2bx^{-n}(ex)^{3n}\arctan\left(e^{c+dx^n}\right)}{den}\\
&\quad-\frac{\left(2ibx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x\log\left(1-ie^{c+dx}\right)\,dx,x,x^n\right)}{den}\\
&\quad+\frac{\left(2ibx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x\log\left(1+ie^{c+dx}\right)\,dx,x,x^n\right)}{den}
\end{aligned}
$$

$$= \frac{a(ex)^{3n}}{3en} + \frac{2bx^{-n}(ex)^{3n} \arctan\left(e^{c+dx^n}\right)}{den} - \frac{2ibx^{-2n}(ex)^{3n} \operatorname{PolyLog}\left(2, -ie^{c+dx^n}\right)}{d^2en} + \frac{2ibx^{-2n}(ex)^{3n} \operatorname{PolyLog}\left(2, ie^{c+dx^n}\right)}{d^2en} + \frac{(2ibx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2, -ie^{c+dx}\right) dx, x, x^n\right)}{d^2en} - \frac{(2ibx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2, ie^{c+dx}\right) dx, x, x^n\right)}{d^2en}$$

$$= \frac{a(ex)^{3n}}{3en} + \frac{2bx^{-n}(ex)^{3n} \arctan\left(e^{c+dx^n}\right)}{den} - \frac{2ibx^{-2n}(ex)^{3n} \operatorname{PolyLog}\left(2, -ie^{c+dx^n}\right)}{d^2en} + \frac{2ibx^{-2n}(ex)^{3n} \operatorname{PolyLog}\left(2, ie^{c+dx^n}\right)}{d^2en} + \frac{(2ibx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-ix)}{x} dx, x, e^{c+dx^n}\right)}{d^3en} - \frac{(2ibx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,ix)}{x} dx, x, e^{c+dx^n}\right)}{d^3en}$$

$$= \frac{a(ex)^{3n}}{3en} + \frac{2bx^{-n}(ex)^{3n} \arctan\left(e^{c+dx^n}\right)}{den} - \frac{2ibx^{-2n}(ex)^{3n} \operatorname{PolyLog}\left(2, -ie^{c+dx^n}\right)}{d^2en} + \frac{2ibx^{-2n}(ex)^{3n} \operatorname{PolyLog}\left(2, ie^{c+dx^n}\right)}{d^2en} + \frac{2ibx^{-3n}(ex)^{3n} \operatorname{PolyLog}\left(3, -ie^{c+dx^n}\right)}{d^3en} - \frac{2ibx^{-3n}(ex)^{3n} \operatorname{PolyLog}\left(3, ie^{c+dx^n}\right)}{d^3en}$$

Mathematica [F]

$$\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n)) \, dx = \int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n)) \, dx$$

```
[In] Integrate[(e*x)^(-1 + 3*n)*(a + b*Sech[c + d*x^n]),x]
[Out] Integrate[(e*x)^(-1 + 3*n)*(a + b*Sech[c + d*x^n]), x]
```

Maple [F]

$$\int (ex)^{-1+3n} (a + b \operatorname{sech}(c + d x^n)) \, dx$$

[In] int((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n)),x)

[Out] int((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n)),x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1082 vs. 2(200) = 400.

Time = 0.30 (sec) , antiderivative size = 1082, normalized size of antiderivative = 4.99

$$\int (ex)^{-1+3n} (a + b\mathrm{sech}(c + dx^n)) \, dx = \text{Too large to display}$$

[In] integrate((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n)),x, algorithm="fricas")

```
[Out] 1/3*(a*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x))^3 + a*d^3*cosh(n*log(x))^3
*sinh((3*n - 1)*log(e)) + (a*d^3*cosh((3*n - 1)*log(e)) + a*d^3*sinh((3*n -
 1)*log(e)))*sinh(n*log(x))^3 + 3*(a*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(
x)) + a*d^3*cosh(n*log(x))*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 - 6*(-I
*b*d*cosh((3*n - 1)*log(e))*cosh(n*log(x)) - I*b*d*cosh(n*log(x))*sinh((3*n
 - 1)*log(e)) + (-I*b*d*cosh((3*n - 1)*log(e)) - I*b*d*sinh((3*n - 1)*log(e
)))*sinh(n*log(x)))*dilog(I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) +
 I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) - 6*(I*b*d*cosh((3*n - 1)
*log(e))*cosh(n*log(x)) + I*b*d*cosh(n*log(x))*sinh((3*n - 1)*log(e)) + (I*
b*d*cosh((3*n - 1)*log(e)) + I*b*d*sinh((3*n - 1)*log(e)))*sinh(n*log(x)))*
dilog(-I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - I*sinh(d*cosh(n*lo
g(x)) + d*sinh(n*log(x)) + c)) - 3*(-I*b*c^2*cosh((3*n - 1)*log(e)) - I*b*c
^2*sinh((3*n - 1)*log(e)))*log(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c
) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + I) - 3*(I*b*c^2*cosh((3
*n - 1)*log(e)) + I*b*c^2*sinh((3*n - 1)*log(e)))*log(cosh(d*cosh(n*log(x))
 + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) -
I) - 3*(I*b*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x))^2 - I*b*c^2*cosh((3*n
 - 1)*log(e)) + (I*b*d^2*cosh((3*n - 1)*log(e)) + I*b*d^2*sinh((3*n - 1)*lo
g(e)))*sinh(n*log(x))^2 + (I*b*d^2*cosh(n*log(x))^2 - I*b*c^2)*sinh((3*n -
1)*log(e)) + 2*(I*b*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + I*b*d^2*cos
h(n*log(x))*sinh((3*n - 1)*log(e)))*sinh(n*log(x)))*log(I*cosh(d*cosh(n*log
(x)) + d*sinh(n*log(x)) + c) + I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) +
 c) + 1) - 3*(-I*b*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x))^2 + I*b*c^2*co
sh((3*n - 1)*log(e)) + (-I*b*d^2*cosh((3*n - 1)*log(e)) - I*b*d^2*sinh((3*n
 - 1)*log(e)))*sinh(n*log(x))^2 + (-I*b*d^2*cosh(n*log(x))^2 + I*b*c^2)*sin
h((3*n - 1)*log(e)) + 2*(-I*b*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x)) - I
```

```
*b*d^2*cosh(n*log(x))*sinh((3*n - 1)*log(e)))*sinh(n*log(x)))*log(-I*cosh(d
*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - I*sinh(d*cosh(n*log(x)) + d*sinh(
n*log(x)) + c) + 1) - 6*(I*b*cosh((3*n - 1)*log(e)) + I*b*sinh((3*n - 1)*lo
g(e)))*polylog(3, I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + I*sinh(
d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) - 6*(-I*b*cosh((3*n - 1)*log(e))
- I*b*sinh((3*n - 1)*log(e)))*polylog(3, -I*cosh(d*cosh(n*log(x)) + d*sinh(
n*log(x)) + c) - I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) + 3*(a*d^
3*cosh((3*n - 1)*log(e))*cosh(n*log(x))^2 + a*d^3*cosh(n*log(x))^2*sinh((3*
n - 1)*log(e)))*sinh(n*log(x)))/(d^3*n)
```

Sympy [F]

$$\int (ex)^{-1+3n}(a+b\text{sech}(c+dx^n))\,dx=\int (ex)^{3n-1}\left(a+b\operatorname{sech}\left(c+dx^n\right)\right)\,dx$$

```
[In] integrate((e*x)**(-1+3*n)*(a+b*sech(c+d*x**n)),x)
[Out] Integral((e*x)**(3*n - 1)*(a + b*sech(c + d*x**n)), x)
```

Maxima [F]

$$\int (ex)^{-1+3n}(a+b\text{sech}(c+dx^n))\,dx=\int (b\operatorname{sech}\left(dx^n+c\right)+a)(ex)^{3n-1}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n)),x, algorithm="maxima")
[Out] 2*b*integrate((e*x)^(3*n - 1)/(e^(d*x^n + c) + e^(-d*x^n - c)), x) + 1/3*(e
*x)^(3*n)*a/(e*n)
```

Giac [F]

$$\int (ex)^{-1+3n}(a+b\text{sech}(c+dx^n))\,dx=\int (b\operatorname{sech}\left(dx^n+c\right)+a)(ex)^{3n-1}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n)),x, algorithm="giac")
[Out] integrate((b*sech(d*x^n + c) + a)*(e*x)^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n}(a + b\operatorname{sech}(c + dx^n))\,dx = \int \left(a + \frac{b}{\cosh(c + d\,x^n)}\right)(e\,x)^{3\,n-1}\,dx$$

```
[In] int((a + b/cosh(c + d*x^n))*(e*x)^(3*n - 1),x)
[Out] int((a + b/cosh(c + d*x^n))*(e*x)^(3*n - 1), x)
```

3.76 $\int (ex)^{-1+n} (a + b\text{sech}(c + dx^n))^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 79

$$\int (ex)^{-1+n} (a + b\text{sech}(c + dx^n))^2 \, dx = \frac{a^2(ex)^n}{en} + \frac{2abx^{-n}(ex)^n \arctan(\sinh(c + dx^n))}{den} + \frac{b^2x^{-n}(ex)^n \tanh(c + dx^n)}{den}$$

```
[Out] a^2*(e*x)^n/e/n+2*a*b*(e*x)^n*arctan(sinh(c+d*x^n))/d/e/n/(x^n)+b^2*(e*x)^n
*tanh(c+d*x^n)/d/e/n/(x^n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5548, 5544, 3858, 3855, 3852, 8}

$$\int (ex)^{-1+n} (a + b\text{sech}(c + dx^n))^2 \, dx = \frac{a^2(ex)^n}{en} + \frac{2abx^{-n}(ex)^n \arctan(\sinh(c + dx^n))}{den} + \frac{b^2x^{-n}(ex)^n \tanh(c + dx^n)}{den}$$

```
[In] Int[(e*x)^(-1 + n)*(a + b*Sech[c + d*x^n])^2,x]
```

```
[Out] (a^2*(e*x)^n)/(e*n) + (2*a*b*(e*x)^n*ArcTan[Sinh[c + d*x^n]])/(d*e*n*x^n) +
 (b^2*(e*x)^n*Tanh[c + d*x^n])/(d*e*n*x^n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5548

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(x^{-n}(ex)^n)\int x^{-1+n}(a+b\operatorname{sech}(c+dx^n))^2\,dx}{e}\\
&= \frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int (a+b\operatorname{sech}(c+dx))^2\,dx,x,x^n\right)}{en}\\
&= \frac{a^2(ex)^n}{en}+\frac{(2abx^{-n}(ex)^n)\,\text{Subst}\left(\int \operatorname{sech}(c+dx)\,dx,x,x^n\right)}{en}\\
&\quad+\frac{(b^2x^{-n}(ex)^n)\,\text{Subst}\left(\int \operatorname{sech}^2(c+dx)\,dx,x,x^n\right)}{en}\\
&= \frac{a^2(ex)^n}{en}+\frac{2abx^{-n}(ex)^n\arctan\left(\sinh\left(c+dx^n\right)\right)}{den}\\
&\quad+\frac{(ib^2x^{-n}(ex)^n)\,\text{Subst}\left(\int 1\,dx,x,-i\tanh\left(c+dx^n\right)\right)}{den}\\
&= \frac{a^2(ex)^n}{en}+\frac{2abx^{-n}(ex)^n\arctan\left(\sinh\left(c+dx^n\right)\right)}{den}+\frac{b^2x^{-n}(ex)^n\tanh\left(c+dx^n\right)}{den}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$\int (ex)^{-1+n}\left(a+b\mathrm{sech}(c+dx^n)\right)^2\,dx = \frac{x^{-n}(ex)^n\left(a(a(c+dx^n)+2b\arctan\left(\sinh\left(c+dx^n\right)\right))+b^2\tanh\left(c+dx^n\right)\right)}{den}$$

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Sech[c + d*x^n])^2,x]
[Out] ((e*x)^n*(a*(a*(c + d*x^n) + 2*b*ArcTan[Sinh[c + d*x^n]]) + b^2*Tanh[c + d*
x^n]))/(d*e*n*x^n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 17.01 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.43

method	result
risch	$\frac{a^2 x\,\mathrm{e}^{\frac{(-1+n)\left(-i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)\pi+i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^2\pi+i\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^2\pi-i\,\mathrm{csgn}(iex)^3\pi+2\ln(e)+2\ln(x)\right)}{2}}}{n} - 2x\,x^{-n}b^2\mathrm{e}^{(-1+}$

```
[In] int((e*x)^(-1+n)*(a+b*sech(c+d*x^n))^2,x,method=_RETURNVERBOSE)
[Out] a^2/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*c
sgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*l
n(x)))-2/d/n*x/(x^n)*b^2*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)
*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)
^3*Pi+2*ln(e)+2*ln(x)))/(1+exp(2*c+2*d*x^n))+4*arctan(exp(c+d*x^n))/d/e*e^n
/n*a*b*exp(1/2*I*Pi*csgn(I*e*x)*(-1+n)*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x
)+csgn(I*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. 2(79) = 158.

Time = 0.27 (sec) , antiderivative size = 646, normalized size of antiderivative = 8.18

$$\int (ex)^{-1+n}\left(a+b\mathrm{sech}(c+dx^n)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+n)*(a+b*sech(c+d*x^n))^2,x, algorithm="fricas")
[Out] (a^2*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^2*d*cosh((n - 1)*log(e))*co
sh(n*log(x)) + a^2*d*cosh(n*log(x))*sinh((n - 1)*log(e)) + (a^2*d*cosh((n -
```

```
 1)*log(e)) + a^2*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log
(x)) + d*sinh(n*log(x)) + c)^2 - 2*b^2*cosh((n - 1)*log(e)) + 2*(a^2*d*cosh
((n - 1)*log(e))*cosh(n*log(x)) + a^2*d*cosh(n*log(x))*sinh((n - 1)*log(e))
 + (a^2*d*cosh((n - 1)*log(e)) + a^2*d*sinh((n - 1)*log(e)))*sinh(n*log(x))
)*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*s
inh(n*log(x)) + c) + (a^2*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + a^2*d*cos
h(n*log(x))*sinh((n - 1)*log(e)) + (a^2*d*cosh((n - 1)*log(e)) + a^2*d*sinh
((n - 1)*log(e)))*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x))
+ c)^2 + 4*((a*b*cosh((n - 1)*log(e)) + a*b*sinh((n - 1)*log(e)))*cosh(d*co
sh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + a*b*cosh((n - 1)*log(e)) + 2*(a*b*
cosh((n - 1)*log(e)) + a*b*sinh((n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*
sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a*b*co
sh((n - 1)*log(e)) + a*b*sinh((n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d*si
nh(n*log(x)) + c)^2 + a*b*sinh((n - 1)*log(e)))*arctan(cosh(d*cosh(n*log(x)
) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c))
+ (a^2*d*cosh(n*log(x)) - 2*b^2)*sinh((n - 1)*log(e)) + (a^2*d*cosh((n - 1)
*log(e)) + a^2*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))/(d*n*cosh(d*cosh(n*l
og(x)) + d*sinh(n*log(x)) + c)^2 + 2*d*n*cosh(d*cosh(n*log(x)) + d*sinh(n*l
og(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + d*n*sinh(d*cosh
(n*log(x)) + d*sinh(n*log(x)) + c)^2 + d*n)
```

Sympy [F]

$$\int (ex)^{-1+n}\,(a+b\text{sech}(c+dx^n))^2\,dx=\int (ex)^{n-1}\,(a+b\,\text{sech}\,(c+dx^n))^2\,dx$$

```
[In] integrate((e*x)**(-1+n)*(a+b*sech(c+d*x**n))**2,x)
```

```
[Out] Integral((e*x)**(n - 1)*(a + b*sech(c + d*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^{-1+n}\,(a+b\text{sech}(c+dx^n))^2\,dx=\int (b\,\text{sech}\,(dx^n+c)+a)^2(ex)^{n-1}\,dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*sech(c+d*x^n))^2,x, algorithm="maxima")
```

```
[Out] 4*a*b*e^n*integrate(e^(d*x^n + n*log(x) + c)/(e*x*e^(2*d*x^n + 2*c) + e*x),
 x) - 2*b^2*e^n/(d*e*n*e^(2*d*x^n + 2*c) + d*e*n) + (e*x)^n*a^2/(e*n)
```

Giac [F]

$$\int (ex)^{-1+n} (a + b\text{sech}(c + dx^n))^2 \, dx = \int (b \,\text{sech}\,(dx^n + c) + a)^2 (ex)^{n-1} \, dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*sech(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((b*sech(d*x^n + c) + a)^2*(e*x)^(n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.00

$$\int (ex)^{-1+n} (a + b\text{sech}(c + dx^n))^2 \, dx = \frac{4\,\text{atan}\left(\frac{a\,b\,x\,e^{d\,x^n}\,e^c\,(e\,x)^{n-1}\,\sqrt{d^2\,n^2\,x^{2\,n}}}{d\,n\,x^n\,\sqrt{a^2\,b^2\,x^2\,(e\,x)^{2\,n-2}}}\right)\,\sqrt{a^2\,b^2\,x^2\,(e\,x)^{2\,n-2}}}{\sqrt{d^2\,n^2\,x^{2\,n}}} + \frac{a^2\,x\,(e\,x)^{n-1}}{n} - \frac{2\,b^2\,x\,(e\,x)^{n-1}}{d\,n\,x^n\,\left(e^{2\,c+2\,d\,x^n}+1\right)}$$

```
[In] int((a + b/cosh(c + d*x^n))^2*(e*x)^(n - 1),x)
[Out] (4*atan((a*b*x*exp(d*x^n)*exp(c)*(e*x)^(n - 1)*(d^2*n^2*x^(2*n))^(1/2))/(d*
n*x^n*(a^2*b^2*x^2*(e*x)^(2*n - 2))^(1/2)))*(a^2*b^2*x^2*(e*x)^(2*n - 2))^(
1/2))/(d^2*n^2*x^(2*n))^(1/2) + (a^2*x*(e*x)^(n - 1))/n - (2*b^2*x*(e*x)^(n
 - 1))/(d*n*x^n*(exp(2*c + 2*d*x^n) + 1))
```

3.77 $\int (ex)^{-1+2n} (a+b\text{sech}(c+dx^n))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 208

$$\int (ex)^{-1+2n} (a+b\text{sech}(c+dx^n))^2\,dx = \frac{a^2(ex)^{2n}}{2en} + \frac{4abx^{-n}(ex)^{2n}\arctan\left(e^{c+dx^n}\right)}{den} - \frac{b^2x^{-2n}(ex)^{2n}\log\left(\cosh\left(c+dx^n\right)\right)}{d^2en} - \frac{2iabx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-ie^{c+dx^n}\right)}{d^2en} + \frac{2iabx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,ie^{c+dx^n}\right)}{d^2en} + \frac{b^2x^{-n}(ex)^{2n}\tanh\left(c+dx^n\right)}{den}$$

```
[Out] 1/2*a^2*(e*x)^(2*n)/e/n+4*a*b*(e*x)^(2*n)*arctan(exp(c+d*x^n))/d/e/n/(x^n)-
b^2*(e*x)^(2*n)*ln(cosh(c+d*x^n))/d^2/e/n/(x^(2*n))-2*I*a*b*(e*x)^(2*n)*pol
ylog(2,-I*exp(c+d*x^n))/d^2/e/n/(x^(2*n))+2*I*a*b*(e*x)^(2*n)*polylog(2,I*e
xp(c+d*x^n))/d^2/e/n/(x^(2*n))+b^2*(e*x)^(2*n)*tanh(c+d*x^n)/d/e/n/(x^n)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

= {5548, 5544, 4275, 4265, 2317, 2438, 4269, 3556}

$$\int (ex)^{-1+2n} (a + b\mathrm{sech}(c + dx^n))^2 \, dx = \frac{a^2(ex)^{2n}}{2en} + \frac{4abx^{-n}(ex)^{2n} \arctan\left(e^{c+dx^n}\right)}{den} - \frac{2iabx^{-2n}(ex)^{2n} \operatorname{PolyLog}\left(2, -ie^{dx^n+c}\right)}{d^2en} + \frac{2iabx^{-2n}(ex)^{2n} \operatorname{PolyLog}\left(2, ie^{dx^n+c}\right)}{d^2en} - \frac{b^2x^{-2n}(ex)^{2n} \log\left(\cosh\left(c + dx^n\right)\right)}{d^2en} + \frac{b^2x^{-n}(ex)^{2n} \tanh\left(c + dx^n\right)}{den}$$

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Sech[c + d*x^n])^2,x]
```

```
[Out] (a^2*(e*x)^(2*n))/(2*e*n) + (4*a*b*(e*x)^(2*n)*ArcTan[E^(c + d*x^n)])/(d*e*
n*x^n) - (b^2*(e*x)^(2*n)*Log[Cosh[c + d*x^n]])/(d^2*e*n*x^(2*n)) - ((2*I)*
a*b*(e*x)^(2*n)*PolyLog[2, (-I)*E^(c + d*x^n)])/(d^2*e*n*x^(2*n)) + ((2*I)*
a*b*(e*x)^(2*n)*PolyLog[2, I*E^(c + d*x^n)])/(d^2*e*n*x^(2*n)) + (b^2*(e*x)
^(2*n)*Tanh[c + d*x^n])/(d*e*n*x^n)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5548

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(x^{-2n}(ex)^{2n})\int x^{-1+2n}(a+b\operatorname{sech}(c+dx^n))^2\,dx}{e}\\
&= \frac{(x^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int x(a+b\operatorname{sech}(c+dx))^2\,dx,x,x^n\right)}{en}\\
&= \frac{(x^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\left(a^2x+2abx\operatorname{sech}(c+dx)+b^2x\operatorname{sech}^2(c+dx)\right)\,dx,x,x^n\right)}{en}\\
&= \frac{a^2(ex)^{2n}}{2en}+\frac{(2abx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int x\operatorname{sech}(c+dx)\,dx,x,x^n\right)}{en}\\
&\quad+\frac{(b^2x^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int x\operatorname{sech}^2(c+dx)\,dx,x,x^n\right)}{en}\\
&= \frac{a^2(ex)^{2n}}{2en}+\frac{4abx^{-n}(ex)^{2n}\arctan\left(e^{c+dx^n}\right)}{den}+\frac{b^2x^{-n}(ex)^{2n}\tanh\left(c+dx^n\right)}{den}\\
&\quad-\frac{(2iabx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\log\left(1-ie^{c+dx}\right)\,dx,x,x^n\right)}{den}\\
&\quad+\frac{(2iabx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\log\left(1+ie^{c+dx}\right)\,dx,x,x^n\right)}{den}\\
&\quad-\frac{(b^2x^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\tanh(c+dx)\,dx,x,x^n\right)}{den}
\end{aligned}
$$

$$= \frac{a^2(ex)^{2n}}{2en} + \frac{4abx^{-n}(ex)^{2n}\arctan\left(e^{c+dx^n}\right)}{den} - \frac{b^2x^{-2n}(ex)^{2n}\log\left(\cosh\left(c+dx^n\right)\right)}{d^2en} + \frac{b^2x^{-n}(ex)^{2n}\tanh\left(c+dx^n\right)}{den} - \frac{\left(2iabx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx, x, e^{c+dx^n}\right)}{d^2en} + \frac{\left(2iabx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx, x, e^{c+dx^n}\right)}{d^2en}$$

$$= \frac{a^2(ex)^{2n}}{2en} + \frac{4abx^{-n}(ex)^{2n}\arctan\left(e^{c+dx^n}\right)}{den} - \frac{b^2x^{-2n}(ex)^{2n}\log\left(\cosh\left(c+dx^n\right)\right)}{d^2en} - \frac{2iabx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-ie^{c+dx^n}\right)}{d^2en} + \frac{2iabx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,ie^{c+dx^n}\right)}{d^2en} + \frac{b^2x^{-n}(ex)^{2n}\tanh\left(c+dx^n\right)}{den}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 501 vs. 2(208) = 416.

Time = 5.47 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.41

$$\int (ex)^{-1+2n}\left(a+b\text{sech}(c+dx^n)\right)^2\,dx$$

$$= \frac{x^{-2n}(ex)^{2n}\text{csch}^5(c)\text{sech}(c+dx^n)\left(-2b^2dx^n\cosh\left(dx^n\right)\sqrt{-\text{csch}^2(c)} + 2b^2dx^n\cosh\left(2c+dx^n\right)\sqrt{-\text{csch}^2(c)} \right.}{}$$

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sech[c + d*x^n])^2,x]
```

```
[Out] ((e*x)^(2*n)*Csch[c]^5*Sech[c + d*x^n]*(-2*b^2*d*x^n*Cosh[d*x^n]*Sqrt[-Csch
[c]^2] + 2*b^2*d*x^n*Cosh[2*c + d*x^n]*Sqrt[-Csch[c]^2] + 8*a*b*d*x^n*Cosh[
c + d*x^n]*Log[1 - E^(-(d*x^n) - ArcTanh[Coth[c]])] + 8*a*b*ArcTanh[Coth[c]
]*Cosh[c + d*x^n]*Log[1 - E^(-(d*x^n) - ArcTanh[Coth[c]])] - 8*a*b*d*x^n*Co
sh[c + d*x^n]*Log[1 + E^(-(d*x^n) - ArcTanh[Coth[c]])] - 8*a*b*ArcTanh[Coth
[c]]*Cosh[c + d*x^n]*Log[1 + E^(-(d*x^n) - ArcTanh[Coth[c]])] + 8*a*b*Cosh[
c + d*x^n]*PolyLog[2, -E^(-(d*x^n) - ArcTanh[Coth[c]])] - 8*a*b*Cosh[c + d*
x^n]*PolyLog[2, E^(-(d*x^n) - ArcTanh[Coth[c]])] - a^2*d^2*x^(2*n)*Sqrt[-Cs
ch[c]^2]*Sinh[d*x^n] + 8*a*b*ArcTan[Sinh[c] + Cosh[c]*Tanh[(d*x^n)/2]]*ArcT
anh[Coth[c]]*Sqrt[-Csch[c]^2]*Sinh[d*x^n] + 2*b^2*Sqrt[-Csch[c]^2]*Log[Cosh
[c + d*x^n]]*Sinh[d*x^n] + a^2*d^2*x^(2*n)*Sqrt[-Csch[c]^2]*Sinh[2*c + d*x^
n] - 8*a*b*ArcTan[Sinh[c] + Cosh[c]*Tanh[(d*x^n)/2]]*ArcTanh[Coth[c]]*Sqrt[
-Csch[c]^2]*Sinh[2*c + d*x^n] - 2*b^2*Sqrt[-Csch[c]^2]*Log[Cosh[c + d*x^n]]
*Sinh[2*c + d*x^n]))/(4*d^2*e*n*x^(2*n)*(-Csch[c]^2)^(5/2))
```

Maple [F]

$$\int (ex)^{2n-1} \left(a + b\,\mathrm{sech}\left(c + d\,x^{n}\right)\right)^{2} dx$$

```
[In] int((e*x)^(2*n-1)*(a+b*sech(c+d*x^n))^2,x)
```

```
[Out] int((e*x)^(2*n-1)*(a+b*sech(c+d*x^n))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2972 vs. 2(199) = 398.

Time = 0.32 (sec) , antiderivative size = 2972, normalized size of antiderivative = 14.29

$$\int (ex)^{-1+2n} (a + b\mathrm{sech}(c + dx^n))^2 \ dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sech(c+d*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + 4*b^2*c*cosh((2*n -
1)*log(e)) + (a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + 4*b^2*d*cos
h((2*n - 1)*log(e))*cosh(n*log(x)) + 4*b^2*c*cosh((2*n - 1)*log(e)) + (a^2*
d^2*cosh((2*n - 1)*log(e)) + a^2*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x))
^2 + (a^2*d^2*cosh(n*log(x))^2 + 4*b^2*d*cosh(n*log(x)) + 4*b^2*c)*sinh((2*
n - 1)*log(e)) + 2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + 2*b^2*d
*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh(n*log(x)) + 2*b^2*d)*sinh((2*n - 1)
*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 +
 2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + 4*b^2*d*cosh((2*n - 1
)*log(e))*cosh(n*log(x)) + 4*b^2*c*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh((
2*n - 1)*log(e)) + a^2*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x))^2 + (a^2*
d^2*cosh(n*log(x))^2 + 4*b^2*d*cosh(n*log(x)) + 4*b^2*c)*sinh((2*n - 1)*log
(e)) + 2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + 2*b^2*d*cosh((2*n
 - 1)*log(e)) + (a^2*d^2*cosh(n*log(x)) + 2*b^2*d)*sinh((2*n - 1)*log(e)))*
sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n
*log(x)) + d*sinh(n*log(x)) + c) + (a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*l
og(x))^2 + 4*b^2*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + 4*b^2*c*cosh((2*
n - 1)*log(e)) + (a^2*d^2*cosh((2*n - 1)*log(e)) + a^2*d^2*sinh((2*n - 1)*l
og(e)))*sinh(n*log(x))^2 + (a^2*d^2*cosh(n*log(x))^2 + 4*b^2*d*cosh(n*log(x
)) + 4*b^2*c)*sinh((2*n - 1)*log(e)) + 2*(a^2*d^2*cosh((2*n - 1)*log(e))*co
sh(n*log(x)) + 2*b^2*d*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh(n*log(x)) + 2
*b^2*d)*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) + d*s
inh(n*log(x)) + c)^2 + (a^2*d^2*cosh((2*n - 1)*log(e)) + a^2*d^2*sinh((2*n
- 1)*log(e)))*sinh(n*log(x))^2 - 4*((-I*a*b*cosh((2*n - 1)*log(e)) - I*a*b*
sinh((2*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 - I
*a*b*cosh((2*n - 1)*log(e)) + 2*(-I*a*b*cosh((2*n - 1)*log(e)) - I*a*b*sinh
```

```
((2*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*co
sh(n*log(x)) + d*sinh(n*log(x)) + c) + (-I*a*b*cosh((2*n - 1)*log(e)) - I*a
*b*sinh((2*n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2
- I*a*b*sinh((2*n - 1)*log(e)))*dilog(I*cosh(d*cosh(n*log(x)) + d*sinh(n*lo
g(x)) + c) + I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) - 4*((I*a*b*c
osh((2*n - 1)*log(e)) + I*a*b*sinh((2*n - 1)*log(e)))*cosh(d*cosh(n*log(x))
 + d*sinh(n*log(x)) + c)^2 + I*a*b*cosh((2*n - 1)*log(e)) + 2*(I*a*b*cosh((
2*n - 1)*log(e)) + I*a*b*sinh((2*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*
sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (I*a*b*
cosh((2*n - 1)*log(e)) + I*a*b*sinh((2*n - 1)*log(e)))*sinh(d*cosh(n*log(x)
) + d*sinh(n*log(x)) + c)^2 + I*a*b*sinh((2*n - 1)*log(e)))*dilog(-I*cosh(d
*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - I*sinh(d*cosh(n*log(x)) + d*sinh(
n*log(x)) + c)) - 2*(((2*I*a*b*c + b^2)*cosh((2*n - 1)*log(e)) + (2*I*a*b*c
 + b^2)*sinh((2*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) +
c)^2 + 2*((2*I*a*b*c + b^2)*cosh((2*n - 1)*log(e)) + (2*I*a*b*c + b^2)*sinh
((2*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*co
sh(n*log(x)) + d*sinh(n*log(x)) + c) + ((2*I*a*b*c + b^2)*cosh((2*n - 1)*lo
g(e)) + (2*I*a*b*c + b^2)*sinh((2*n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d
*sinh(n*log(x)) + c)^2 + (2*I*a*b*c + b^2)*cosh((2*n - 1)*log(e)) + (2*I*a*
b*c + b^2)*sinh((2*n - 1)*log(e)))*log(cosh(d*cosh(n*log(x)) + d*sinh(n*log
(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + I) - 2*(((-2*I*
a*b*c + b^2)*cosh((2*n - 1)*log(e)) + (-2*I*a*b*c + b^2)*sinh((2*n - 1)*log
(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + 2*((-2*I*a*b*c + b^
2)*cosh((2*n - 1)*log(e)) + (-2*I*a*b*c + b^2)*sinh((2*n - 1)*log(e)))*cosh
(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*
log(x)) + c) + ((-2*I*a*b*c + b^2)*cosh((2*n - 1)*log(e)) + (-2*I*a*b*c + b
^2)*sinh((2*n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2
 + (-2*I*a*b*c + b^2)*cosh((2*n - 1)*log(e)) + (-2*I*a*b*c + b^2)*sinh((2*n
 - 1)*log(e)))*log(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*c
osh(n*log(x)) + d*sinh(n*log(x)) + c) - I) - 4*(I*a*b*d*cosh((2*n - 1)*log(
e))*cosh(n*log(x)) + I*a*b*c*cosh((2*n - 1)*log(e)) + (I*a*b*d*cosh((2*n -
1)*log(e))*cosh(n*log(x)) + I*a*b*c*cosh((2*n - 1)*log(e)) + (I*a*b*d*cosh(
n*log(x)) + I*a*b*c)*sinh((2*n - 1)*log(e)) + (I*a*b*d*cosh((2*n - 1)*log(e
)) + I*a*b*d*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x))
+ d*sinh(n*log(x)) + c)^2 + 2*(I*a*b*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)
) + I*a*b*c*cosh((2*n - 1)*log(e)) + (I*a*b*d*cosh(n*log(x)) + I*a*b*c)*sin
h((2*n - 1)*log(e)) + (I*a*b*d*cosh((2*n - 1)*log(e)) + I*a*b*d*sinh((2*n -
 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*
sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (I*a*b*d*cosh((2*n - 1)*log
(e))*cosh(n*log(x)) + I*a*b*c*cosh((2*n - 1)*log(e)) + (I*a*b*d*cosh(n*log(
x)) + I*a*b*c)*sinh((2*n - 1)*log(e)) + (I*a*b*d*cosh((2*n - 1)*log(e)) + I
*a*b*d*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) + d*si
nh(n*log(x)) + c)^2 + (I*a*b*d*cosh(n*log(x)) + I*a*b*c)*sinh((2*n - 1)*log
(e)) + (I*a*b*d*cosh((2*n - 1)*log(e)) + I*a*b*d*sinh((2*n - 1)*log(e)))*si
nh(n*log(x)))*log(I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + I*sinh(
```

```
d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 1) - 4*(-I*a*b*d*cosh((2*n - 1)*
log(e))*cosh(n*log(x)) - I*a*b*c*cosh((2*n - 1)*log(e)) + (-I*a*b*d*cosh((2
*n - 1)*log(e))*cosh(n*log(x)) - I*a*b*c*cosh((2*n - 1)*log(e)) + (-I*a*b*d
*cosh(n*log(x)) - I*a*b*c)*sinh((2*n - 1)*log(e)) + (-I*a*b*d*cosh((2*n - 1
)*log(e)) - I*a*b*d*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*l
og(x)) + d*sinh(n*log(x)) + c)^2 + 2*(-I*a*b*d*cosh((2*n - 1)*log(e))*cosh(
n*log(x)) - I*a*b*c*cosh((2*n - 1)*log(e)) + (-I*a*b*d*cosh(n*log(x)) - I*a
*b*c)*sinh((2*n - 1)*log(e)) + (-I*a*b*d*cosh((2*n - 1)*log(e)) - I*a*b*d*s
inh((2*n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log
(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (-I*a*b*d*cosh((2
*n - 1)*log(e))*cosh(n*log(x)) - I*a*b*c*cosh((2*n - 1)*log(e)) + (-I*a*b*d
*cosh(n*log(x)) - I*a*b*c)*sinh((2*n - 1)*log(e)) + (-I*a*b*d*cosh((2*n - 1
)*log(e)) - I*a*b*d*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*sinh(d*cosh(n*l
og(x)) + d*sinh(n*log(x)) + c)^2 + (-I*a*b*d*cosh(n*log(x)) - I*a*b*c)*sinh
((2*n - 1)*log(e)) + (-I*a*b*d*cosh((2*n - 1)*log(e)) - I*a*b*d*sinh((2*n -
 1)*log(e)))*sinh(n*log(x)))*log(-I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)
) + c) - I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 1) + (a^2*d^2*co
sh(n*log(x))^2 + 4*b^2*c)*sinh((2*n - 1)*log(e)) + 2*(a^2*d^2*cosh((2*n - 1
)*log(e))*cosh(n*log(x)) + a^2*d^2*cosh(n*log(x))*sinh((2*n - 1)*log(e)))*s
inh(n*log(x)))/(d^2*n*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + 2*d
^2*n*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)) +
d*sinh(n*log(x)) + c) + d^2*n*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)
^2 + d^2*n)
```

Sympy [F]

$$\int (ex)^{-1+2n}(a+b\operatorname{sech}(c+dx^n))^2\,dx = \int (ex)^{2n-1}\left(a+b\operatorname{sech}\left(c+dx^n\right)\right)^2\,dx$$

```
[In] integrate((e*x)**(-1+2*n)*(a+b*sech(c+d*x**n))**2,x)
[Out] Integral((e*x)**(2*n - 1)*(a + b*sech(c + d*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^{-1+2n}(a+b\operatorname{sech}(c+dx^n))^2\,dx = \int \left(b\operatorname{sech}\left(dx^n+c\right)+a\right)^2(ex)^{2n-1}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sech(c+d*x^n))^2,x, algorithm="maxima")
[Out] 4*a*b*e^(2*n)*integrate(e^(d*x^n + 2*n*log(x) + c)/(e*x*e^(2*d*x^n + 2*c) +
 e*x), x) + b^2*(2*e^(2*n)*e^(2*d*x^n + n*log(x) + 2*c)/(d*e*n*e^(2*d*x^n +
 2*c) + d*e*n) - e^(2*n - 1)*log((e^(2*d*x^n + 2*c) + 1)*e^(-2*c))/(d^2*n))
 + 1/2*(e*x)^(2*n)*a^2/(e*n)
```

Giac [F]

$$\int (ex)^{-1+2n}(a+b\text{sech}(c+dx^n))^2\,dx = \int (b\,\text{sech}\,(dx^n+c)+a)^2(ex)^{2\,n-1}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sech(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((b*sech(d*x^n + c) + a)^2*(e*x)^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}(a+b\text{sech}(c+dx^n))^2\,dx = \int \left(a+\frac{b}{\cosh\left(c+d\,x^n\right)}\right)^2(e\,x)^{2\,n-1}\,dx$$

```
[In] int((a + b/cosh(c + d*x^n))^2*(e*x)^(2*n - 1),x)
[Out] int((a + b/cosh(c + d*x^n))^2*(e*x)^(2*n - 1), x)
```

3.78 $\int (ex)^{-1+3n} (a + b\,\mathrm{sech}(c + dx^n))^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 363

$$
\begin{aligned}
\int (ex)^{-1+3n} (a + b\,\mathrm{sech}(c + dx^n))^2 \, dx = {} & \frac{a^2 (ex)^{3n}}{3en} + \frac{b^2 x^{-n} (ex)^{3n}}{den} \\
& + \frac{4abx^{-n} (ex)^{3n} \arctan\left(e^{c+dx^n}\right)}{den} \\
& - \frac{2b^2 x^{-2n} (ex)^{3n} \log\left(1 + e^{2(c+dx^n)}\right)}{d^2 en} \\
& - \frac{4iabx^{-2n} (ex)^{3n} \operatorname{PolyLog}\left(2, -ie^{c+dx^n}\right)}{d^2 en} \\
& + \frac{4iabx^{-2n} (ex)^{3n} \operatorname{PolyLog}\left(2, ie^{c+dx^n}\right)}{d^2 en} \\
& - \frac{b^2 x^{-3n} (ex)^{3n} \operatorname{PolyLog}\left(2, -e^{2(c+dx^n)}\right)}{d^3 en} \\
& + \frac{4iabx^{-3n} (ex)^{3n} \operatorname{PolyLog}\left(3, -ie^{c+dx^n}\right)}{d^3 en} \\
& - \frac{4iabx^{-3n} (ex)^{3n} \operatorname{PolyLog}\left(3, ie^{c+dx^n}\right)}{d^3 en} \\
& + \frac{b^2 x^{-n} (ex)^{3n} \tanh\left(c + dx^n\right)}{den}
\end{aligned}
$$

```
[Out] 1/3*a^2*(e*x)^(3*n)/e/n+b^2*(e*x)^(3*n)/d/e/n/(x^n)+4*a*b*(e*x)^(3*n)*arcta
n(exp(c+d*x^n))/d/e/n/(x^n)-2*b^2*(e*x)^(3*n)*ln(1+exp(2*c+2*d*x^n))/d^2/e/
n/(x^(2*n))-4*I*a*b*(e*x)^(3*n)*polylog(2,-I*exp(c+d*x^n))/d^2/e/n/(x^(2*n)
)+4*I*a*b*(e*x)^(3*n)*polylog(2,I*exp(c+d*x^n))/d^2/e/n/(x^(2*n))-b^2*(e*x)
^(3*n)*polylog(2,-exp(2*c+2*d*x^n))/d^3/e/n/(x^(3*n))+4*I*a*b*(e*x)^(3*n)*p
olylog(3,-I*exp(c+d*x^n))/d^3/e/n/(x^(3*n))-4*I*a*b*(e*x)^(3*n)*polylog(3,I
*exp(c+d*x^n))/d^3/e/n/(x^(3*n))+b^2*(e*x)^(3*n)*tanh(c+d*x^n)/d/e/n/(x^n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5548, 5544, 4275, 4265, 2611, 2320, 6724, 4269, 3799, 2221, 2317, 2438}

$$\int (ex)^{-1+3n}\left(a+b\text{sech}(c+dx^n)\right)^2\,dx=\frac{a^2(ex)^{3n}}{3en}+\frac{4abx^{-n}(ex)^{3n}\arctan\left(e^{c+dx^n}\right)}{den}+\frac{4iabx^{-3n}(ex)^{3n}\text{PolyLog}\left(3,-ie^{dx^n+c}\right)}{d^3en}-\frac{4iabx^{-3n}(ex)^{3n}\text{PolyLog}\left(3,ie^{dx^n+c}\right)}{d^3en}-\frac{4iabx^{-2n}(ex)^{3n}\text{PolyLog}\left(2,-ie^{dx^n+c}\right)}{d^2en}+\frac{4iabx^{-2n}(ex)^{3n}\text{PolyLog}\left(2,ie^{dx^n+c}\right)}{d^2en}-\frac{b^2x^{-3n}(ex)^{3n}\text{PolyLog}\left(2,-e^{2(dx^n+c)}\right)}{d^3en}-\frac{2b^2x^{-2n}(ex)^{3n}\log\left(e^{2(c+dx^n)}+1\right)}{d^2en}+\frac{b^2x^{-n}(ex)^{3n}\tanh(c+dx^n)}{den}+\frac{b^2x^{-n}(ex)^{3n}}{den}$$

```
[In] Int[(e*x)^(-1 + 3*n)*(a + b*Sech[c + d*x^n])^2,x]
```

```
[Out] (a^2*(e*x)^(3*n))/(3*e*n) + (b^2*(e*x)^(3*n))/(d*e*n*x^n) + (4*a*b*(e*x)^(3
*n)*ArcTan[E^(c + d*x^n)])/(d*e*n*x^n) - (2*b^2*(e*x)^(3*n)*Log[1 + E^(2*(c
 + d*x^n))])/(d^2*e*n*x^(2*n)) - ((4*I)*a*b*(e*x)^(3*n)*PolyLog[2, (-I)*E^(
c + d*x^n)])/(d^2*e*n*x^(2*n)) + ((4*I)*a*b*(e*x)^(3*n)*PolyLog[2, I*E^(c +
 d*x^n)])/(d^2*e*n*x^(2*n)) - (b^2*(e*x)^(3*n)*PolyLog[2, -E^(2*(c + d*x^n)
)])/(d^3*e*n*x^(3*n)) + ((4*I)*a*b*(e*x)^(3*n)*PolyLog[3, (-I)*E^(c + d*x^n
)])/(d^3*e*n*x^(3*n)) - ((4*I)*a*b*(e*x)^(3*n)*PolyLog[3, I*E^(c + d*x^n)])
/(d^3*e*n*x^(3*n)) + (b^2*(e*x)^(3*n)*Tanh[c + d*x^n])/(d*e*n*x^n)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5548

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(x^{-3n}(ex)^{3n}\right)\int x^{-1+3n}(a+b\operatorname{sech}(c+dx^n))^2\,dx}{e}\\
&= \frac{\left(x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x^2(a+b\operatorname{sech}(c+dx))^2\,dx,x,x^n\right)}{en}\\
&= \frac{\left(x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int \left(a^2x^2+2abx^2\operatorname{sech}(c+dx)+b^2x^2\operatorname{sech}^2(c+dx)\right)\,dx,x,x^n\right)}{en}\\
&= \frac{a^2(ex)^{3n}}{3en}+\frac{\left(2abx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x^2\operatorname{sech}(c+dx)\,dx,x,x^n\right)}{en}\\
&\quad+\frac{\left(b^2x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x^2\operatorname{sech}^2(c+dx)\,dx,x,x^n\right)}{en}\\
&= \frac{a^2(ex)^{3n}}{3en}+\frac{4abx^{-n}(ex)^{3n}\arctan\left(e^{c+dx^n}\right)}{den}+\frac{b^2x^{-n}(ex)^{3n}\tanh\left(c+dx^n\right)}{den}\\
&\quad-\frac{\left(4iabx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x\log\left(1-ie^{c+dx}\right)\,dx,x,x^n\right)}{den}\\
&\quad+\frac{\left(4iabx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x\log\left(1+ie^{c+dx}\right)\,dx,x,x^n\right)}{den}\\
&\quad-\frac{\left(2b^2x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int x\tanh(c+dx)\,dx,x,x^n\right)}{den}
\end{aligned}
$$

$$=\frac{a^2(ex)^{3n}}{3en}+\frac{b^2x^{-n}(ex)^{3n}}{den}+\frac{4abx^{-n}(ex)^{3n}\arctan\left(e^{c+dx^n}\right)}{den}-\frac{4iabx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-ie^{c+dx^n}\right)}{d^2en}+\frac{4iabx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,ie^{c+dx^n}\right)}{d^2en}+\frac{b^2x^{-n}(ex)^{3n}\tanh\left(c+dx^n\right)}{den}+\frac{(4iabx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int\text{PolyLog}\left(2,-ie^{c+dx}\right)dx,x,x^n\right)}{d^2en}-\frac{(4iabx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int\text{PolyLog}\left(2,ie^{c+dx}\right)dx,x,x^n\right)}{d^2en}-\frac{(4b^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int\frac{e^{2(c+dx)}x}{1+e^{2(c+dx)}}dx,x,x^n\right)}{den}$$

$$=\frac{a^2(ex)^{3n}}{3en}+\frac{b^2x^{-n}(ex)^{3n}}{den}+\frac{4abx^{-n}(ex)^{3n}\arctan\left(e^{c+dx^n}\right)}{den}-\frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+e^{2(c+dx^n)}\right)}{d^2en}-\frac{4iabx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-ie^{c+dx^n}\right)}{d^2en}+\frac{4iabx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,ie^{c+dx^n}\right)}{d^2en}+\frac{b^2x^{-n}(ex)^{3n}\tanh\left(c+dx^n\right)}{den}+\frac{(4iabx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int\frac{\text{PolyLog}(2,-ix)}{x}dx,x,e^{c+dx^n}\right)}{d^3en}-\frac{(4iabx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int\frac{\text{PolyLog}(2,ix)}{x}dx,x,e^{c+dx^n}\right)}{d^3en}+\frac{(2b^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int\log\left(1+e^{2(c+dx)}\right)dx,x,x^n\right)}{d^2en}$$

$$=\frac{a^2(ex)^{3n}}{3en}+\frac{b^2x^{-n}(ex)^{3n}}{den}+\frac{4abx^{-n}(ex)^{3n}\arctan\left(e^{c+dx^n}\right)}{den}-\frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+e^{2(c+dx^n)}\right)}{d^2en}-\frac{4iabx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-ie^{c+dx^n}\right)}{d^2en}+\frac{4iabx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,ie^{c+dx^n}\right)}{d^2en}+\frac{4iabx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-ie^{c+dx^n}\right)}{d^3en}-\frac{4iabx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,ie^{c+dx^n}\right)}{d^3en}+\frac{b^2x^{-n}(ex)^{3n}\tanh\left(c+dx^n\right)}{den}+\frac{(b^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int\frac{\log(1+x)}{x}dx,x,e^{2(c+dx^n)}\right)}{d^3en}$$

$$
\begin{aligned}
&= \frac{a^2(ex)^{3n}}{3en} + \frac{b^2x^{-n}(ex)^{3n}}{den} + \frac{4abx^{-n}(ex)^{3n}\arctan\left(e^{c+dx^n}\right)}{den} \\
&\quad - \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+e^{2(c+dx^n)}\right)}{d^2en} - \frac{4iabx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-ie^{c+dx^n}\right)}{d^2en} \\
&\quad + \frac{4iabx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,ie^{c+dx^n}\right)}{d^2en} - \frac{b^2x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-e^{2(c+dx^n)}\right)}{d^3en} \\
&\quad + \frac{4iabx^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,-ie^{c+dx^n}\right)}{d^3en} \\
&\quad - \frac{4iabx^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,ie^{c+dx^n}\right)}{d^3en} + \frac{b^2x^{-n}(ex)^{3n}\tanh\left(c+dx^n\right)}{den}
\end{aligned}
$$

Mathematica [F]

$$
\int (ex)^{-1+3n}\left(a+b\mathrm{sech}(c+dx^n)\right)^2\,dx = \int (ex)^{-1+3n}\left(a+b\mathrm{sech}(c+dx^n)\right)^2\,dx
$$

```
[In] Integrate[(e*x)^(-1 + 3*n)*(a + b*Sech[c + d*x^n])^2,x]
[Out] Integrate[(e*x)^(-1 + 3*n)*(a + b*Sech[c + d*x^n])^2, x]
```

Maple [F]

$$
\int (ex)^{-1+3n}\left(a+b\,\mathrm{sech}\left(c+d\,x^n\right)\right)^2dx
$$

```
[In] int((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n))^2,x)
[Out] int((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n))^2,x)
```

Fricas [F(-1)]

Timed out.

$$
\int (ex)^{-1+3n}\left(a+b\mathrm{sech}(c+dx^n)\right)^2\,dx = \text{Timed out}
$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int (ex)^{-1+3n} (a + b\text{sech}(c + dx^n))^2 \, dx = \int (ex)^{3n-1} (a + b \operatorname{sech}(c + dx^n))^2 \, dx$$

```
[In] integrate((e*x)**(-1+3*n)*(a+b*sech(c+d*x**n))**2,x)
[Out] Integral((e*x)**(3*n - 1)*(a + b*sech(c + d*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^{-1+3n} (a + b\text{sech}(c + dx^n))^2 \, dx = \int (b \operatorname{sech}(dx^n + c) + a)^2 (ex)^{3n-1} \, dx$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n))^2,x, algorithm="maxima")
[Out] -2*b^2*e^(3*n)*x^(2*n)/(d*e*n*e^(2*d*x^n + 2*c) + d*e*n) + 1/3*(e*x)^(3*n)*
a^2/(e*n) + integrate(4*(a*b*d*e^(3*n)*e^(d*x^n + 3*n*log(x) + c) + b^2*e^(
3*n)*x^(2*n))/(d*e*x*e^(2*d*x^n + 2*c) + d*e*x), x)
```

Giac [F]

$$\int (ex)^{-1+3n} (a + b\text{sech}(c + dx^n))^2 \, dx = \int (b \operatorname{sech}(dx^n + c) + a)^2 (ex)^{3n-1} \, dx$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((b*sech(d*x^n + c) + a)^2*(e*x)^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n} (a + b\text{sech}(c + dx^n))^2 \, dx = \int \left(a + \frac{b}{\cosh(c + d\, x^n)}\right)^2 (e\, x)^{3n-1} \, dx$$

```
[In] int((a + b/cosh(c + d*x^n))^2*(e*x)^(3*n - 1),x)
[Out] int((a + b/cosh(c + d*x^n))^2*(e*x)^(3*n - 1), x)
```

3.79 $\int \frac{(ex)^{-1+n}}{a+b\,\text{sech}(c+dx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{(ex)^{-1+n}}{a+b\,\text{sech}\,(c+dx^n)}\,dx=\frac{(ex)^n}{aen}-\frac{2bx^{-n}(ex)^n\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}den}$$

```
[Out] (e*x)^n/a/e/n-2*b*(e*x)^n*arctan((a-b)^(1/2)*tanh(1/2*c+1/2*d*x^n)/(a+b)^(1
/2))/a/d/e/n/(x^n)/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5548, 5544, 3868, 2738, 214}

$$\int \frac{(ex)^{-1+n}}{a+b\,\text{sech}\,(c+dx^n)}\,dx=\frac{(ex)^n}{aen}-\frac{2bx^{-n}(ex)^n\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{aden\sqrt{a-b}\sqrt{a+b}}$$

```
[In] Int[(e*x)^(-1 + n)/(a + b*Sech[c + d*x^n]),x]
```

```
[Out] (e*x)^n/(a*e*n) - (2*b*(e*x)^n*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x^n)/2])/Sqr
t[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d*e*n*x^n)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5548

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(x^{-n}(ex)^n)\int \frac{x^{-1+n}}{a+b\operatorname{sech}(c+dx^n)}\,dx}{e}\\
&= \frac{(x^{-n}(ex)^n)\operatorname{Subst}\left(\int \frac{1}{a+b\operatorname{sech}(c+dx)}\,dx, x, x^n\right)}{en}\\
&= \frac{(ex)^n}{aen} - \frac{(x^{-n}(ex)^n)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a\cosh(c+dx)}{b}}\,dx, x, x^n\right)}{aen}\\
&= \frac{(ex)^n}{aen} + \frac{(2ix^{-n}(ex)^n)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, i\tanh\left(\frac{1}{2}(c+dx^n)\right)\right)}{aden}\\
&= \frac{(ex)^n}{aen} - \frac{2bx^{-n}(ex)^n\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}den}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{(ex)^{-1+n}}{a+b\operatorname{sech}(c+dx^n)}\,dx=\frac{(ex)^n\left(d+cx^{-n}+\frac{2bx^{-n}\arctan\left(\frac{(-a+b)\tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{aden}$$

```
[In] Integrate[(e*x)^(-1 + n)/(a + b*Sech[c + d*x^n]),x]
```

```
[Out] ((e*x)^n*(d + c/x^n + (2*b*ArcTan[((-a + b)*Tanh[(c + d*x^n)/2])/Sqrt[a^2 -
 b^2]])/(Sqrt[a^2 - b^2]*x^n)))/(a*d*e*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.85 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.64

method	result
risch	$\frac{x\,\mathrm{e}^{\frac{(-1+n)\left(-i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)\pi+i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^2\pi+i\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^2\pi-i\,\mathrm{csgn}(iex)^3\pi+2\ln(e)+2\ln(x)\right)}{2}}}{an} - \frac{2b\,\mathrm{e}^{-\frac{i\pi n\,\mathrm{csgn}(ie)\,\mathrm{cs}}{}}}{}$

```
[In] int((e*x)^(-1+n)/(a+b*sech(c+d*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*c
sgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*l
n(x)))-2*b/a/n*exp(-1/2*I*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*P
i*n*csgn(I*e)*csgn(I*e*x)^2)*exp(1/2*I*Pi*n*csgn(I*x)*csgn(I*e*x)^2)*exp(-1
/2*I*Pi*n*csgn(I*e*x)^3)*exp(1/2*I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(
-1/2*I*Pi*csgn(I*e)*csgn(I*e*x)^2)*exp(-1/2*I*Pi*csgn(I*x)*csgn(I*e*x)^2)*e
xp(1/2*I*Pi*csgn(I*e*x)^3)*e^n/e*exp(c)/d/(a^2*exp(2*c)-exp(2*c)*b^2)^(1/2)
*arctan(1/2*(2*a*exp(2*c+d*x^n)+2*exp(c)*b)/(a^2*exp(2*c)-exp(2*c)*b^2)^(1/
2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(78) = 156$.

Time = 0.30 (sec) , antiderivative size = 511, normalized size of antiderivative = 5.87

$$\int \frac{(ex)^{-1+n}}{a + b\text{sech}(c + dx^n)} dx$$
$$= \left[\frac{(a^2 - b^2)d\cosh((n-1)\log(e))\cosh(n\log(x)) + (a^2 - b^2)d\cosh(n\log(x))\sinh((n-1)\log(e)) - (\sqrt{-}}{} \right.$$

```
[In] integrate((e*x)^(-1+n)/(a+b*sech(c+d*x^n)),x, algorithm="fricas")
```

```
[Out] [((a^2 - b^2)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^2 - b^2)*d*cosh(n*
log(x))*sinh((n - 1)*log(e)) - (sqrt(-a^2 + b^2)*b*cosh((n - 1)*log(e)) + s
qrt(-a^2 + b^2)*b*sinh((n - 1)*log(e)))*log((a*b + (b^2 + sqrt(-a^2 + b^2)*
b)*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a^2 - b^2 - sqrt(-a^2 +
 b^2)*b)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sqrt(-a^2 + b^2)*a
)/(a*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + b)) + ((a^2 - b^2)*d*c
osh((n - 1)*log(e)) + (a^2 - b^2)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))/(
(a^3 - a*b^2)*d*n), ((a^2 - b^2)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a
^2 - b^2)*d*cosh(n*log(x))*sinh((n - 1)*log(e)) + 2*(sqrt(a^2 - b^2)*b*cosh
((n - 1)*log(e)) + sqrt(a^2 - b^2)*b*sinh((n - 1)*log(e)))*arctan(-(sqrt(a^
2 - b^2)*a*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sqrt(a^2 - b^2)*
a*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sqrt(a^2 - b^2)*b)/(a^2 -
 b^2)) + ((a^2 - b^2)*d*cosh((n - 1)*log(e)) + (a^2 - b^2)*d*sinh((n - 1)*l
og(e)))*sinh(n*log(x)))/((a^3 - a*b^2)*d*n)]
```

Sympy [F]

$$\int \frac{(ex)^{-1+n}}{a + b\text{sech}(c + dx^n)} dx = \int \frac{(ex)^{n-1}}{a + b\,\text{sech}(c + dx^n)} dx$$

```
[In] integrate((e*x)**(-1+n)/(a+b*sech(c+d*x**n)),x)
```

```
[Out] Integral((e*x)**(n - 1)/(a + b*sech(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+n}}{a+b\text{sech}\left(c+dx^n\right)}\,dx=\int \frac{(ex)^{n-1}}{b\,\text{sech}\left(dx^n+c\right)+a}\,dx$$

[In] integrate((e*x)^(-1+n)/(a+b*sech(c+d*x^n)),x, algorithm="maxima")

[Out] -2*b*e^n*integrate(e^(d*x^n + n*log(x) + c)/(a^2*e*x*e^(2*d*x^n + 2*c) + 2*a*b*e*x*e^(d*x^n + c) + a^2*e*x), x) + e^(n - 1)*x^n/(a*n)

Giac [F]

$$\int \frac{(ex)^{-1+n}}{a+b\text{sech}\left(c+dx^n\right)}\,dx=\int \frac{(ex)^{n-1}}{b\,\text{sech}\left(dx^n+c\right)+a}\,dx$$

[In] integrate((e*x)^(-1+n)/(a+b*sech(c+d*x^n)),x, algorithm="giac")

[Out] integrate((e*x)^(n - 1)/(b*sech(d*x^n + c) + a), x)

Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 409, normalized size of antiderivative = 4.70

$$\int \frac{(ex)^{-1+n}}{a+b\text{sech}\left(c+dx^n\right)}\,dx=\frac{x\,{\left(e\,x\right)}^{n-1}}{a\,n}-\frac{\left(2\,\mathrm{atan}\left(\frac{a^2\,\mathrm{e}^{d\,x^n}\,\mathrm{e}^c\,\left(\frac{2\,b\,x\,{\left(e\,x\right)}^{n-1}}{a^4\,d\,n\,x^n\,\sqrt{b^2\,x^2\,{\left(e\,x\right)}^{2\,n-2}}}+\frac{2\,b\,d\,n\,x^n\,{\left(e\,x\right)}^{1-n}\,\sqrt{b^2\,x^2\,{\left(e\,x\right)}^{2\,n-2}}}{a^2\,x\,\sqrt{a^4\,d^2\,n^2\,x^{2\,n}-a^2\,b^2\,d^2\,n^2\,x^{2\,n}}\,\sqrt{a^2\,d^2\,n^2\,x^{2\,n}\,\left(a^2-b^2\right)}}\right)\,\sqrt{a^4\,d^2\,n^2\,x^{2\,n}-a^2\,b^2\,d^2\,n^2\,x \cdots}}{2}\cdots\right)\cdots\right)}{\sqrt{a^4\,d^2\,n^2\,x^{2\,n}-a^2\,b^2\,\cdots}}$$

[In] int((e*x)^(n - 1)/(a + b/cosh(c + d*x^n)),x)

[Out] (x*(e*x)^(n - 1))/(a*n) - ((2*atan((a^2*exp(d*x^n)*exp(c)*((2*b*x*(e*x)^(n - 1))/(a^4*d*n*x^n*(b^2*x^2*(e*x)^(2*n - 2))^(1/2)) + (2*b*d*n*x^n*(e*x)^(1 - n)*(b^2*x^2*(e*x)^(2*n - 2))^(1/2))/(a^2*x*(a^4*d^2*n^2*x^(2*n) - a^2*b^2*d^2*n^2*x^(2*n))^(1/2)*(a^2*d^2*n^2*x^(2*n)*(a^2 - b^2))^(1/2)))*(a^4*d^2*n^2*x^(2*n) - a^2*b^2*d^2*n^2*x^(2*n))^(1/2))/2 + (a*d*n*x^n*(e*x)^(1 - n)*(b^2*x^2*(e*x)^(2*n - 2))^(1/2))/(x*(a^2*d^2*n^2*x^(2*n)*(a^2 - b^2))^(1/2))) + 2*atan((x*(e*x)^(n - 1)*(a^2*d^2*n^2*x^(2*n)*(a^2 - b^2))^(1/2))/(a*d*n*x^n*(b^2*x^2*(e*x)^(2*n - 2))^(1/2))))*(b^2*x^2*(e*x)^(2*n - 2))^(1/2))/(a^4*d^2*n^2*x^(2*n) - a^2*b^2*d^2*n^2*x^(2*n))^(1/2)

3.80 $\int \frac{(ex)^{-1+2n}}{a+b\,\mathbf{sech}(c+dx^n)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 307

$$\begin{aligned}\int \frac{(ex)^{-1+2n}}{a+b\operatorname{sech}(c+dx^n)}\,dx &= \frac{(ex)^{2n}}{2aen} - \frac{bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} \\ &+ \frac{bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} \\ &- \frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} \\ &+ \frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}\end{aligned}$$

```
[Out] 1/2*(e*x)^(2*n)/a/e/n-b*(e*x)^(2*n)*ln(1+a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)
))/a/d/e/n/(x^n)/(-a^2+b^2)^(1/2)+b*(e*x)^(2*n)*ln(1+a*exp(c+d*x^n)/(b+(-a^
2+b^2)^(1/2)))/a/d/e/n/(x^n)/(-a^2+b^2)^(1/2)-b*(e*x)^(2*n)*polylog(2,-a*ex
p(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a/d^2/e/n/(x^(2*n))/(-a^2+b^2)^(1/2)+b*(e*
x)^(2*n)*polylog(2,-a*exp(c+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a/d^2/e/n/(x^(2*n)
)/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5548, 5544, 4276, 3401, 2296, 2221, 2317, 2438}

$$\int \frac{(ex)^{-1+2n}}{a+b\text{sech}(c+dx^n)}\,dx = -\frac{bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} + \frac{bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} - \frac{bx^{-n}(ex)^{2n}\log\left(\frac{ae^{c+dx^n}}{b-\sqrt{b^2-a^2}}+1\right)}{aden\sqrt{b^2-a^2}} + \frac{bx^{-n}(ex)^{2n}\log\left(\frac{ae^{c+dx^n}}{\sqrt{b^2-a^2}+b}+1\right)}{aden\sqrt{b^2-a^2}} + \frac{(ex)^{2n}}{2aen}$$

```
[In] Int[(e*x)^(-1 + 2*n)/(a + b*Sech[c + d*x^n]),x]
```

```
[Out] (e*x)^(2*n)/(2*a*e*n) - (b*(e*x)^(2*n)*Log[1 + (a*E^(c + d*x^n))/(b - Sqrt[
-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d*e*n*x^n) + (b*(e*x)^(2*n)*Log[1 + (a*E
^(c + d*x^n))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d*e*n*x^n) - (b*
(e*x)^(2*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqr
t[-a^2 + b^2]*d^2*e*n*x^(2*n)) + (b*(e*x)^(2*n)*PolyLog[2, -((a*E^(c + d*x^
n))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2*e*n*x^(2*n))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5548

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(x^{-2n}(ex)^{2n}\right)\int \frac{x^{-1+2n}}{a+b\operatorname{sech}(c+dx^n)}\,dx}{e}$$
$$= \frac{\left(x^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int \frac{x}{a+b\operatorname{sech}(c+dx)}\,dx, x, x^n\right)}{en}$$
$$= \frac{\left(x^{-2n}(ex)^{2n}\right)\operatorname{Subst}\left(\int \left(\frac{x}{a}-\frac{bx}{a(b+a\cosh(c+dx))}\right)\,dx, x, x^n\right)}{en}$$

$$= \frac{(ex)^{2n}}{2aen} - \frac{(bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{x}{b+a\cosh(c+dx)}\,dx, x, x^n\right)}{aen}$$

$$= \frac{(ex)^{2n}}{2aen} - \frac{(2bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{e^{c+dx}x}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^n\right)}{aen}$$

$$= \frac{(ex)^{2n}}{2aen} - \frac{(2bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{e^{c+dx}x}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, x^n\right)}{\sqrt{-a^2+b^2}en} + \frac{(2bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{e^{c+dx}x}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, x^n\right)}{\sqrt{-a^2+b^2}en}$$

$$= \frac{(ex)^{2n}}{2aen} - \frac{bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{(bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, x^n\right)}{a\sqrt{-a^2+b^2}den} - \frac{(bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, x^n\right)}{a\sqrt{-a^2+b^2}den}$$

$$= \frac{(ex)^{2n}}{2aen} - \frac{bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{(bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{\log\left(1+\frac{2ax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{c+dx^n}\right)}{a\sqrt{-a^2+b^2}d^2en} - \frac{(bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{\log\left(1+\frac{2ax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{c+dx^n}\right)}{a\sqrt{-a^2+b^2}d^2en}$$

$$= \frac{(ex)^{2n}}{2aen} - \frac{bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} - \frac{bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} + \frac{bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.70 (sec) , antiderivative size = 859, normalized size of antiderivative = 2.80

$$\int \frac{(ex)^{-1+2n}}{a+b\text{sech}(c+dx^n)}\,dx$$
$$= \frac{(ex)^{2n}(b+a\cosh(c+dx^n))\left(1+\frac{2bx^{-2n}\left(2(c+dx^n)\arctan\left(\frac{(a+b)\coth\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right)+2\left(c-i\arccos\left(-\frac{b}{a}\right)\right)\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}\right.}{\sqrt{a^2-}}\right.\right.}{}\right.}{}$$

```
[In] Integrate[(e*x)^(-1 + 2*n)/(a + b*Sech[c + d*x^n]),x]
```

```
[Out] ((e*x)^(2*n)*(b + a*Cosh[c + d*x^n])*(1 + (2*b*(2*(c + d*x^n)*ArcTan[((a +
b)*Coth[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + 2*(c - I*ArcCos[-(b/a)])*ArcTan[
((a - b)*Tanh[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + (ArcCos[-(b/a)] + 2*(ArcTa
n[((a + b)*Coth[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + ArcTan[((a - b)*Tanh[(c
+ d*x^n)/2])/Sqrt[a^2 - b^2]]))*Log[(Sqrt[a^2 - b^2]*E^(-1/2*c - (d*x^n)/2)
)/(Sqrt[2]*Sqrt[a]*Sqrt[b + a*Cosh[c + d*x^n]])] + (ArcCos[-(b/a)] - 2*(Arc
Tan[((a + b)*Coth[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + ArcTan[((a - b)*Tanh[(
c + d*x^n)/2])/Sqrt[a^2 - b^2]]))*Log[(Sqrt[a^2 - b^2]*E^((c + d*x^n)/2))/(
Sqrt[2]*Sqrt[a]*Sqrt[b + a*Cosh[c + d*x^n]])] - (ArcCos[-(b/a)] + 2*ArcTan[
((a - b)*Tanh[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Log[((a + b)*(-a + b + I*Sq
rt[a^2 - b^2])*(-1 + Tanh[(c + d*x^n)/2]))/(a*(a + b + I*Sqrt[a^2 - b^2]*Ta
nh[(c + d*x^n)/2]))] - (ArcCos[-(b/a)] - 2*ArcTan[((a - b)*Tanh[(c + d*x^n)
/2])/Sqrt[a^2 - b^2]])*Log[((a + b)*(a - b + I*Sqrt[a^2 - b^2])*(1 + Tanh[(
c + d*x^n)/2]))/(a*(a + b + I*Sqrt[a^2 - b^2]*Tanh[(c + d*x^n)/2]))] + I*(P
olyLog[2, ((b - I*Sqrt[a^2 - b^2])*(a + b - I*Sqrt[a^2 - b^2]*Tanh[(c + d*x
^n)/2]))/(a*(a + b + I*Sqrt[a^2 - b^2]*Tanh[(c + d*x^n)/2]))] - PolyLog[2,
((b + I*Sqrt[a^2 - b^2])*(a + b - I*Sqrt[a^2 - b^2]*Tanh[(c + d*x^n)/2]))/(
a*(a + b + I*Sqrt[a^2 - b^2]*Tanh[(c + d*x^n)/2]))])))/(Sqrt[a^2 - b^2]*d^2
*x^(2*n)))*Sech[c + d*x^n])/(2*a*e*n*(a + b*Sech[c + d*x^n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.78 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.91

method	result
risch	$\frac{x\,\mathrm{e}^{\frac{(2n-1)\left(-i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)\pi+i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^2\pi+i\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^2\pi-i\,\mathrm{csgn}(iex)^3\pi+2\ln(e)+2\ln(x)\right)}{2}}}{2an} - \frac{2b\,\mathrm{e}^{-i\pi n\,\mathrm{csgn}(ie)\,\cdots}}{}$

```
[In] int((e*x)^(2*n-1)/(a+b*sech(c+d*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a/n*x*exp(1/2*(2*n-1)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e
)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+
2*ln(x)))-2*b/a*exp(-I*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(I*Pi*n*csg
n(I*e)*csgn(I*e*x)^2)*exp(I*Pi*n*csgn(I*x)*csgn(I*e*x)^2)*exp(-I*Pi*n*csgn(
I*e*x)^3)*exp(1/2*I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(-1/2*I*Pi*csgn(
I*e)*csgn(I*e*x)^2)*exp(-1/2*I*Pi*csgn(I*x)*csgn(I*e*x)^2)*exp(1/2*I*Pi*csg
n(I*e*x)^3)*(e^n)^2/e*exp(c)/n/d^2*(1/2*x^n*d*(ln((-a*exp(2*c+d*x^n)-exp(c)
*b+(exp(2*c)*b^2-a^2*exp(2*c))^(1/2))/(-exp(c)*b+(exp(2*c)*b^2-a^2*exp(2*c)
)^(1/2)))-ln((a*exp(2*c+d*x^n)+exp(c)*b+(exp(2*c)*b^2-a^2*exp(2*c))^(1/2))/
(exp(c)*b+(exp(2*c)*b^2-a^2*exp(2*c))^(1/2))))/(exp(2*c)*b^2-a^2*exp(2*c))^
(1/2)+1/2*(dilog((-a*exp(2*c+d*x^n)-exp(c)*b+(exp(2*c)*b^2-a^2*exp(2*c))^(1
/2))/(-exp(c)*b+(exp(2*c)*b^2-a^2*exp(2*c))^(1/2)))-dilog((a*exp(2*c+d*x^n)
+exp(c)*b+(exp(2*c)*b^2-a^2*exp(2*c))^(1/2))/(exp(c)*b+(exp(2*c)*b^2-a^2*ex
p(2*c))^(1/2))))/(exp(2*c)*b^2-a^2*exp(2*c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1286 vs. $2(287) = 574$.

Time = 0.30 (sec) , antiderivative size = 1286, normalized size of antiderivative = 4.19

$$\int \frac{(ex)^{-1+2n}}{a+b\operatorname{sech}(c+dx^n)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*sech(c+d*x^n)),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 - b^2)*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + (a^2 - b^2)*
d^2*cosh(n*log(x))^2*sinh((2*n - 1)*log(e)) + ((a^2 - b^2)*d^2*cosh((2*n -
1)*log(e)) + (a^2 - b^2)*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x))^2 + 2*(
a*b*sqrt(-(a^2 - b^2)/a^2)*cosh((2*n - 1)*log(e)) + a*b*sqrt(-(a^2 - b^2)/a
^2)*sinh((2*n - 1)*log(e)))*dilog(-((a*sqrt(-(a^2 - b^2)/a^2) + b)*cosh(d*c
osh(n*log(x)) + d*sinh(n*log(x)) + c) + (a*sqrt(-(a^2 - b^2)/a^2) + b)*sinh
(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + a)/a + 1) - 2*(a*b*sqrt(-(a^2 -
 b^2)/a^2)*cosh((2*n - 1)*log(e)) + a*b*sqrt(-(a^2 - b^2)/a^2)*sinh((2*n -
1)*log(e)))*dilog(((a*sqrt(-(a^2 - b^2)/a^2) - b)*cosh(d*cosh(n*log(x)) + d
*sinh(n*log(x)) + c) + (a*sqrt(-(a^2 - b^2)/a^2) - b)*sinh(d*cosh(n*log(x))
 + d*sinh(n*log(x)) + c) - a)/a + 1) + 2*(a*b*c*sqrt(-(a^2 - b^2)/a^2)*cosh
((2*n - 1)*log(e)) + a*b*c*sqrt(-(a^2 - b^2)/a^2)*sinh((2*n - 1)*log(e)))*l
og(2*a*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 2*a*sinh(d*cosh(n*lo
g(x)) + d*sinh(n*log(x)) + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) - 2*(a*b*
c*sqrt(-(a^2 - b^2)/a^2)*cosh((2*n - 1)*log(e)) + a*b*c*sqrt(-(a^2 - b^2)/a
^2)*sinh((2*n - 1)*log(e)))*log(2*a*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)
) + c) + 2*a*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - 2*a*sqrt(-(a^2
 - b^2)/a^2) + 2*b) + 2*(a*b*d*sqrt(-(a^2 - b^2)/a^2)*cosh((2*n - 1)*log(e)
```

```
)*cosh(n*log(x)) + a*b*c*sqrt(-(a^2 - b^2)/a^2)*cosh((2*n - 1)*log(e)) + (a
*b*d*sqrt(-(a^2 - b^2)/a^2)*cosh(n*log(x)) + a*b*c*sqrt(-(a^2 - b^2)/a^2))*
sinh((2*n - 1)*log(e)) + (a*b*d*sqrt(-(a^2 - b^2)/a^2)*cosh((2*n - 1)*log(e
)) + a*b*d*sqrt(-(a^2 - b^2)/a^2)*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*l
og(((a*sqrt(-(a^2 - b^2)/a^2) + b)*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x))
 + c) + (a*sqrt(-(a^2 - b^2)/a^2) + b)*sinh(d*cosh(n*log(x)) + d*sinh(n*log
(x)) + c) + a)/a) - 2*(a*b*d*sqrt(-(a^2 - b^2)/a^2)*cosh((2*n - 1)*log(e))*
cosh(n*log(x)) + a*b*c*sqrt(-(a^2 - b^2)/a^2)*cosh((2*n - 1)*log(e)) + (a*b
*d*sqrt(-(a^2 - b^2)/a^2)*cosh(n*log(x)) + a*b*c*sqrt(-(a^2 - b^2)/a^2))*si
nh((2*n - 1)*log(e)) + (a*b*d*sqrt(-(a^2 - b^2)/a^2)*cosh((2*n - 1)*log(e))
 + a*b*d*sqrt(-(a^2 - b^2)/a^2)*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*log
(-((a*sqrt(-(a^2 - b^2)/a^2) - b)*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x))
+ c) + (a*sqrt(-(a^2 - b^2)/a^2) - b)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(
x)) + c) - a)/a) + 2*((a^2 - b^2)*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))
 + (a^2 - b^2)*d^2*cosh(n*log(x))*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))/(
(a^3 - a*b^2)*d^2*n)
```

Sympy [F]

$$\int \frac{(ex)^{-1+2n}}{a + b\text{sech}\,(c + dx^n)}\,dx = \int \frac{(ex)^{2n-1}}{a + b\,\text{sech}\,(c + dx^n)}\,dx$$

```
[In] integrate((e*x)**(-1+2*n)/(a+b*sech(c+d*x**n)),x)
[Out] Integral((e*x)**(2*n - 1)/(a + b*sech(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+2n}}{a + b\text{sech}\,(c + dx^n)}\,dx = \int \frac{(ex)^{2\,n-1}}{b\,\text{sech}\,(dx^n + c) + a}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*sech(c+d*x^n)),x, algorithm="maxima")
[Out] -2*b*e^(2*n)*integrate(e^(d*x^n + 2*n*log(x) + c)/(a^2*e*x*e^(2*d*x^n + 2*c
) + 2*a*b*e*x*e^(d*x^n + c) + a^2*e*x), x) + 1/2*e^(2*n - 1)*x^(2*n)/(a*n)
```

Giac [F]

$$\int \frac{(ex)^{-1+2n}}{a+b\text{sech}\left(c+dx^n\right)}\,dx = \int \frac{(ex)^{2\,n-1}}{b\,\text{sech}\left(dx^n+c\right)+a}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*sech(c+d*x^n)),x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)/(b*sech(d*x^n + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+2n}}{a+b\text{sech}\left(c+dx^n\right)}\,dx = \int \frac{(e\,x)^{2\,n-1}}{a+\frac{b}{\cosh(c+d\,x^n)}}\,dx$$

```
[In] int((e*x)^(2*n - 1)/(a + b/cosh(c + d*x^n)),x)
[Out] int((e*x)^(2*n - 1)/(a + b/cosh(c + d*x^n)), x)
```

3.81 $\int \frac{(ex)^{-1+3n}}{a+b\operatorname{sech}(c+dx^n)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 452

$$\begin{aligned}\int \frac{(ex)^{-1+3n}}{a+b\operatorname{sech}(c+dx^n)}\,dx &= \frac{(ex)^{3n}}{3aen}-\frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}\\
&+\frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}\\
&-\frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}\\
&+\frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}\\
&+\frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en}\\
&-\frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en}\end{aligned}$$

```
[Out] 1/3*(e*x)^(3*n)/a/e/n-b*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)
))/a/d/e/n/(x^n)/(-a^2+b^2)^(1/2)+b*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b+(-a^
2+b^2)^(1/2)))/a/d/e/n/(x^n)/(-a^2+b^2)^(1/2)-2*b*(e*x)^(3*n)*polylog(2,-a*
exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a/d^2/e/n/(x^(2*n))/(-a^2+b^2)^(1/2)+2*b
*(e*x)^(3*n)*polylog(2,-a*exp(c+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a/d^2/e/n/(x^(
2*n))/(-a^2+b^2)^(1/2)+2*b*(e*x)^(3*n)*polylog(3,-a*exp(c+d*x^n)/(b-(-a^2+b
^2)^(1/2)))/a/d^3/e/n/(x^(3*n))/(-a^2+b^2)^(1/2)-2*b*(e*x)^(3*n)*polylog(3,
-a*exp(c+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a/d^3/e/n/(x^(3*n))/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5548, 5544, 4276, 3401, 2296, 2221, 2611, 2320, 6724}

$$\int \frac{(ex)^{-1+3n}}{a+b\text{sech}(c+dx^n)}\,dx = \frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)}{ad^3en\sqrt{b^2-a^2}} - \frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)}{ad^3en\sqrt{b^2-a^2}} - \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} - \frac{bx^{-n}(ex)^{3n}\log\left(\frac{ae^{c+dx^n}}{b-\sqrt{b^2-a^2}}+1\right)}{aden\sqrt{b^2-a^2}} + \frac{bx^{-n}(ex)^{3n}\log\left(\frac{ae^{c+dx^n}}{\sqrt{b^2-a^2}+b}+1\right)}{aden\sqrt{b^2-a^2}} + \frac{(ex)^{3n}}{3aen}$$

```
[In] Int[(e*x)^(-1 + 3*n)/(a + b*Sech[c + d*x^n]),x]
```

```
[Out] (e*x)^(3*n)/(3*a*e*n) - (b*(e*x)^(3*n)*Log[1 + (a*E^(c + d*x^n))/(b - Sqrt[
-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d*e*n*x^n) + (b*(e*x)^(3*n)*Log[1 + (a*E
^(c + d*x^n))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d*e*n*x^n) - (2*
b*(e*x)^(3*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]))])/(a*S
qrt[-a^2 + b^2]*d^2*e*n*x^(2*n)) + (2*b*(e*x)^(3*n)*PolyLog[2, -((a*E^(c +
d*x^n))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2*e*n*x^(2*n)) + (2
*b*(e*x)^(3*n)*PolyLog[3, -((a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]))])/(a*
Sqrt[-a^2 + b^2]*d^3*e*n*x^(3*n)) - (2*b*(e*x)^(3*n)*PolyLog[3, -((a*E^(c +
 d*x^n))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3*e*n*x^(3*n))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5548

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
```

```
a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\left(x^{-3n}(ex)^{3n}\right)\int \frac{x^{-1+3n}}{a+b\,\text{sech}(c+dx^n)}\,dx}{e}$$

$$= \frac{\left(x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{x^2}{a+b\,\text{sech}(c+dx)}\,dx, x, x^n\right)}{en}$$

$$= \frac{\left(x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \left(\frac{x^2}{a}-\frac{bx^2}{a(b+a\cosh(c+dx))}\right)dx, x, x^n\right)}{en}$$

$$= \frac{(ex)^{3n}}{3aen} - \frac{\left(bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{x^2}{b+a\cosh(c+dx)}\,dx, x, x^n\right)}{aen}$$

$$= \frac{(ex)^{3n}}{3aen} - \frac{\left(2bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^n\right)}{aen}$$

$$= \frac{(ex)^{3n}}{3aen} - \frac{\left(2bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, x^n\right)}{\sqrt{-a^2+b^2}\,en} + \frac{\left(2bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{e^{c+dx}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, x^n\right)}{\sqrt{-a^2+b^2}\,en}$$

$$= \frac{(ex)^{3n}}{3aen} - \frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,den} + \frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}\,den} + \frac{\left(2bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, x^n\right)}{a\sqrt{-a^2+b^2}\,den} - \frac{\left(2bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, x^n\right)}{a\sqrt{-a^2+b^2}\,den}$$

$$= \frac{(ex)^{3n}}{3aen} - \frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}$$
$$- \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}$$
$$+ \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}$$
$$+ \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,x^n\right)}{a\sqrt{-a^2+b^2}d^2en}$$
$$- \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,x^n\right)}{a\sqrt{-a^2+b^2}d^2en}$$

$$= \frac{(ex)^{3n}}{3aen} - \frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}$$
$$- \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}$$
$$+ \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}$$
$$+ \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{c+dx^n}\right)}{a\sqrt{-a^2+b^2}d^3en}$$
$$- \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{c+dx^n}\right)}{a\sqrt{-a^2+b^2}d^3en}$$

$$= \frac{(ex)^{3n}}{3aen} - \frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}$$
$$- \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}$$
$$+ \frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en} - \frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en}$$

Mathematica [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\text{sech}\left(c+dx^n\right)}\,dx=\int \frac{(ex)^{-1+3n}}{a+b\text{sech}\left(c+dx^n\right)}\,dx$$

```
[In] Integrate[(e*x)^(-1 + 3*n)/(a + b*Sech[c + d*x^n]),x]
[Out] Integrate[(e*x)^(-1 + 3*n)/(a + b*Sech[c + d*x^n]), x]
```

Maple [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\,\text{sech}\left(c+d\,x^{n}\right)}dx$$

```
[In] int((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n)),x)
[Out] int((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2005 vs. $2(426) = 852$.

Time = 0.32 (sec) , antiderivative size = 2005, normalized size of antiderivative = 4.44

$$\int \frac{(ex)^{-1+3n}}{a+b\text{sech}\left(c+dx^n\right)}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n)),x, algorithm="fricas")
[Out] 1/3*((a^2 - b^2)*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x))^3 + (a^2 - b^2)*
d^3*cosh(n*log(x))^3*sinh((3*n - 1)*log(e)) + ((a^2 - b^2)*d^3*cosh((3*n -
1)*log(e)) + (a^2 - b^2)*d^3*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^3 + 3*(
(a^2 - b^2)*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + (a^2 - b^2)*d^3*cos
h(n*log(x))*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 + 6*(a*b*d*sqrt(-(a^2
- b^2)/a^2)*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + a*b*d*sqrt(-(a^2 - b^2)
/a^2)*cosh(n*log(x))*sinh((3*n - 1)*log(e)) + (a*b*d*sqrt(-(a^2 - b^2)/a^2)
*cosh((3*n - 1)*log(e)) + a*b*d*sqrt(-(a^2 - b^2)/a^2)*sinh((3*n - 1)*log(e
)))*sinh(n*log(x)))*dilog(-((a*sqrt(-(a^2 - b^2)/a^2) + b)*cosh(d*cosh(n*lo
g(x)) + d*sinh(n*log(x)) + c) + (a*sqrt(-(a^2 - b^2)/a^2) + b)*sinh(d*cosh(
n*log(x)) + d*sinh(n*log(x)) + c) + a)/a + 1) - 6*(a*b*d*sqrt(-(a^2 - b^2)/
a^2)*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + a*b*d*sqrt(-(a^2 - b^2)/a^2)*c
osh(n*log(x))*sinh((3*n - 1)*log(e)) + (a*b*d*sqrt(-(a^2 - b^2)/a^2)*cosh((
3*n - 1)*log(e)) + a*b*d*sqrt(-(a^2 - b^2)/a^2)*sinh((3*n - 1)*log(e)))*sin
h(n*log(x)))*dilog(((a*sqrt(-(a^2 - b^2)/a^2) - b)*cosh(d*cosh(n*log(x)) +
d*sinh(n*log(x)) + c) + (a*sqrt(-(a^2 - b^2)/a^2) - b)*sinh(d*cosh(n*log(x)
```

```
) + d*sinh(n*log(x)) + c) - a)/a + 1) - 3*(a*b*c^2*sqrt(-(a^2 - b^2)/a^2)*c
osh((3*n - 1)*log(e)) + a*b*c^2*sqrt(-(a^2 - b^2)/a^2)*sinh((3*n - 1)*log(e
)))*log(2*a*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 2*a*sinh(d*cosh
(n*log(x)) + d*sinh(n*log(x)) + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) + 3*
(a*b*c^2*sqrt(-(a^2 - b^2)/a^2)*cosh((3*n - 1)*log(e)) + a*b*c^2*sqrt(-(a^2
 - b^2)/a^2)*sinh((3*n - 1)*log(e)))*log(2*a*cosh(d*cosh(n*log(x)) + d*sinh
(n*log(x)) + c) + 2*a*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - 2*a*s
qrt(-(a^2 - b^2)/a^2) + 2*b) + 3*(a*b*d^2*sqrt(-(a^2 - b^2)/a^2)*cosh((3*n
- 1)*log(e))*cosh(n*log(x))^2 - a*b*c^2*sqrt(-(a^2 - b^2)/a^2)*cosh((3*n -
1)*log(e)) + (a*b*d^2*sqrt(-(a^2 - b^2)/a^2)*cosh((3*n - 1)*log(e)) + a*b*d
^2*sqrt(-(a^2 - b^2)/a^2)*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 + (a*b*d
^2*sqrt(-(a^2 - b^2)/a^2)*cosh(n*log(x))^2 - a*b*c^2*sqrt(-(a^2 - b^2)/a^2)
)*sinh((3*n - 1)*log(e)) + 2*(a*b*d^2*sqrt(-(a^2 - b^2)/a^2)*cosh((3*n - 1)
*log(e))*cosh(n*log(x)) + a*b*d^2*sqrt(-(a^2 - b^2)/a^2)*cosh(n*log(x))*sin
h((3*n - 1)*log(e)))*sinh(n*log(x)))*log(((a*sqrt(-(a^2 - b^2)/a^2) + b)*co
sh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a*sqrt(-(a^2 - b^2)/a^2) + b
)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + a)/a) - 3*(a*b*d^2*sqrt(-
(a^2 - b^2)/a^2)*cosh((3*n - 1)*log(e))*cosh(n*log(x))^2 - a*b*c^2*sqrt(-(a
^2 - b^2)/a^2)*cosh((3*n - 1)*log(e)) + (a*b*d^2*sqrt(-(a^2 - b^2)/a^2)*cos
h((3*n - 1)*log(e)) + a*b*d^2*sqrt(-(a^2 - b^2)/a^2)*sinh((3*n - 1)*log(e))
)*sinh(n*log(x))^2 + (a*b*d^2*sqrt(-(a^2 - b^2)/a^2)*cosh(n*log(x))^2 - a*b
*c^2*sqrt(-(a^2 - b^2)/a^2))*sinh((3*n - 1)*log(e)) + 2*(a*b*d^2*sqrt(-(a^2
 - b^2)/a^2)*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + a*b*d^2*sqrt(-(a^2 - b
^2)/a^2)*cosh(n*log(x))*sinh((3*n - 1)*log(e)))*sinh(n*log(x)))*log(-((a*sq
rt(-(a^2 - b^2)/a^2) - b)*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (
a*sqrt(-(a^2 - b^2)/a^2) - b)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)
 - a)/a) - 6*(a*b*sqrt(-(a^2 - b^2)/a^2)*cosh((3*n - 1)*log(e)) + a*b*sqrt(
-(a^2 - b^2)/a^2)*sinh((3*n - 1)*log(e)))*polylog(3, -((a*sqrt(-(a^2 - b^2)
/a^2) + b)*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a*sqrt(-(a^2 -
b^2)/a^2) + b)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c))/a) + 6*(a*b*s
qrt(-(a^2 - b^2)/a^2)*cosh((3*n - 1)*log(e)) + a*b*sqrt(-(a^2 - b^2)/a^2)*s
inh((3*n - 1)*log(e)))*polylog(3, ((a*sqrt(-(a^2 - b^2)/a^2) - b)*cosh(d*co
sh(n*log(x)) + d*sinh(n*log(x)) + c) + (a*sqrt(-(a^2 - b^2)/a^2) - b)*sinh(
d*cosh(n*log(x)) + d*sinh(n*log(x)) + c))/a) + 3*((a^2 - b^2)*d^3*cosh((3*n
 - 1)*log(e))*cosh(n*log(x))^2 + (a^2 - b^2)*d^3*cosh(n*log(x))^2*sinh((3*n
 - 1)*log(e)))*sinh(n*log(x)))/((a^3 - a*b^2)*d^3*n)
```

Sympy [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\text{sech}\,(c+dx^n)}\,dx = \int \frac{(ex)^{3n-1}}{a+b\,\text{sech}\,(c+dx^n)}\,dx$$

```
[In] integrate((e*x)**(-1+3*n)/(a+b*sech(c+d*x**n)),x)
[Out] Integral((e*x)**(3*n - 1)/(a + b*sech(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\text{sech}\,(c+dx^n)}\,dx = \int \frac{(ex)^{3n-1}}{b\,\text{sech}\,(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n)),x, algorithm="maxima")
[Out] -2*b*e^(3*n)*integrate(e^(d*x^n + 3*n*log(x) + c)/(a^2*e*x*e^(2*d*x^n + 2*c
) + 2*a*b*e*x*e^(d*x^n + c) + a^2*e*x), x) + 1/3*e^(3*n - 1)*x^(3*n)/(a*n)
```

Giac [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\text{sech}\,(c+dx^n)}\,dx = \int \frac{(ex)^{3n-1}}{b\,\text{sech}\,(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n)),x, algorithm="giac")
[Out] integrate((e*x)^(3*n - 1)/(b*sech(d*x^n + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+3n}}{a+b\text{sech}\,(c+dx^n)}\,dx = \int \frac{(e\,x)^{3\,n-1}}{a+\frac{b}{\cosh(c+d\,x^n)}}\,dx$$

```
[In] int((e*x)^(3*n - 1)/(a + b/cosh(c + d*x^n)),x)
[Out] int((e*x)^(3*n - 1)/(a + b/cosh(c + d*x^n)), x)
```

3.82 $\int \frac{(ex)^{-1+n}}{(a+b\,\text{sech}(c+dx^n))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{(ex)^{-1+n}}{(a+b\,\text{sech}(c+dx^n))^2}\,dx = \frac{(ex)^n}{a^2en} - \frac{2b(2a^2-b^2)\,x^{-n}(ex)^n\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}den} + \frac{b^2x^{-n}(ex)^n\tanh(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\,\text{sech}(c+dx^n))}$$

```
[Out] (e*x)^n/a^2/e/n-2*b*(2*a^2-b^2)*(e*x)^n*arctan((a-b)^(1/2)*tanh(1/2*c+1/2*d
*x^n)/(a+b)^(1/2))/a^2/(a-b)^(3/2)/(a+b)^(3/2)/d/e/n/(x^n)+b^2*(e*x)^n*tanh
(c+d*x^n)/a/(a^2-b^2)/d/e/n/(x^n)/(a+b*sech(c+d*x^n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5548, 5544, 3870, 4004, 3916, 2738, 214}

$$\int \frac{(ex)^{-1+n}}{(a+b\,\text{sech}(c+dx^n))^2}\,dx = -\frac{2b(2a^2-b^2)\,x^{-n}(ex)^n\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a^2den(a-b)^{3/2}(a+b)^{3/2}} + \frac{b^2x^{-n}(ex)^n\tanh(c+dx^n)}{aden\,(a^2-b^2)\,(a+b\,\text{sech}(c+dx^n))} + \frac{(ex)^n}{a^2en}$$

```
[In] Int[(e*x)^(-1 + n)/(a + b*Sech[c + d*x^n])^2,x]
```

```
[Out] (e*x)^n/(a^2*e*n) - (2*b*(2*a^2 - b^2)*(e*x)^n*ArcTan[(Sqrt[a - b]*Tanh[(c
+ d*x^n)/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d*e*n*x^n) + (b
```

```
^2*(e*x)^n*Tanh[c + d*x^n])/(a*(a^2 - b^2)*d*e*n*x^n*(a + b*Sech[c + d*x^n]
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5548

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
```

```
a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{(x^{-n}(ex)^n)\int \frac{x^{-1+n}}{(a+b\text{sech}(c+dx^n))^2}\,dx}{e}$$

$$= \frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{1}{(a+b\text{sech}(c+dx))^2}\,dx, x, x^n\right)}{en}$$

$$= \frac{b^2x^{-n}(ex)^n\tanh(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\text{sech}(c+dx^n))} - \frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{-a^2+b^2+ab\text{sech}(c+dx)}{a+b\text{sech}(c+dx)}\,dx, x, x^n\right)}{a(a^2-b^2)\,en}$$

$$= \frac{(ex)^n}{a^2en} + \frac{b^2x^{-n}(ex)^n\tanh(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\text{sech}(c+dx^n))} + \frac{((-a^2b+b(-a^2+b^2))\,x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{\text{sech}(c+dx)}{a+b\text{sech}(c+dx)}\,dx, x, x^n\right)}{a^2(a^2-b^2)\,en}$$

$$= \frac{(ex)^n}{a^2en} + \frac{b^2x^{-n}(ex)^n\tanh(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\text{sech}(c+dx^n))} + \frac{((-a^2b+b(-a^2+b^2))\,x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{1}{1+\frac{a\cosh(c+dx)}{b}}\,dx, x, x^n\right)}{a^2b(a^2-b^2)\,en}$$

$$= \frac{(ex)^n}{a^2en} + \frac{b^2x^{-n}(ex)^n\tanh(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\text{sech}(c+dx^n))} - \frac{(2i(-a^2b+b(-a^2+b^2))\,x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, i\tanh\left(\frac{1}{2}(c+dx^n)\right)\right)}{a^2b(a^2-b^2)\,den}$$

$$= \frac{(ex)^n}{a^2en} - \frac{2b(2a^2-b^2)\,x^{-n}(ex)^n\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}den} + \frac{b^2x^{-n}(ex)^n\tanh(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\text{sech}(c+dx^n))}$$

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.48

$$\int \frac{(ex)^{-1+n}}{(a+b\operatorname{sech}(c+dx^n))^2}\,dx$$
$$=\frac{x^{-n}(ex)^n\left(a\left((a^2-b^2)^{3/2}(c+dx^n)+(4a^2b-2b^3)\arctan\left(\frac{(-a+b)\tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right)\right)\cosh(c+dx^n)+b\left((a^2-\right.\right.}{a^2(a-b)(a+b)\sqrt{a^2-b^2}den\,(b+}$$

```
[In] Integrate[(e*x)^(-1 + n)/(a + b*Sech[c + d*x^n])^2,x]
```

```
[Out] ((e*x)^n*(a*((a^2 - b^2)^(3/2)*(c + d*x^n) + (4*a^2*b - 2*b^3)*ArcTan[((-a
+ b)*Tanh[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Cosh[c + d*x^n] + b*((a^2 - b^2
)^(3/2)*(c + d*x^n) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*x^n)/2
])/Sqrt[a^2 - b^2]] + a*b*Sqrt[a^2 - b^2]*Sinh[c + d*x^n])))/(a^2*(a - b)*(
a + b)*Sqrt[a^2 - b^2]*d*e*n*x^n*(b + a*Cosh[c + d*x^n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.45 (sec) , antiderivative size = 491, normalized size of antiderivative = 3.13

method	result
risch	$\frac{x\,\mathrm{e}^{\frac{(-1+n)\left(-i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)\pi+i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^2\pi+i\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^2\pi-i\,\mathrm{csgn}(iex)^3\pi+2\ln(e)+2\ln(x)\right)}{2}}}{a^2n}-\frac{2b^2\mathrm{e}^{\frac{(-1+n)\left(-i\,\mathrm{csg}\right.}{}}}{}$

```
[In] int((e*x)^(-1+n)/(a+b*sech(c+d*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)
*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2
*ln(x)))-2*b^2*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn
(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln
(e)+2*ln(x)))*x*(b*exp(c+d*x^n)+a)/a^2/(a^2-b^2)/d/n/(x^n)/(2*b*exp(c+d*x^n
)+a*exp(2*c+2*d*x^n)+a)-2*b/a^2*(2*a^2-b^2)/(a^2-b^2)/n*exp(-1/2*I*Pi*n*csg
n(I*e)*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*Pi*n*csgn(I*e)*csgn(I*e*x)^2)*exp(1
/2*I*Pi*n*csgn(I*x)*csgn(I*e*x)^2)*exp(-1/2*I*Pi*n*csgn(I*e*x)^3)*exp(1/2*I
*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(-1/2*I*Pi*csgn(I*e)*csgn(I*e*x)^2)
*exp(-1/2*I*Pi*csgn(I*x)*csgn(I*e*x)^2)*exp(1/2*I*Pi*csgn(I*e*x)^3)*e^n/e*e
xp(c)/d/(a^2*exp(2*c)-exp(2*c)*b^2)^(1/2)*arctan(1/2*(2*a*exp(2*c+d*x^n)+2*
exp(c)*b)/(a^2*exp(2*c)-exp(2*c)*b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1758 vs. 2(148) = 296.

Time = 0.32 (sec) , antiderivative size = 3547, normalized size of antiderivative = 22.59

$$\int \frac{(ex)^{-1+n}}{(a+b\operatorname{sech}(c+dx^n))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+n)/(a+b*sech(c+d*x^n))^2,x, algorithm="fricas")
```

```
[Out] [((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + ((a^5 -
 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^5 - 2*a^3*b^
2 + a*b^4)*d*cosh(n*log(x))*sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*b^2 + a*b^
4)*d*cosh((n - 1)*log(e)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e)
))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + ((a^5
- 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^5 - 2*a^3*b
^2 + a*b^4)*d*cosh(n*log(x))*sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*b^2 + a*b
^4)*d*cosh((n - 1)*log(e)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e
)))*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + 2*((a
^4*b - 2*a^2*b^3 + b^5)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) - (a^2*b^3 -
b^5)*cosh((n - 1)*log(e)) - (a^2*b^3 - b^5 - (a^4*b - 2*a^2*b^3 + b^5)*d*co
sh(n*log(x)))*sinh((n - 1)*log(e)) + ((a^4*b - 2*a^2*b^3 + b^5)*d*cosh((n -
 1)*log(e)) + (a^4*b - 2*a^2*b^3 + b^5)*d*sinh((n - 1)*log(e)))*sinh(n*log(
x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - 2*(a^3*b^2 - a*b^4)*co
sh((n - 1)*log(e)) - (((2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*cosh((n - 1)*log(
e)) + (2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*sinh((n - 1)*log(e)))*cosh(d*cosh(
n*log(x)) + d*sinh(n*log(x)) + c)^2 + ((2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*c
osh((n - 1)*log(e)) + (2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*sinh((n - 1)*log(e
)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + (2*a^3*b - a*b^3)*sqr
t(-a^2 + b^2)*cosh((n - 1)*log(e)) + (2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*sin
h((n - 1)*log(e)) + 2*((2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2)*cosh((n - 1)*log(
e)) + (2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2)*sinh((n - 1)*log(e)))*cosh(d*cosh(
n*log(x)) + d*sinh(n*log(x)) + c) + 2*((2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2)*c
osh((n - 1)*log(e)) + (2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2)*sinh((n - 1)*log(e
)) + ((2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*cosh((n - 1)*log(e)) + (2*a^3*b -
a*b^3)*sqrt(-a^2 + b^2)*sinh((n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sin
h(n*log(x)) + c))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c))*log((a*b +
 (b^2 + sqrt(-a^2 + b^2)*b)*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) +
 (a^2 - b^2 - sqrt(-a^2 + b^2)*b)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x))
+ c) + sqrt(-a^2 + b^2)*a)/(a*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)
 + b)) + 2*((a^4*b - 2*a^2*b^3 + b^5)*d*cosh((n - 1)*log(e))*cosh(n*log(x))
 + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^5
- 2*a^3*b^2 + a*b^4)*d*cosh(n*log(x))*sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*
b^2 + a*b^4)*d*cosh((n - 1)*log(e)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n -
 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)
```

```
- (a^2*b^3 - b^5)*cosh((n - 1)*log(e)) - (a^2*b^3 - b^5 - (a^4*b - 2*a^2*b^
3 + b^5)*d*cosh(n*log(x)))*sinh((n - 1)*log(e)) + ((a^4*b - 2*a^2*b^3 + b^5
)*d*cosh((n - 1)*log(e)) + (a^4*b - 2*a^2*b^3 + b^5)*d*sinh((n - 1)*log(e))
)*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - (2*a^3*b^
2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(n*log(x)))*sinh((n - 1)*log(
e)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e)) + (a^5 - 2*a^3*b^2
+ a*b^4)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))/((a^7 - 2*a^5*b^2 + a^3*b^
4)*d*n*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + (a^7 - 2*a^5*b^2 +
 a^3*b^4)*d*n*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + 2*(a^6*b -
2*a^4*b^3 + a^2*b^5)*d*n*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a
^7 - 2*a^5*b^2 + a^3*b^4)*d*n + 2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*cosh(d*c
osh(n*log(x)) + d*sinh(n*log(x)) + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d*n)*
sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)), ((a^5 - 2*a^3*b^2 + a*b^4)*
d*cosh((n - 1)*log(e))*cosh(n*log(x)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((
n - 1)*log(e))*cosh(n*log(x)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(n*log(x))*
sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e)) +
(a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*co
sh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(
(n - 1)*log(e))*cosh(n*log(x)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(n*log(x))
*sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e)) +
 (a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))*sinh(d*c
osh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + 2*(((2*a^3*b - a*b^3)*sqrt(a^2 -
b^2)*cosh((n - 1)*log(e)) + (2*a^3*b - a*b^3)*sqrt(a^2 - b^2)*sinh((n - 1)*
log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + ((2*a^3*b - a*b^
3)*sqrt(a^2 - b^2)*cosh((n - 1)*log(e)) + (2*a^3*b - a*b^3)*sqrt(a^2 - b^2)
*sinh((n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + (2
*a^3*b - a*b^3)*sqrt(a^2 - b^2)*cosh((n - 1)*log(e)) + (2*a^3*b - a*b^3)*sq
rt(a^2 - b^2)*sinh((n - 1)*log(e)) + 2*((2*a^2*b^2 - b^4)*sqrt(a^2 - b^2)*c
osh((n - 1)*log(e)) + (2*a^2*b^2 - b^4)*sqrt(a^2 - b^2)*sinh((n - 1)*log(e)
))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 2*((2*a^2*b^2 - b^4)*sqr
t(a^2 - b^2)*cosh((n - 1)*log(e)) + (2*a^2*b^2 - b^4)*sqrt(a^2 - b^2)*sinh(
(n - 1)*log(e)) + ((2*a^3*b - a*b^3)*sqrt(a^2 - b^2)*cosh((n - 1)*log(e)) +
 (2*a^3*b - a*b^3)*sqrt(a^2 - b^2)*sinh((n - 1)*log(e)))*cosh(d*cosh(n*log(
x)) + d*sinh(n*log(x)) + c))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c))
*arctan(-(sqrt(a^2 - b^2)*a*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) +
 sqrt(a^2 - b^2)*a*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sqrt(a^2
 - b^2)*b)/(a^2 - b^2)) + 2*((a^4*b - 2*a^2*b^3 + b^5)*d*cosh((n - 1)*log(e
))*cosh(n*log(x)) - (a^2*b^3 - b^5)*cosh((n - 1)*log(e)) - (a^2*b^3 - b^5 -
 (a^4*b - 2*a^2*b^3 + b^5)*d*cosh(n*log(x)))*sinh((n - 1)*log(e)) + ((a^4*b
 - 2*a^2*b^3 + b^5)*d*cosh((n - 1)*log(e)) + (a^4*b - 2*a^2*b^3 + b^5)*d*si
nh((n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)
) + c) - 2*(a^3*b^2 - a*b^4)*cosh((n - 1)*log(e)) + 2*((a^4*b - 2*a^2*b^3 +
 b^5)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*
cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(n*lo
g(x))*sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(
```

```
e)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))*cos
h(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - (a^2*b^3 - b^5)*cosh((n - 1)*l
og(e)) - (a^2*b^3 - b^5 - (a^4*b - 2*a^2*b^3 + b^5)*d*cosh(n*log(x)))*sinh(
(n - 1)*log(e)) + ((a^4*b - 2*a^2*b^3 + b^5)*d*cosh((n - 1)*log(e)) + (a^4*
b - 2*a^2*b^3 + b^5)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))*sinh(d*cosh(n*
log(x)) + d*sinh(n*log(x)) + c) - (2*a^3*b^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 +
 a*b^4)*d*cosh(n*log(x)))*sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*b^2 + a*b^4)
*d*cosh((n - 1)*log(e)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e)))
*sinh(n*log(x)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*cosh(d*cosh(n*log(x)) + d
*sinh(n*log(x)) + c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*sinh(d*cosh(n*log(
x)) + d*sinh(n*log(x)) + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*n*cosh(d*
cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d*n +
2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x))
 + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d*n)*sinh(d*cosh(n*log(x)) + d*sinh(n
*log(x)) + c))]
```

Sympy [F]

$$\int \frac{(ex)^{-1+n}}{(a+b\operatorname{sech}(c+dx^n))^2}\,dx = \int \frac{(ex)^{n-1}}{(a+b\operatorname{sech}(c+dx^n))^2}\,dx$$

```
[In] integrate((e*x)**(-1+n)/(a+b*sech(c+d*x**n))**2,x)
[Out] Integral((e*x)**(n - 1)/(a + b*sech(c + d*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+n}}{(a+b\operatorname{sech}(c+dx^n))^2}\,dx = \int \frac{(ex)^{n-1}}{(b\operatorname{sech}(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+n)/(a+b*sech(c+d*x^n))^2,x, algorithm="maxima")
[Out] -2*(2*a^2*b*e^n*e^c - b^3*e^n*e^c)*integrate(e^(d*x^n + n*log(x))/((a^5*e*e
^(2*c) - a^3*b^2*e*e^(2*c))*x*e^(2*d*x^n) + 2*(a^4*b*e*e^c - a^2*b^3*e*e^c)
*x*e^(d*x^n) + (a^5*e - a^3*b^2*e)*x), x) - (2*a*b^2*e^n - (a^3*d*e^n - a*b
^2*d*e^n)*x^n - (a^3*d*e^n*e^(2*c) - a*b^2*d*e^n*e^(2*c))*e^(2*d*x^n + n*lo
g(x)) + 2*(b^3*e^n*e^c - (a^2*b*d*e^n*e^c - b^3*d*e^n*e^c)*x^n)*e^(d*x^n))/
(a^5*d*e*n - a^3*b^2*d*e*n + (a^5*d*e*n*e^(2*c) - a^3*b^2*d*e*n*e^(2*c))*e^
(2*d*x^n) + 2*(a^4*b*d*e*n*e^c - a^2*b^3*d*e*n*e^c)*e^(d*x^n))
```

Giac [F]

$$\int \frac{(ex)^{-1+n}}{(a+b\text{sech}\,(c+dx^n))^2}\,dx = \int \frac{(ex)^{n-1}}{(b\,\text{sech}\,(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+n)/(a+b*sech(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((e*x)^(n - 1)/(b*sech(d*x^n + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+n}}{(a+b\text{sech}\,(c+dx^n))^2}\,dx = \int \frac{(e\,x)^{n-1}}{\left(a+\frac{b}{\cosh(c+d\,x^n)}\right)^2}\,dx$$

```
[In] int((e*x)^(n - 1)/(a + b/cosh(c + d*x^n))^2,x)
[Out] int((e*x)^(n - 1)/(a + b/cosh(c + d*x^n))^2, x)
```

3.83 $\int \frac{(ex)^{-1+2n}}{\left(a+b\,\mathrm{sech}(c+dx^n)\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 717

$$\int \frac{(ex)^{-1+2n}}{(a+b\operatorname{sech}(c+dx^n))^2}\,dx = \frac{(ex)^{2n}}{2a^2en}+\frac{b^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}den} - \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{b^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}den} + \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{b^2x^{-2n}(ex)^{2n}\log(b+a\cosh(c+dx^n))}{a^2(a^2-b^2)d^2en} + \frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2en} - \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} - \frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2en} + \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} + \frac{b^2x^{-n}(ex)^{2n}\sinh(c+dx^n)}{a(a^2-b^2)den(b+a\cosh(c+dx^n))}$$

```
[Out] 1/2*(e*x)^(2*n)/a^2/e/n-b^2*(e*x)^(2*n)*ln(b+a*cosh(c+d*x^n))/a^2/(a^2-b^2)
/d^2/e/n/(x^(2*n))+b^3*(e*x)^(2*n)*ln(1+a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2))
)/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)-b^3*(e*x)^(2*n)*ln(1+a*exp(c+d*x^n)/(b+(
-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)+b^3*(e*x)^(2*n)*polylog(
2,-a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2/e/n/(x^(2*
n))-b^3*(e*x)^(2*n)*polylog(2,-a*exp(c+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a^2/(-a
^2+b^2)^(3/2)/d^2/e/n/(x^(2*n))+b^2*(e*x)^(2*n)*sinh(c+d*x^n)/a/(a^2-b^2)/d
/e/n/(x^n)/(b+a*cosh(c+d*x^n))-2*b*(e*x)^(2*n)*ln(1+a*exp(c+d*x^n)/(b-(-a^2
+b^2)^(1/2)))/a^2/d/e/n/(x^n)/(-a^2+b^2)^(1/2)+2*b*(e*x)^(2*n)*ln(1+a*exp(c
+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a^2/d/e/n/(x^n)/(-a^2+b^2)^(1/2)-2*b*(e*x)^(2
*n)*polylog(2,-a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a^2/d^2/e/n/(x^(2*n))/(
-a^2+b^2)^(1/2)+2*b*(e*x)^(2*n)*polylog(2,-a*exp(c+d*x^n)/(b+(-a^2+b^2)^(1/
2)))/a^2/d^2/e/n/(x^(2*n))/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 717, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5548, 5544, 4276, 3405, 3401, 2296, 2221, 2317, 2438, 2747, 31}

$$\int \frac{(ex)^{-1+2n}}{(a+b\text{sech}(c+dx^n))^2} dx = -\frac{2bx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2en\sqrt{b^2-a^2}} + \frac{2bx^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2en\sqrt{b^2-a^2}} - \frac{b^2x^{-2n}(ex)^{2n}\log(a\cosh(c+dx^n)+b)}{a^2d^2en(a^2-b^2)} - \frac{2bx^{-n}(ex)^{2n}\log\left(\frac{ae^{c+dx^n}}{b-\sqrt{b^2-a^2}}+1\right)}{a^2den\sqrt{b^2-a^2}} + \frac{2bx^{-n}(ex)^{2n}\log\left(\frac{ae^{c+dx^n}}{\sqrt{b^2-a^2}+b}+1\right)}{a^2den\sqrt{b^2-a^2}} + \frac{b^2x^{-n}(ex)^{2n}\sinh(c+dx^n)}{aden(a^2-b^2)(a\cosh(c+dx^n)+b)} + \frac{b^3x^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2en(b^2-a^2)^{3/2}} - \frac{b^3x^{-2n}(ex)^{2n}\text{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2en(b^2-a^2)^{3/2}} + \frac{b^3x^{-n}(ex)^{2n}\log\left(\frac{ae^{c+dx^n}}{b-\sqrt{b^2-a^2}}+1\right)}{a^2den(b^2-a^2)^{3/2}} - \frac{b^3x^{-n}(ex)^{2n}\log\left(\frac{ae^{c+dx^n}}{\sqrt{b^2-a^2}+b}+1\right)}{a^2den(b^2-a^2)^{3/2}} + \frac{(ex)^{2n}}{2a^2en}$$

```
[In] Int[(e*x)^(-1 + 2*n)/(a + b*Sech[c + d*x^n])^2,x]
```

```
[Out] (e*x)^(2*n)/(2*a^2*e*n) + (b^3*(e*x)^(2*n)*Log[1 + (a*E^(c + d*x^n))/(b - S
qrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d*e*n*x^n) - (2*b*(e*x)^(2*n)*Lo
g[1 + (a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d*e*
n*x^n) - (b^3*(e*x)^(2*n)*Log[1 + (a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2])]
)/(a^2*(-a^2 + b^2)^(3/2)*d*e*n*x^n) + (2*b*(e*x)^(2*n)*Log[1 + (a*E^(c + d
*x^n))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d*e*n*x^n) - (b^2*(e*
x)^(2*n)*Log[b + a*Cosh[c + d*x^n]])/(a^2*(a^2 - b^2)*d^2*e*n*x^(2*n)) + (b
^3*(e*x)^(2*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]))])/(a^
2*(-a^2 + b^2)^(3/2)*d^2*e*n*x^(2*n)) - (2*b*(e*x)^(2*n)*PolyLog[2, -((a*E^
```

```
(c + d*x^n))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2*e*n*x^(2*n
)) - (b^3*(e*x)^(2*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2])
)])/(a^2*(-a^2 + b^2)^(3/2)*d^2*e*n*x^(2*n)) + (2*b*(e*x)^(2*n)*PolyLog[2,
-((a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2*e*n
*x^(2*n)) + (b^2*(e*x)^(2*n)*Sinh[c + d*x^n])/(a*(a^2 - b^2)*d*e*n*x^n*(b +
 a*Cosh[c + d*x^n]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5548

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(x^{-2n}(ex)^{2n}\right)\int \frac{x^{-1+2n}}{\left(a+b\,\text{sech}(c+dx^n)\right)^2}\,dx}{e}$$

$$= \frac{\left(x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{x}{(a+b\,\text{sech}(c+dx))^2}\,dx, x, x^n\right)}{en}$$

$$= \frac{\left(x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \left(\frac{x}{a^2}+\frac{b^2x}{a^2(b+a\cosh(c+dx))^2}-\frac{2bx}{a^2(b+a\cosh(c+dx))}\right)dx, x, x^n\right)}{en}$$

$$= \frac{(ex)^{2n}}{2a^2en} - \frac{(2bx^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{x}{b+a\cosh(c+dx)}\,dx, x, x^n\right)}{a^2en} + \frac{(b^2x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{x}{(b+a\cosh(c+dx))^2}\,dx, x, x^n\right)}{a^2en}$$

$$= \frac{(ex)^{2n}}{2a^2en} + \frac{b^2x^{-n}(ex)^{2n}\sinh(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\cosh(c+dx^n))} - \frac{(4bx^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{e^{c+dx}x}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^n\right)}{a^2en} - \frac{(b^3x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{x}{b+a\cosh(c+dx)}\,dx, x, x^n\right)}{a^2(a^2-b^2)\,en} - \frac{(b^2x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{\sinh(c+dx)}{b+a\cosh(c+dx)}\,dx, x, x^n\right)}{a(a^2-b^2)\,den}$$

$$= \frac{(ex)^{2n}}{2a^2en} + \frac{b^2x^{-n}(ex)^{2n}\sinh(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\cosh(c+dx^n))} - \frac{(2b^3x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{e^{c+dx}x}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx, x, x^n\right)}{a^2(a^2-b^2)\,en} - \frac{(4bx^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{e^{c+dx}x}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, x^n\right)}{a\sqrt{-a^2+b^2}\,en} + \frac{(4bx^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{e^{c+dx}x}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx, x, x^n\right)}{a\sqrt{-a^2+b^2}\,en} - \frac{(b^2x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{1}{b+x}\,dx, x, a\cosh(c+dx^n)\right)}{a^2(a^2-b^2)\,d^2en}$$

$$= \frac{(ex)^{2n}}{2a^2en} - \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den}$$
$$- \frac{b^2x^{-2n}(ex)^{2n}\log(b+a\cosh(c+dx^n))}{a^2(a^2-b^2)d^2en} + \frac{b^2x^{-n}(ex)^{2n}\sinh(c+dx^n)}{a(a^2-b^2)den(b+a\cosh(c+dx^n))}$$
$$- \frac{(2b^3x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int\frac{e^{c+dx}x}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx,x,x^n\right)}{a(a^2-b^2)\sqrt{-a^2+b^2}en}$$
$$+ \frac{(2b^3x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int\frac{e^{c+dx}x}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx,x,x^n\right)}{a(a^2-b^2)\sqrt{-a^2+b^2}en}$$
$$+ \frac{(2bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\sqrt{-a^2+b^2}den}$$
$$- \frac{(2bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\sqrt{-a^2+b^2}den}$$

$$= \frac{(ex)^{2n}}{2a^2en} + \frac{b^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}den} - \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den}$$
$$- \frac{b^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}den} + \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den}$$
$$- \frac{b^2x^{-2n}(ex)^{2n}\log(b+a\cosh(c+dx^n))}{a^2(a^2-b^2)d^2en} + \frac{b^2x^{-n}(ex)^{2n}\sinh(c+dx^n)}{a(a^2-b^2)den(b+a\cosh(c+dx^n))}$$
$$+ \frac{(2bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{c+dx^n}\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$
$$- \frac{(2bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{c+dx^n}\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$
$$+ \frac{(b^3x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}den}$$
$$- \frac{(b^3x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}den}$$

$$= \frac{(ex)^{2n}}{2a^2en} + \frac{b^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}den} - \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{b^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}den} + \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{b^2x^{-2n}(ex)^{2n}\log\left(b+a\cosh\left(c+dx^n\right)\right)}{a^2\left(a^2-b^2\right)d^2en} - \frac{2bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} + \frac{2bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} + \frac{b^2x^{-n}(ex)^{2n}\sinh\left(c+dx^n\right)}{a\left(a^2-b^2\right)den\left(b+a\cosh\left(c+dx^n\right)\right)} + \frac{\left(b^3x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{c+dx^n}\right)}{a^2\left(a^2-b^2\right)\sqrt{-a^2+b^2}d^2en} - \frac{\left(b^3x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{2ax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{c+dx^n}\right)}{a^2\left(a^2-b^2\right)\sqrt{-a^2+b^2}d^2en}$$

$$= \frac{(ex)^{2n}}{2a^2en} + \frac{b^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}den} - \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{b^3x^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}den} + \frac{2bx^{-n}(ex)^{2n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{b^2x^{-2n}(ex)^{2n}\log\left(b+a\cosh\left(c+dx^n\right)\right)}{a^2\left(a^2-b^2\right)d^2en} + \frac{b^3x^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2en} - \frac{2bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} - \frac{b^3x^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2en} + \frac{2bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} + \frac{b^2x^{-n}(ex)^{2n}\sinh\left(c+dx^n\right)}{a\left(a^2-b^2\right)den\left(b+a\cosh\left(c+dx^n\right)\right)}$$

Mathematica [A] (verified)

Time = 6.46 (sec) , antiderivative size = 469, normalized size of antiderivative = 0.65

$$\int \frac{(ex)^{-1+2n}}{(a+b\text{sech}(c+dx^n))^2}\,dx$$

$$= \frac{x^{-2n}(ex)^{2n}(b+a\cosh(c+dx^n))\,\text{sech}^2(c+dx^n)\left(\frac{4b^2de^{2c}x^n(b+a\cosh(c+dx^n))}{(a^2-b^2)(1+e^{2c})}+\frac{2b(b+a\cosh(c+dx^n))\left(b\sqrt{-a^2+b^2}\log\left(a\right.\right.}{}\right.}{}$$

```
[In] Integrate[(e*x)^(-1 + 2*n)/(a + b*Sech[c + d*x^n])^2,x]
```

```
[Out] ((e*x)^(2*n)*(b + a*Cosh[c + d*x^n])*Sech[c + d*x^n]^2*((4*b^2*d*E^(2*c)*x^
n*(b + a*Cosh[c + d*x^n]))/((a^2 - b^2)*(1 + E^(2*c))) + (2*b*(b + a*Cosh[c
 + d*x^n])*(b*Sqrt[-a^2 + b^2]*Log[a + 2*b*E^(c + d*x^n) + a*E^(2*(c + d*x^
n))] + (2*a^2 - b^2)*(d*x^n*Log[1 + (a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]
)] + PolyLog[2, (a*E^(c + d*x^n))/(-b + Sqrt[-a^2 + b^2])]) - (2*a^2 - b^2)
*(d*x^n*Log[1 + (a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2])] + PolyLog[2, -((a
*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2]))])))/(-a^2 + b^2)^(3/2) + (2*b^2*d*x
^n*Sech[c]*(b*Sinh[c] - a*Sinh[d*x^n]))/((-a + b)*(a + b)) + (2*b^2*d*x^n*(
b + a*Cosh[c + d*x^n])*Tanh[c])/(-a^2 + b^2) + (d*x^n*(b + a*Cosh[c + d*x^n
])*((a^2 - b^2)*d*x^n + 2*b^2*Tanh[c]))/((a - b)*(a + b))))/(2*a^2*d^2*e*n*
x^(2*n)*(a + b*Sech[c + d*x^n])^2)
```

Maple [F]

$$\int \frac{(ex)^{2n-1}}{(a+b\,\text{sech}(c+d\,x^n))^2}dx$$

```
[In] int((e*x)^(2*n-1)/(a+b*sech(c+d*x^n))^2,x)
```

```
[Out] int((e*x)^(2*n-1)/(a+b*sech(c+d*x^n))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9020 vs. 2(681) = 1362.

Time = 0.42 (sec) , antiderivative size = 9020, normalized size of antiderivative = 12.58

$$\int \frac{(ex)^{-1+2n}}{(a+b\text{sech}(c+dx^n))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*sech(c+d*x^n))^2,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{(ex)^{-1+2n}}{(a+b\mathrm{sech}\,(c+dx^n))^2}\,dx = \int \frac{(ex)^{2n-1}}{(a+b\,\mathrm{sech}\,(c+dx^n))^2}\,dx$$

```
[In] integrate((e*x)**(-1+2*n)/(a+b*sech(c+d*x**n))**2,x)
[Out] Integral((e*x)**(2*n - 1)/(a + b*sech(c + d*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+2n}}{(a+b\mathrm{sech}\,(c+dx^n))^2}\,dx = \int \frac{(ex)^{2\,n-1}}{(b\,\mathrm{sech}\,(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*sech(c+d*x^n))^2,x, algorithm="maxima")
[Out] -1/2*(4*a*b^2*e^(2*n)*x^n - (a^3*d*e^(2*n) - a*b^2*d*e^(2*n))*x^(2*n) - (a^
3*d*e^(2*n)*e^(2*c) - a*b^2*d*e^(2*n)*e^(2*c))*e^(2*d*x^n + 2*n*log(x)) + 2
*(2*b^3*e^(2*n)*e^(n*log(x) + c) - (a^2*b*d*e^(2*n)*e^c - b^3*d*e^(2*n)*e^c
)*x^(2*n))*e^(d*x^n))/(a^5*d*e*n - a^3*b^2*d*e*n + (a^5*d*e*n*e^(2*c) - a^3
*b^2*d*e*n*e^(2*c))*e^(2*d*x^n) + 2*(a^4*b*d*e*n*e^c - a^2*b^3*d*e*n*e^c)*e
^(d*x^n)) - integrate(-2*(a*b^2*e^(2*n)*x^n + (b^3*e^(2*n)*e^(n*log(x) + c)
 - (2*a^2*b*d*e^(2*n)*e^c - b^3*d*e^(2*n)*e^c)*x^(2*n))*e^(d*x^n))/((a^5*d*
e*e^(2*c) - a^3*b^2*d*e*e^(2*c))*x*e^(2*d*x^n) + 2*(a^4*b*d*e*e^c - a^2*b^3
*d*e*e^c)*x*e^(d*x^n) + (a^5*d*e - a^3*b^2*d*e)*x), x)
```

Giac [F]

$$\int \frac{(ex)^{-1+2n}}{(a+b\mathrm{sech}\,(c+dx^n))^2}\,dx = \int \frac{(ex)^{2\,n-1}}{(b\,\mathrm{sech}\,(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*sech(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)/(b*sech(d*x^n + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+2n}}{(a+b\text{sech}\,(c+dx^n))^2}\,dx = \int \frac{(e\,x)^{2\,n-1}}{\left(a+\frac{b}{\cosh(c+d\,x^n)}\right)^2}\,dx$$

```
[In] int((e*x)^(2*n - 1)/(a + b/cosh(c + d*x^n))^2,x)
[Out] int((e*x)^(2*n - 1)/(a + b/cosh(c + d*x^n))^2, x)
```

3.84 $\int \frac{(ex)^{-1+3n}}{\left(a+b\,\mathbf{sech}(c+dx^n)\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1284

$$\int \frac{(ex)^{-1+3n}}{(a+b\mathrm{sech}(c+dx^n))^2}\,dx = \frac{(ex)^{3n}}{3a^2en}+\frac{b^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} - \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^2en} + \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} - \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^2en} - \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} + \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{2b^2x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^3en} + \frac{2b^3x^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} - \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} - \frac{2b^2x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^3en} - \frac{2b^3x^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} + \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} - \frac{2b^3x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3en} + \frac{4bx^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3en} + \frac{2b^3x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3en}$$

$$4bx^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)$$

```
[Out] 1/3*(e*x)^(3*n)/a^2/e/n+b^2*(e*x)^(3*n)/a^2/(a^2-b^2)/d/e/n/(x^n)-2*b^2*(e*
x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^2/e/n/(x
^(2*n))+b^3*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2
+b^2)^(3/2)/d/e/n/(x^n)-2*b^2*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b+(-a^2+b^2)
^(1/2)))/a^2/(a^2-b^2)/d^2/e/n/(x^(2*n))-b^3*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n
)/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)-2*b^2*(e*x)^(3*n)*
polylog(2,-a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^3/e/n/(x^(3
*n))+2*b^3*(e*x)^(3*n)*polylog(2,-a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a^2/
(-a^2+b^2)^(3/2)/d^2/e/n/(x^(2*n))-2*b^2*(e*x)^(3*n)*polylog(2,-a*exp(c+d*x
^n)/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^3/e/n/(x^(3*n))-2*b^3*(e*x)^(3*n)
*polylog(2,-a*exp(c+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2/e
/n/(x^(2*n))-2*b^3*(e*x)^(3*n)*polylog(3,-a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2
)))/a^2/(-a^2+b^2)^(3/2)/d^3/e/n/(x^(3*n))+2*b^3*(e*x)^(3*n)*polylog(3,-a*e
xp(c+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3/e/n/(x^(3*n))+b^
2*(e*x)^(3*n)*sinh(c+d*x^n)/a/(a^2-b^2)/d/e/n/(x^n)/(b+a*cosh(c+d*x^n))-2*b
*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a^2/d/e/n/(x^n)/(-a^
2+b^2)^(1/2)+2*b*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a^2/
d/e/n/(x^n)/(-a^2+b^2)^(1/2)-4*b*(e*x)^(3*n)*polylog(2,-a*exp(c+d*x^n)/(b-(
-a^2+b^2)^(1/2)))/a^2/d^2/e/n/(x^(2*n))/(-a^2+b^2)^(1/2)+4*b*(e*x)^(3*n)*po
lylog(2,-a*exp(c+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/e/n/(x^(2*n))/(-a^2+b
^2)^(1/2)+4*b*(e*x)^(3*n)*polylog(3,-a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a
^2/d^3/e/n/(x^(3*n))/(-a^2+b^2)^(1/2)-4*b*(e*x)^(3*n)*polylog(3,-a*exp(c+d*
x^n)/(b+(-a^2+b^2)^(1/2)))/a^2/d^3/e/n/(x^(3*n))/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 1284, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules

used = {5548, 5544, 4276, 3405, 3401, 2296, 2221, 2611, 2320, 6724, 5681, 2317, 2438}

$$\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{sech}(c+dx^n))^2}\,dx = -\frac{2b^2(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2(a^2-b^2)d^3en} - \frac{2b^2(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2(a^2-b^2)d^3en} + \frac{4b(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2\sqrt{b^2-a^2}d^3en} - \frac{2b^3(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2(b^2-a^2)^{3/2}d^3en} - \frac{4b(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2\sqrt{b^2-a^2}d^3en} + \frac{2b^3(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2(b^2-a^2)^{3/2}d^3en} - \frac{2b^2(ex)^{3n}\log\left(\frac{e^{dx^n+c}a}{b-\sqrt{b^2-a^2}}+1\right)x^{-2n}}{a^2(a^2-b^2)d^2en} - \frac{2b^2(ex)^{3n}\log\left(\frac{e^{dx^n+c}a}{b+\sqrt{b^2-a^2}}+1\right)x^{-2n}}{a^2(a^2-b^2)d^2en} - \frac{4b(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)x^{-2n}}{a^2\sqrt{b^2-a^2}d^2en} + \frac{2b^3(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)x^{-2n}}{a^2(b^2-a^2)^{3/2}d^2en} + \frac{4b(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)x^{-2n}}{a^2\sqrt{b^2-a^2}d^2en} - \frac{2b^3(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)x^{-2n}}{a^2(b^2-a^2)^{3/2}d^2en} + \frac{b^2(ex)^{3n}x^{-n}}{a^2(a^2-b^2)den} - \frac{2b(ex)^{3n}\log\left(\frac{e^{dx^n+c}a}{b-\sqrt{b^2-a^2}}+1\right)x^{-n}}{a^2\sqrt{b^2-a^2}den} + \frac{b^3(ex)^{3n}\log\left(\frac{e^{dx^n+c}a}{b-\sqrt{b^2-a^2}}+1\right)x^{-n}}{a^2(b^2-a^2)^{3/2}den} + \frac{2b(ex)^{3n}\log\left(\frac{e^{dx^n+c}a}{b+\sqrt{b^2-a^2}}+1\right)x^{-n}}{a^2\sqrt{b^2-a^2}den} - \frac{b^3(ex)^{3n}\log\left(\frac{e^{dx^n+c}a}{b+\sqrt{b^2-a^2}}+1\right)x^{-n}}{a^2(b^2-a^2)^{3/2}den} + \frac{b^2(ex)^{3n}\sinh(dx^n+c)x^{-n}}{a(a^2-b^2)den(b+a\cosh(dx^n+c))} + \frac{(ex)^{3n}}{3a^2en}$$

```
[In] Int[(e*x)^(-1 + 3*n)/(a + b*Sech[c + d*x^n])^2,x]
```

```
[Out] (e*x)^(3*n)/(3*a^2*e*n) + (b^2*(e*x)^(3*n))/(a^2*(a^2 - b^2)*d*e*n*x^n) - (
2*b^2*(e*x)^(3*n)*Log[1 + (a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2])])/(a^2*(
a^2 - b^2)*d^2*e*n*x^(2*n)) + (b^3*(e*x)^(3*n)*Log[1 + (a*E^(c + d*x^n))/(b
 - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d*e*n*x^n) - (2*b*(e*x)^(3*n
)*Log[1 + (a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*
d*e*n*x^n) - (2*b^2*(e*x)^(3*n)*Log[1 + (a*E^(c + d*x^n))/(b + Sqrt[-a^2 +
b^2])])/(a^2*(a^2 - b^2)*d^2*e*n*x^(2*n)) - (b^3*(e*x)^(3*n)*Log[1 + (a*E^(
c + d*x^n))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d*e*n*x^n) + (
2*b*(e*x)^(3*n)*Log[1 + (a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqr
t[-a^2 + b^2]*d*e*n*x^n) - (2*b^2*(e*x)^(3*n)*PolyLog[2, -((a*E^(c + d*x^n)
)/(b - Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^3*e*n*x^(3*n)) + (2*b^3*(e*x
)^(3*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2
 + b^2)^(3/2)*d^2*e*n*x^(2*n)) - (4*b*(e*x)^(3*n)*PolyLog[2, -((a*E^(c + d*
x^n))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2*e*n*x^(2*n)) - (2
*b^2*(e*x)^(3*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2]))])/(
a^2*(a^2 - b^2)*d^3*e*n*x^(3*n)) - (2*b^3*(e*x)^(3*n)*PolyLog[2, -((a*E^(c
+ d*x^n))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2*e*n*x^(2*n)
) + (4*b*(e*x)^(3*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2]))
])/(a^2*Sqrt[-a^2 + b^2]*d^2*e*n*x^(2*n)) - (2*b^3*(e*x)^(3*n)*PolyLog[3, -
((a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3*e*
n*x^(3*n)) + (4*b*(e*x)^(3*n)*PolyLog[3, -((a*E^(c + d*x^n))/(b - Sqrt[-a^2
 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^3*e*n*x^(3*n)) + (2*b^3*(e*x)^(3*n)*Pol
yLog[3, -((a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/
2)*d^3*e*n*x^(3*n)) - (4*b*(e*x)^(3*n)*PolyLog[3, -((a*E^(c + d*x^n))/(b +
Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^3*e*n*x^(3*n)) + (b^2*(e*x)^(3
*n)*Sinh[c + d*x^n])/(a*(a^2 - b^2)*d*e*n*x^n*(b + a*Cosh[c + d*x^n]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
```

```
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5544

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 5548

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(
a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^{-3n}(ex)^{3n}\right)\int \frac{x^{-1+3n}}{\left(a+b\operatorname{sech}(c+dx^n)\right)^2}\,dx}{e}\\
&= \frac{\left(x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int \frac{x^2}{(a+b\operatorname{sech}(c+dx))^2}\,dx,x,x^n\right)}{en}\\
&= \frac{\left(x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int \left(\frac{x^2}{a^2}+\frac{b^2x^2}{a^2(b+a\cosh(c+dx))^2}-\frac{2bx^2}{a^2(b+a\cosh(c+dx))}\right)dx,x,x^n\right)}{en}\\
&= \frac{(ex)^{3n}}{3a^2en}-\frac{\left(2bx^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int \frac{x^2}{b+a\cosh(c+dx)}\,dx,x,x^n\right)}{a^2en}\\
&\quad+\frac{\left(b^2x^{-3n}(ex)^{3n}\right)\operatorname{Subst}\left(\int \frac{x^2}{(b+a\cosh(c+dx))^2}\,dx,x,x^n\right)}{a^2en}
\end{aligned}$$

$$= \frac{(ex)^{3n}}{3a^2en} + \frac{b^2x^{-n}(ex)^{3n}\sinh(c+dx^n)}{a(a^2-b^2)den(b+a\cosh(c+dx^n))} - \frac{(4bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx,x,x^n\right)}{a^2en} - \frac{(b^3x^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \frac{x^2}{b+a\cosh(c+dx)}\,dx,x,x^n\right)}{a^2(a^2-b^2)en} - \frac{(2b^2x^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \frac{x\sinh(c+dx)}{b+a\cosh(c+dx)}\,dx,x,x^n\right)}{a(a^2-b^2)den}$$

$$= \frac{(ex)^{3n}}{3a^2en} + \frac{b^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)den} + \frac{b^2x^{-n}(ex)^{3n}\sinh(c+dx^n)}{a(a^2-b^2)den(b+a\cosh(c+dx^n))} - \frac{(2b^3x^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{a+2be^{c+dx}+ae^{2(c+dx)}}\,dx,x,x^n\right)}{a^2(a^2-b^2)en} - \frac{(4bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx,x,x^n\right)}{a\sqrt{-a^2+b^2}en} + \frac{(4bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \frac{e^{c+dx}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx,x,x^n\right)}{a\sqrt{-a^2+b^2}en} - \frac{(2b^2x^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \frac{e^{c+dx}x}{b-\sqrt{-a^2+b^2}+ae^{c+dx}}\,dx,x,x^n\right)}{a(a^2-b^2)den} - \frac{(2b^2x^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int \frac{e^{c+dx}x}{b+\sqrt{-a^2+b^2}+ae^{c+dx}}\,dx,x,x^n\right)}{a(a^2-b^2)den}$$

$$= \frac{(ex)^{3n}}{3a^2en} + \frac{b^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} - \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^2en}$$

$$- \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^2en}$$

$$+ \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{b^2x^{-n}(ex)^{3n}\sinh(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\cosh(c+dx^n))}$$

$$- \frac{\left(2b^3x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int\frac{e^{c+dx}x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx,x,x^n\right)}{a(a^2-b^2)\sqrt{-a^2+b^2}en}$$

$$+ \frac{\left(2b^3x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int\frac{e^{c+dx}x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}}\,dx,x,x^n\right)}{a(a^2-b^2)\sqrt{-a^2+b^2}en}$$

$$+ \frac{\left(2b^2x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int\log\left(1+\frac{ae^{c+dx}}{b-\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2(a^2-b^2)\,d^2en}$$

$$+ \frac{\left(2b^2x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int\log\left(1+\frac{ae^{c+dx}}{b+\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2(a^2-b^2)\,d^2en}$$

$$+ \frac{\left(4bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\sqrt{-a^2+b^2}den}$$

$$- \frac{\left(4bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\sqrt{-a^2+b^2}den}$$

$$= \frac{(ex)^{3n}}{3a^2en} + \frac{b^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} - \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^2en}$$

$$+ \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} - \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den}$$

$$- \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^2en} - \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den}$$

$$+ \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{4bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$

$$+ \frac{4bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} + \frac{b^2x^{-n}(ex)^{3n}\sinh(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\cosh(c+dx^n))}$$

$$+ \frac{(2b^2x^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{ax}{b-\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{c+dx^n}\right)}{a^2(a^2-b^2)\,d^3en}$$

$$+ \frac{(2b^2x^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{c+dx^n}\right)}{a^2(a^2-b^2)\,d^3en}$$

$$+ \frac{(4bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, x^n\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$

$$- \frac{(4bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, x^n\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$

$$+ \frac{(2b^3x^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, x^n\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}den}$$

$$- \frac{(2b^3x^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int x\log\left(1+\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, x^n\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}den}$$

$$
\begin{aligned}
&= \frac{(ex)^{3n}}{3a^2en} + \frac{b^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} - \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^2en} \\
&+ \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} - \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} \\
&- \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^2en} - \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} \\
&+ \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{2b^2x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^3en} \\
&+ \frac{2b^3x^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} \\
&- \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} \\
&- \frac{2b^2x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^3en} \\
&- \frac{2b^3x^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} \\
&+ \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} + \frac{b^2x^{-n}(ex)^{3n}\sinh(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\cosh(c+dx^n))} \\
&+ \frac{(4bx^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{c+dx^n}\right)}{a^2\sqrt{-a^2+b^2}d^3en} \\
&- \frac{(4bx^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{c+dx^n}\right)}{a^2\sqrt{-a^2+b^2}d^3en} \\
&+ \frac{(2b^3x^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,x^n\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}d^2en} \\
&- \frac{(2b^3x^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,-\frac{2ae^{c+dx}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,x^n\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}d^2en}
\end{aligned}
$$

$$= \frac{(ex)^{3n}}{3a^2en} + \frac{b^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} - \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^2en}$$
$$+ \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} - \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den}$$
$$- \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^2en} - \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den}$$
$$+ \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{2b^2x^{-3n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^3en}$$
$$+ \frac{2b^3x^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en}$$
$$- \frac{4bx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$
$$- \frac{2b^2x^{-3n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^3en}$$
$$- \frac{2b^3x^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en}$$
$$+ \frac{4bx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$
$$+ \frac{4bx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3en}$$
$$- \frac{4bx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3en} + \frac{b^2x^{-n}(ex)^{3n}\sinh(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\cosh(c+dx^n))}$$
$$+ \frac{(2b^3x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,\frac{ax}{-b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{c+dx^n}\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}d^3en}$$
$$- \frac{(2b^3x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,-\frac{ax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{c+dx^n}\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}d^3en}$$

$$
\begin{aligned}
&= \frac{(ex)^{3n}}{3a^2en} + \frac{b^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} - \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^2en} \\
&+ \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} - \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} \\
&- \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^2en} - \frac{b^3x^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} \\
&+ \frac{2bx^{-n}(ex)^{3n}\log\left(1+\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{2b^2x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^3en} \\
&+ \frac{2b^3x^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} \\
&- \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} \\
&- \frac{2b^2x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)\,d^3en} \\
&- \frac{2b^3x^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} \\
&+ \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} \\
&- \frac{2b^3x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3en} \\
&+ \frac{4bx^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3en} \\
&+ \frac{2b^3x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3en} \\
&- \frac{4bx^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,-\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3en} + \frac{b^2x^{-n}(ex)^{3n}\sinh(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\cosh(c+dx^n))}
\end{aligned}
$$

Mathematica [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\text{sech}\,(c+dx^n))^2}\,dx=\int \frac{(ex)^{-1+3n}}{(a+b\text{sech}\,(c+dx^n))^2}\,dx$$

```
[In] Integrate[(e*x)^(-1 + 3*n)/(a + b*Sech[c + d*x^n])^2,x]
[Out] Integrate[(e*x)^(-1 + 3*n)/(a + b*Sech[c + d*x^n])^2, x]
```

Maple [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\,\mathrm{sech}\,(c+d\,x^n))^2}dx$$

```
[In] int((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n))^2,x)
[Out] int((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n))^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+3n}}{(a+b\text{sech}\,(c+dx^n))^2}\,dx=\text{Timed out}$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\text{sech}\,(c+dx^n))^2}\,dx=\int \frac{(ex)^{3n-1}}{(a+b\,\mathrm{sech}\,(c+dx^n))^2}\,dx$$

```
[In] integrate((e*x)**(-1+3*n)/(a+b*sech(c+d*x**n))**2,x)
[Out] Integral((e*x)**(3*n - 1)/(a + b*sech(c + d*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{sech}(c+dx^n))^2}\,dx = \int \frac{(ex)^{3n-1}}{(b\operatorname{sech}(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(6*a*b^2*e^(3*n)*x^(2*n) - (a^3*d*e^(3*n) - a*b^2*d*e^(3*n))*x^(3*n) -
 (a^3*d*e^(3*n)*e^(2*c) - a*b^2*d*e^(3*n)*e^(2*c))*e^(2*d*x^n + 3*n*log(x))
 + 2*(3*b^3*e^(3*n)*e^(2*n*log(x) + c) - (a^2*b*d*e^(3*n)*e^c - b^3*d*e^(3*
n)*e^c)*x^(3*n))*e^(d*x^n))/(a^5*d*e*n - a^3*b^2*d*e*n + (a^5*d*e*n*e^(2*c)
 - a^3*b^2*d*e*n*e^(2*c))*e^(2*d*x^n) + 2*(a^4*b*d*e*n*e^c - a^2*b^3*d*e*n*
e^c)*e^(d*x^n)) - integrate(-2*(2*a*b^2*e^(3*n)*x^(2*n) + (2*b^3*e^(3*n)*e^
(2*n*log(x) + c) - (2*a^2*b*d*e^(3*n)*e^c - b^3*d*e^(3*n)*e^c)*x^(3*n))*e^(
d*x^n))/((a^5*d*e*e^(2*c) - a^3*b^2*d*e*e^(2*c))*x*e^(2*d*x^n) + 2*(a^4*b*d
*e*e^c - a^2*b^3*d*e*e^c)*x*e^(d*x^n) + (a^5*d*e - a^3*b^2*d*e)*x), x)
```

Giac [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{sech}(c+dx^n))^2}\,dx = \int \frac{(ex)^{3n-1}}{(b\operatorname{sech}(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(3*n - 1)/(b*sech(d*x^n + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{sech}(c+dx^n))^2}\,dx = \int \frac{(e\,x)^{3\,n-1}}{\left(a+\frac{b}{\cosh(c+d\,x^n)}\right)^2}\,dx$$

```
[In] int((e*x)^(3*n - 1)/(a + b/cosh(c + d*x^n))^2,x)
```

```
[Out] int((e*x)^(3*n - 1)/(a + b/cosh(c + d*x^n))^2, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                  end if
              else #result contains complex but optimal is not
                  if debug then
                          print("result contains complex but optimal is not");
                  fi;
                  return "C","Result contains complex when optimal does not.";
              fi;
          else # result do not contain complex
               # this assumes optimal do not as well. No check is needed here.
              if debug then
                     print("result do not contain complex, this assumes optimal do not as well")
              fi;
              if leaf_count_result<=2*leaf_count_optimal then
                  if debug then
                      print("leaf_count_result<=2*leaf_count_optimal");
                  fi;
                  return "A"," ";
              else
                  if debug then
                      print("leaf_count_result>2*leaf_count_optimal");
                  fi;
                  return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
              fi;
          fi;
      else #ExpnType(result) > ExpnType(optimal)
          if debug then
              print("ExpnType(result) > ExpnType(optimal)");
          fi;
          return "C",cat("Result contains higher order function than in optimal. Order ",
                         convert(ExpnType_result,string)," vs. order ",
                         convert(ExpnType_optimal,string),".");
      fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```